Color vision and colorimetry

Photoreceptor types

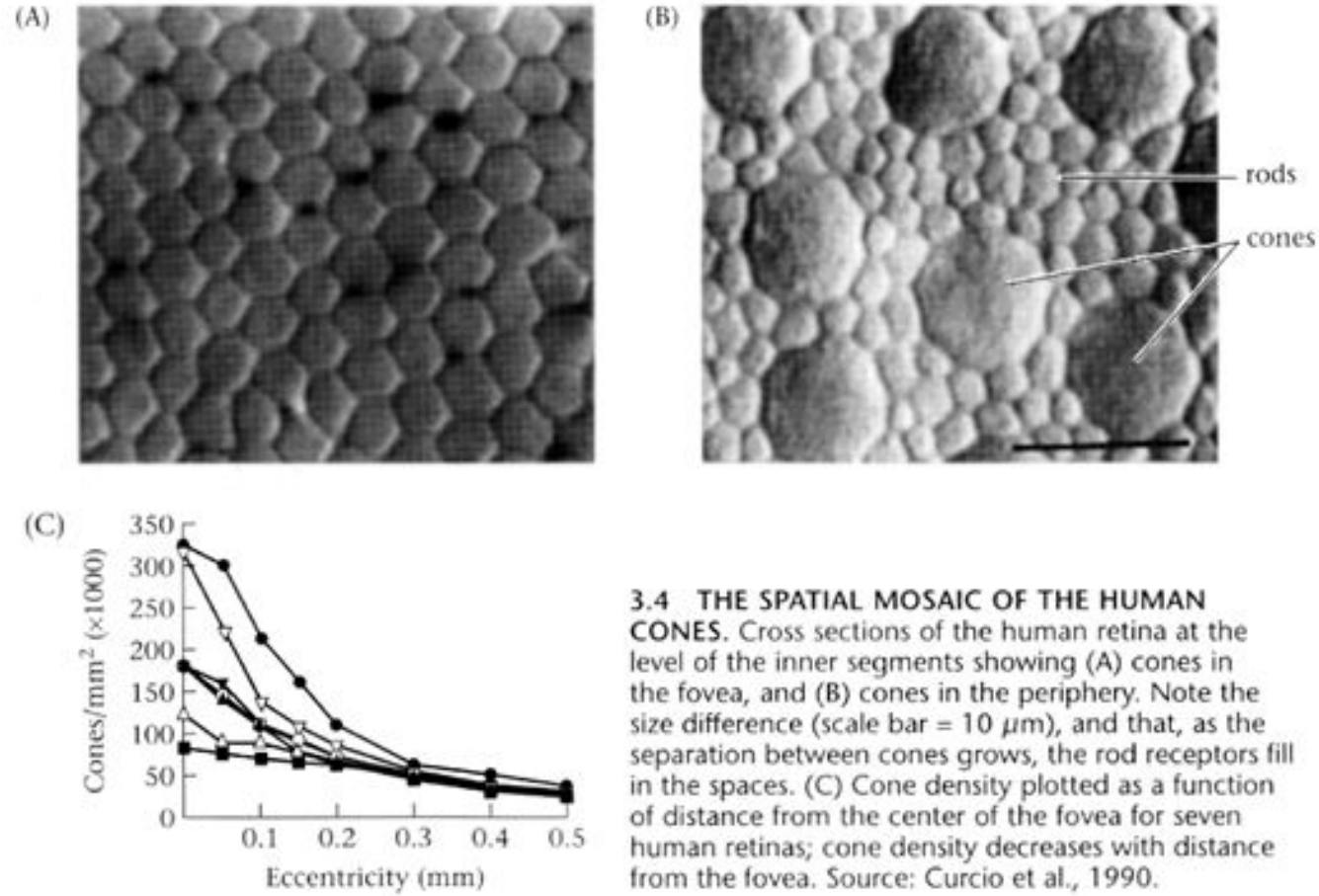
Rods

- *Scotopic* vision (low illumination)
- Do not mediate color perception
- High density in the periphery to capture many quanta
- Low spatial resolution
- Many-to-one structure
  - The information from many rods is conveyed to a single neuron in the retina
- Very sensitive light detectors
  - Reaching high quantum efficiency could be the reason behind the integration of the signals from many receptors to a single output. The price for this is a low spatial resolution
- About 10 millions

