A tour problem on a toroidal chessboard

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Abstract

Let us consider a partially filled $n \times m$ array A, a vector of directions, one for each row, $R := (r_1, \dots, r_n) \in \{\leftarrow, \rightarrow\}^n$ and a vector of directions, one for each column, $C := (c_1, \dots, c_m) \in \{\uparrow, \downarrow\}^m$. For example:

	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
$\rightarrow$	$\bullet$				$\bullet$	$\bullet$	$\bullet$
$\rightarrow$	$\bullet$	$\bullet$				$\bullet$	$\bullet$
$\leftarrow$	$\bullet$	$\bullet$	$\bullet$				$\bullet$
$\leftarrow$	$\bullet$	$\bullet$	$\bullet$	$\bullet$			
$\rightarrow$		$\bullet$	$\bullet$	$\bullet$	$\bullet$		
$\rightarrow$			$\bullet$	$\bullet$	$\bullet$	$\bullet$	
$\leftarrow$				$\bullet$	$\bullet$	$\bullet$	$\bullet$

From a filled position (i, j), we move first in the row i following the direction of r_i (skipping the holes) and then, from the intermediate position (i, j'), we move in the direction of $c_{j'}$ (skipping the holes) arriving in the position $S_{R,C}(i, j)$. We propose the following problem:

Crazy Knight's Tour Problem. *Is it possible to choose R and C so that, starting from a filled position and iterating $S_{R,C}$, we cover all the filled positions of A?*

In this talk we will see how this problem naturally arises studying biembeddings of orthogonal cycle decompositions on surfaces via Heffter arrays. Although the general problem is widely open, we have been able to solve it in some instances and here we will present an overview of the results we achieved and of the open questions we met.

References

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