

- (1) $\vdash (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow (\alpha \wedge \beta) \rightarrow \gamma$
 (2) $\vdash (\alpha \wedge \beta) \rightarrow \gamma \rightarrow (\alpha \rightarrow (\beta \rightarrow \gamma))$

$$\frac{\frac{\frac{[\alpha \wedge \beta]^1}{[\alpha \rightarrow (\beta \rightarrow \gamma)]^2} \quad [\alpha \wedge \beta]^1}{\beta \rightarrow \gamma} \quad \beta}{\gamma}$$

(1)

$$\frac{\frac{\gamma}{(\alpha \wedge \beta) \rightarrow \gamma}^1}{(\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow (\alpha \wedge \beta) \rightarrow \gamma}^2$$

$$\frac{\alpha}{\alpha \vee \beta} \quad \frac{\alpha}{\beta \vee \alpha} \quad \frac{\alpha \vee \beta}{\gamma} \quad \frac{[\alpha] \quad [\beta]}{\gamma}$$

$$\vdash (\alpha \vee \beta) \rightarrow ((\alpha \vee \gamma) \vee \beta)$$

$$\frac{[\alpha] \quad \frac{\alpha \vee \gamma}{(\alpha \vee \gamma) \vee \beta} \quad \frac{[\beta]}{(\alpha \vee \gamma) \vee \beta}}{(\alpha \vee \gamma) \vee \beta} \quad \frac{(\alpha \vee \gamma) \vee \beta}{(\alpha \vee \beta) \rightarrow ((\alpha \vee \gamma) \vee \beta)}$$

$$\vdash (\alpha \wedge \beta) \rightarrow (\alpha \vee \beta) \quad ?$$

$$\frac{\alpha \wedge \beta}{\alpha}$$

$$\frac{\alpha \wedge \beta}{\alpha \vee \beta}$$

$$\vdash (\alpha \wedge \beta) \rightarrow (\gamma \vee \beta)$$

$$\frac{\alpha \wedge \beta}{\beta}$$

$$\frac{\beta}{\gamma \vee \beta}$$

$$\vdash (\alpha \vee \beta) \rightarrow (\beta \vee \alpha)$$

$$\frac{\frac{[\alpha \vee \beta]^2}{\frac{[\alpha]^1}{\beta \vee \alpha}} \quad \frac{[\beta]^1}{\beta \vee \alpha}}{\beta \vee \alpha} 2$$

$$(\alpha \vee \beta) \rightarrow (\beta \vee \alpha)$$

$$\boxed{\vdash (\neg\alpha \vee \beta) \rightarrow (\alpha \rightarrow \beta)}$$

$$\models (\alpha \rightarrow \beta) \leftrightarrow (\neg\alpha \vee \beta)$$

$$\frac{\neg\alpha \vee \beta \quad \begin{array}{c} (\neg\alpha) \quad (\beta) \\ \vdots \end{array} \quad \alpha \rightarrow \beta}{\alpha \rightarrow \beta} 1$$

$$\alpha \rightarrow \beta$$

$$\frac{\neg\alpha \vee \beta \quad \frac{\frac{\frac{[\neg\alpha]^2 \quad [\alpha]^1}{\perp}}{\beta} \quad \frac{[\beta]^2}{\alpha \rightarrow \beta}}{\alpha \rightarrow \beta} 2}{\alpha \rightarrow \beta} 2$$

$$\vdash (\neg\alpha \vee \neg\beta) \rightarrow \neg(\alpha \wedge \beta)$$

$$\frac{\frac{\frac{[\neg\alpha]^3 \quad \frac{[\alpha \wedge \beta]^1}{\alpha}}{\perp} \quad 1}{\neg(\alpha \wedge \beta)}}{(\neg\alpha) \vee (\neg\beta)} \quad \frac{\frac{\frac{[\neg\beta]^3 \quad \frac{[\alpha \wedge \beta]^2}{\beta}}{\perp} \quad 2}{\neg(\alpha \wedge \beta)}}{\neg(\alpha \wedge \beta)} \quad 3$$

$$\vdash \alpha \vee (\neg \alpha)$$

$$\neg(\alpha \vee \neg \alpha)$$

$\mathcal{D}$

$\perp$

$$\frac{\frac{\frac{[\alpha]^1}{\alpha \vee \neg \alpha}}{\perp} \frac{1}{\neg \alpha}}{\alpha \vee \neg \alpha} \frac{[\neg(\alpha \vee \neg \alpha)]^2}{\perp} \frac{[\neg(\alpha \vee \neg \alpha)]^2}{\perp} \frac{RAA}{\alpha \vee \neg \alpha}$$

$$(\neg\alpha \vee \beta) \rightarrow (\alpha \rightarrow \beta)$$

$$\vdash (\alpha \rightarrow \beta) \rightarrow (\neg\alpha \vee \beta)$$

$$\alpha \rightarrow \beta \quad \cdot \quad \alpha \rightarrow \beta \quad \neg(\neg\alpha \vee \beta)$$

$$\vdots \quad \sim$$

$$\neg\alpha \vee \beta$$

$$\perp \quad [\alpha] \quad \alpha \rightarrow \beta$$

$$\frac{[\alpha]^1}{\alpha \vee \neg\alpha} \quad \frac{[\neg(\alpha \vee \neg\alpha)]^2}{\perp}$$

$$\frac{\perp}{\neg\alpha}$$

$$\frac{\alpha \vee \neg\alpha \quad [\neg(\alpha \vee \neg\alpha)]^2}{\perp} \quad \text{RAA}$$

$$\frac{\perp}{\alpha \vee \neg\alpha}$$

$$\frac{\frac{B}{\neg\alpha \vee \beta} \quad [\neg(\neg\alpha \vee \beta)]^1}{\perp}$$

$$\frac{\perp}{\neg\alpha}$$

$$\frac{\neg\alpha \vee \beta \quad [\neg(\neg\alpha \vee \beta)]^1}{\perp}$$

$$\frac{\perp}{\neg\alpha \vee \beta} \quad \text{RAA}$$