Cones

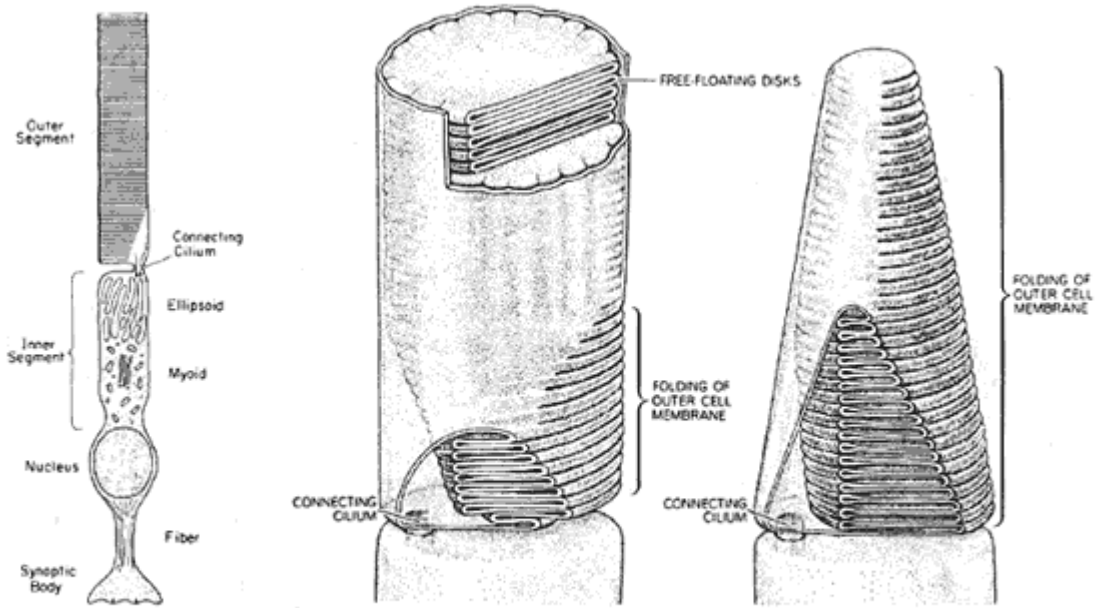
- *Photopic* vision (high illumination)
- Mediate color perception
- High density in the fovea
- One-to-one structure
  - Do not converge into a different single neuron but are communicated along private neural channels to the cortex
- High spatial resolution
  - The lower sensitivity is compensated by the high spatial resolution, providing the eye with good acuity
- About 5 millions
  - 50000 in the central fovea

Cones and Rods mosaic



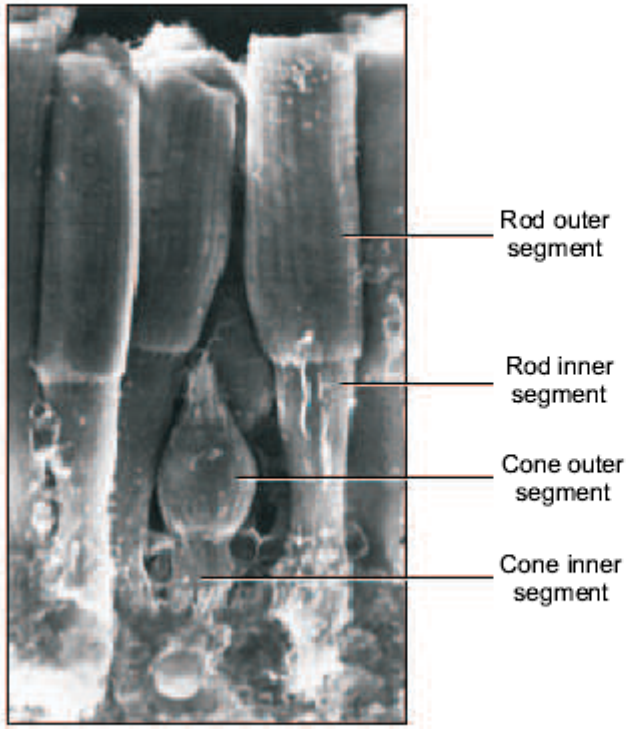
3.4 THE SPATIAL MOSAIC OF THE HUMAN CONES. Cross sections of the human retina at the level of the inner segments showing (A) cones in the fovea, and (B) cones in the periphery. Note the size difference (scale bar = 10 μm), and that, as the separation between cones grows, the rod receptors fill in the spaces. (C) Cone density plotted as a function of distance from the center of the fovea for seven human retinas; cone density decreases with distance from the fovea. Source: Curcio et al., 1990.

