DIFFERENTIAL GEOMETRY AND TOPOLOGY - 1st MODULE

Syllabus a.a. 2012/13

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1. Multilinear algebra. Dual spaces. Inner products (Riesz' representation theorem). Dual homomorphism. Covariance vs contravariance. Algebraic k-forms. Exterior (wedge) product. The Grassmann algebra.

2. Vector fields and differential forms on $\mathbf{R}^n$. Cartan calculus: wedge product, exterior differential, pull-back. Closed and exact forms. de Rham cohomology (hint). The Poincaré lemma (without proof).

3. Submanifolds of $\mathbf{R}^N$. Dini's theory revisited. Examples.

4. Topological manifolds. Differentiable manifolds. Examples: $\mathbf{R}^n$, spheres, real and complex projective spaces. The Riemann sphere.

5. Smooth partitions of unity. Tangent vectors. Tangent and cotangent bundles. Vector fields and differential forms. Diffeomorphisms.

6. The Lie bracket of vector fields. Lie algebras. Examples. Flows of vector fields. Complete vector fields. Escape lemma. Lie derivative of vector fields ("fisherman's derivative").

7. Lie groups and their Lie algebras. Examples.

8. Tensor analysis. Tensor product of vector spaces. Tensors and tensor fields. Covariariance vs contravariance. Example: the metric tensor of a Riemannian manifolds. Lie derivative of tensor fields. Cartan's calculus on manifolds. Exterior differential, contraction, Lie derivative. Cartan's magic formula. Examples. Mechanical digression: symplectic manifolds, Hamiltonian vector fields, Poisson brackets. Integrals of motion. The harmonic oscillator. Completely integrable systems (hint).

9. The inverse function, rank and implicit function theorems for manifolds. k-slices and slice charts. Immersions and embeddings. Examples. Subgroups of Lie groups. The Frobenius theorem and its reintepretation in terms of differential forms. Examples.

The programme of the present module is entirely covered in:

M. SPERA "Lectures on Differential Geometry and Topology" I-XX (available online).

We also point out the following:

References

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D. BACHMAN, A Geometric Approach to Differential Forms, Birkhäuser, Boston, 2006.

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A.T. FOMENKO, T.L. KUNII, Topological Modeling for Visualization. Springer-Verlag, 1997.

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A. HATCHER, Algebraic Topology (scaricabile liberamente dalla pagina web dell'autore)

K. JÄNICH, Topologia, Zanichelli, Bologna, 1994.

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E. SERNESI, Geometria 2 Bollati Boringhieri, Torino, 1994.

I.M. SINGER, J.A. THORPE Lezioni di topologia elementare e di geometria, Boringhieri, Torino, 1980.

M. SPERA "Geometria" [handwritten notes available online at the webpage of: Geometria 2008/09];
"Elementi di Topologia" [handwritten notes available online at the webpage of Ph.D. courses in
Computer Science, 2008/09]

M. SPERA "Topologia e geometria differenziale" [handwritten notes I-XLIII, A1-8 available online,
2009/10-2011/12], together with N. Sansonetto's notes (also available online, at same URL); also, texts
and solutions of earlier exams are provided.