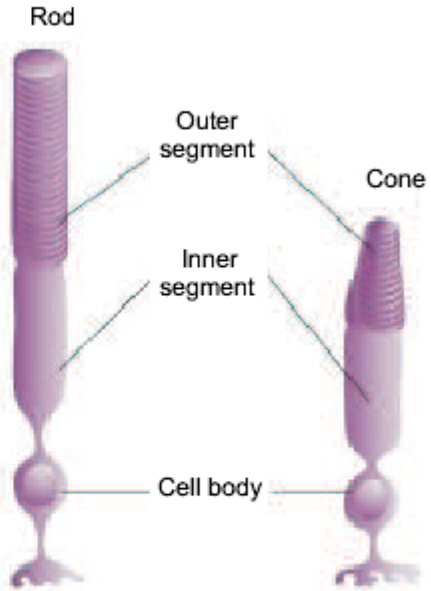
Cones and Rods shape



At the left is a generalized conception of the important structural features of a vertebrate photoreceptor cell. At the right are shown the differences between the structure of rod (left) and cone (right) outer segments. These diagrams are from Young (1970) and Young (1971).

Cones and rods shapes

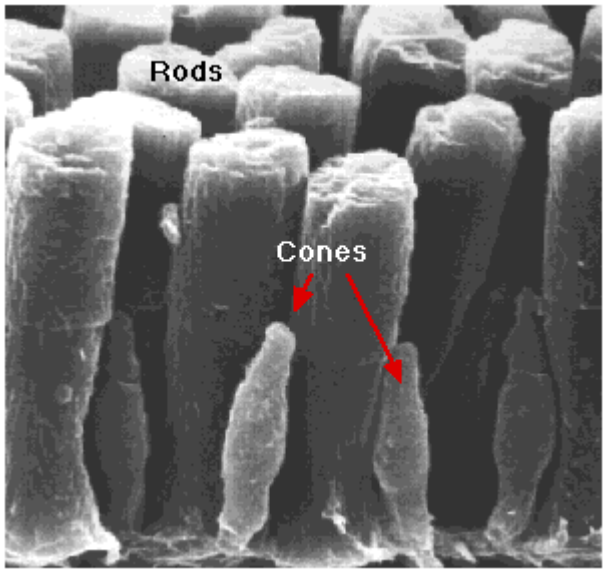


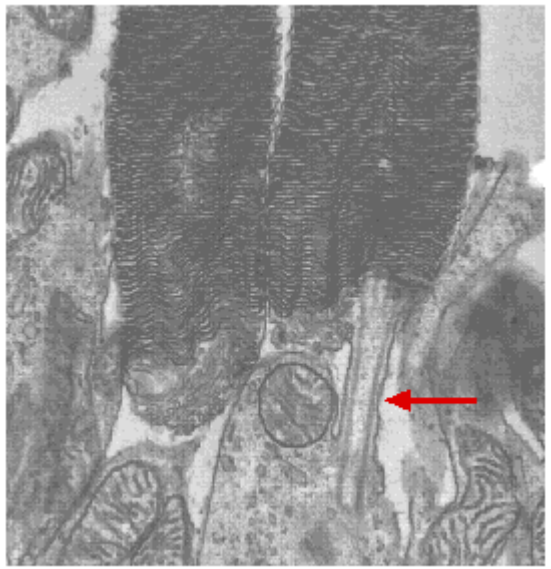


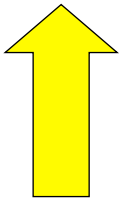
The light enters the inner segment and passes into the outer segment which contains light absorbing photopigments. Less than 10% photons are absorbed by the photopigments [Baylor, 1987].

The rods contain a photopigment called rhodopsin.

Cone and rods



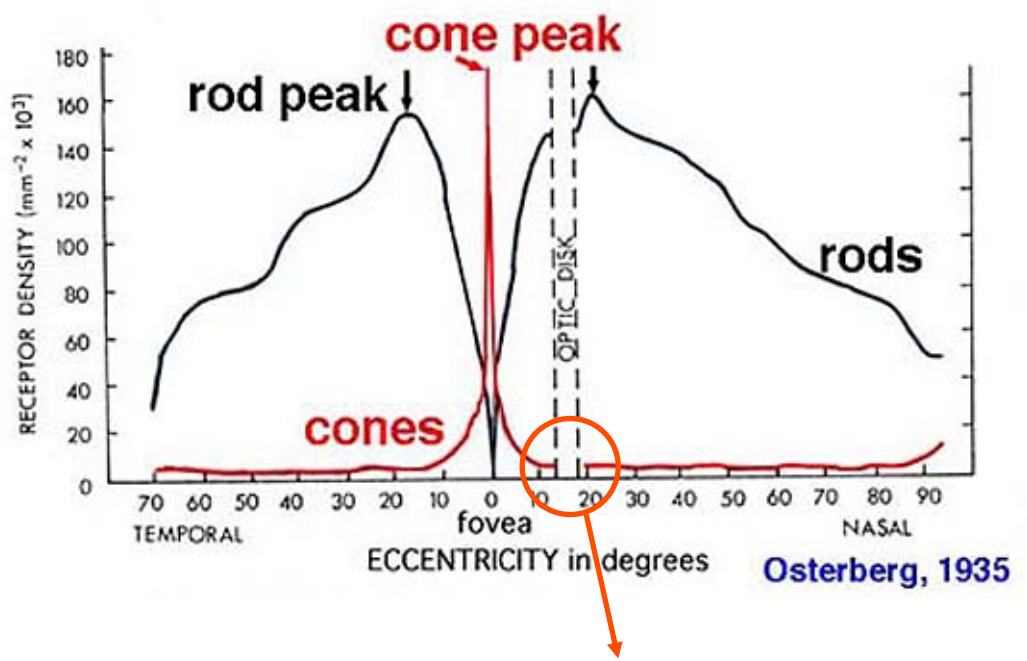




photons

The fovea

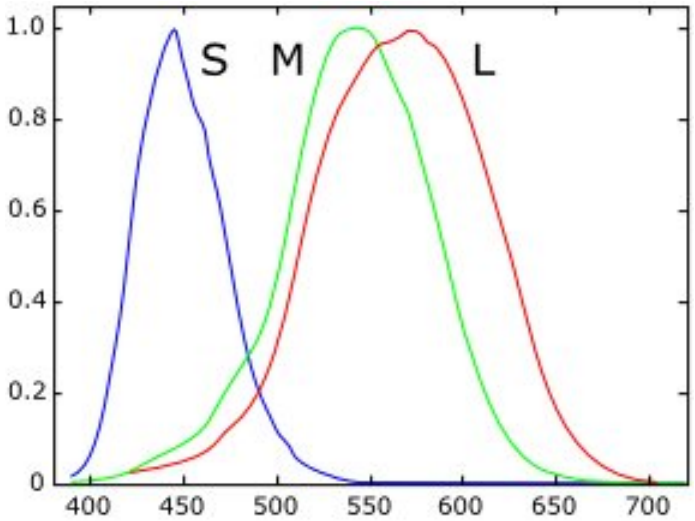
- The fovea is the region of the highest visual acuity. The central fovea contains no rods but does contain the highest concentration of cones.

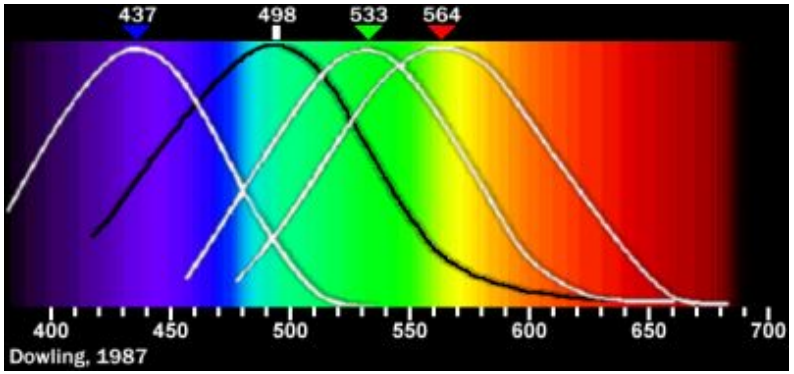


Properties of Rod and Cone Systems

Rods	Cones	Comment
More photopigment	Less photopigment	
Slow response: long integration time	Fast response: short integration time	Temporal integration
High amplification	Less amplification	Single quantum detection in rods (Hecht, Schlaer & Pirenne)
Saturating Response (by 6% bleached)	Non-saturating response (except S-cones)	The rods' response saturates when only a small amount of the pigment is bleached (the absorption of a photon by a pigment molecule is known as bleaching the pigment).
Not directionally selective	Directionally selective	Stiles-Crawford effect (see later this chapter)
Highly convergent retinal pathways	Less convergent retinal pathways	Spatial integration
High sensitivity	Lower absolute sensitivity	
Low acuity	High acuity	Results from degree of spatial integration
Achromatic: one type of pigment	Chromatic: three types of pigment	Color vision results from comparisons between cone responses

Types of cones

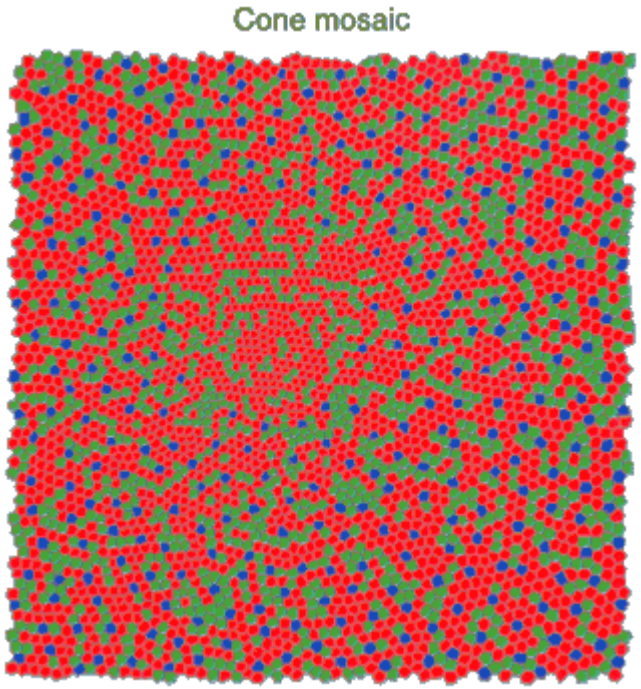




The cones are classified based on their wavelength selectivity as L (long), M (medium) and S (short) wavelength sensors.

L,M and S cones have different sensitivity and spatial distributions. The S cones are far less numerous and more sensitive than the others.

Cone mosaic



Williams (1985) measured the sampling density of the mosaic of the L- and M-cones together. His results are consistent with a sampling frequency of 60 cpd at the central fovea, consistent with a center-to-center spacing of the cones of 30 minutes of degree.

The sampling frequency than decreases when increasing the visual angle, consistently with the decrease in cone density.

This diagram was produced based on histological sections from a human eye to determine the density of the cones. The diagram represents an area of about *1° of visual angle*. The number of S-cones was set to 7% based on estimates from previous studies. The L-cone:M-cone ratio was set to 1.5. This is a reasonable number considering that recent studies have shown wide ranges of cone ratios in people with normal color vision. In the central fovea an area of approximately 0.34° is S-cone free. The S-cones are semi-regularly distributed and the M- and L-cones are randomly distributed.
Throughout the whole retina *the ratio of L- and M- cones to S-cones is about 100:1*.

Wavelength encoding

- Scotopic matching experiment → Scotopic luminosity function V’ (λ)
  - Characterizes vision at low illumination conditions
  - Rod responses
  - One primary light and one test light
  - The intensity of the light beam is the parameter

- Photopic color matching experiment→ Color matching finctions (CMF), photopic luminosity function V(λ)
  - Characterizes vision under high illumination conditions
  - Cones responses
  - Three primary lights and one test light
  - The intensities of each primary lights are the parameters

Brightness matching

Wavelength encoding

The photometric principle

- Basic postulate

  Whatever the visual stimulus, fixed in all respects, of one patch, and whatever the fixed relative spectral distribution of the stimulus on the second patch, a brightness match can always be achieved by varying the absolute value of the second stimulus

- Basic laws of brightness matching
  - Symmetry
    - If A matches B then B matches A
  - Transitivity
    - If A matches B and B matches C than A matches C
  - Proportionality
    - If A matches B then kA matches kB
  - Additivity
    - If A matches B and C matches D than (A+C) matches (B+D)

Brightness match

- Definition
  - *Similar uniform light patches, producing visual stimuli defined by $\{P_\lambda d\lambda\}$ and $\{P'_\lambda d\lambda\}$, respectively, are in brightness match for the standard photopic observer if*

$$\int_\lambda P_\lambda V(\lambda) d\lambda = \int_\lambda P'_\lambda V(\lambda) d\lambda$$

    - For brightness matches, the photopic luminous flux entering the eye per unit solid angle must be the same for the two patches

Matching experiments

- Scotopic matching experiment (brightness matching)
  - Low illumination conditions
  - Rod responses
  - One primary light and one test light
  - The *intensity* of the primary light beam is the parameter

- Measures the scotopic spectral sensitivity function V’ (λ)

- Photopic color matching experiment
  - High illumination conditions
  - Brightness matching
    - Rods
  - Color matching
    - Cones
    - Three primary lights and one test light
    - The intensities of each primary lights are the parameters

- Measures the photopic spectral sensitivity function V(λ)
- Measures the Color Matching Functions (CMFs)

Brightness matching

Necessary and sufficient condition for a brightness match between two stimuli of radiant power distribution $\{P_\lambda d\lambda\}$ and $\{P'_\lambda d\lambda\}$, respectively

$$\int_\lambda P_\lambda \beta(\lambda) d\lambda = \int_\lambda P'_\lambda \beta(\lambda) d\lambda$$

Where $\beta(\lambda)$ is a fixed function characterizing the brightness-matching process depending on

- the spectral radiance power distributions (relative or absolute) of the matching stimuli
- the observational conditions
  - field size, eccentricity of the field of view, state of adaptation as modified by previous or surrounding stimuli
- *Quantum efficiency* of the human visual system

- *Ideal photometric observer*
  - *defined by the CIE by the specification of two fixed functions*
  - *Scotopic* matching $\beta(\lambda) \rightarrow V'(\lambda)$
  - *Photopic* matching $\beta(\lambda) \rightarrow V(\lambda)$

Scotopic brightness matching

Bipartite field

Primary light Test light

The **primary** light has a fixed relative spectral distribution and only the *intensity* can vary

The **test** light can have any spectral distribution. It is common to use an equal-energy spectral light

Task: Adjust the primary light intensity so that the primary and test lights appear indistinguishable.

Scotopic spectral sensitivity function

$$e = \begin{bmatrix} r_1 & r_2 & \dots & r_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

r : system vector (transfer function)

t : spectral distribution of the test light

e : response of the observer

Assuming that the system is linear (homogeneity and superposition hold), the system vector can be measured by feeding it with n_λ monochromatic lights.

It is common to choose an **equal-energy spectral** light (*reference white*) as test light.

$$e = \begin{bmatrix} r_1 & r_2 & \dots & r_{n_\lambda} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = r_1$$

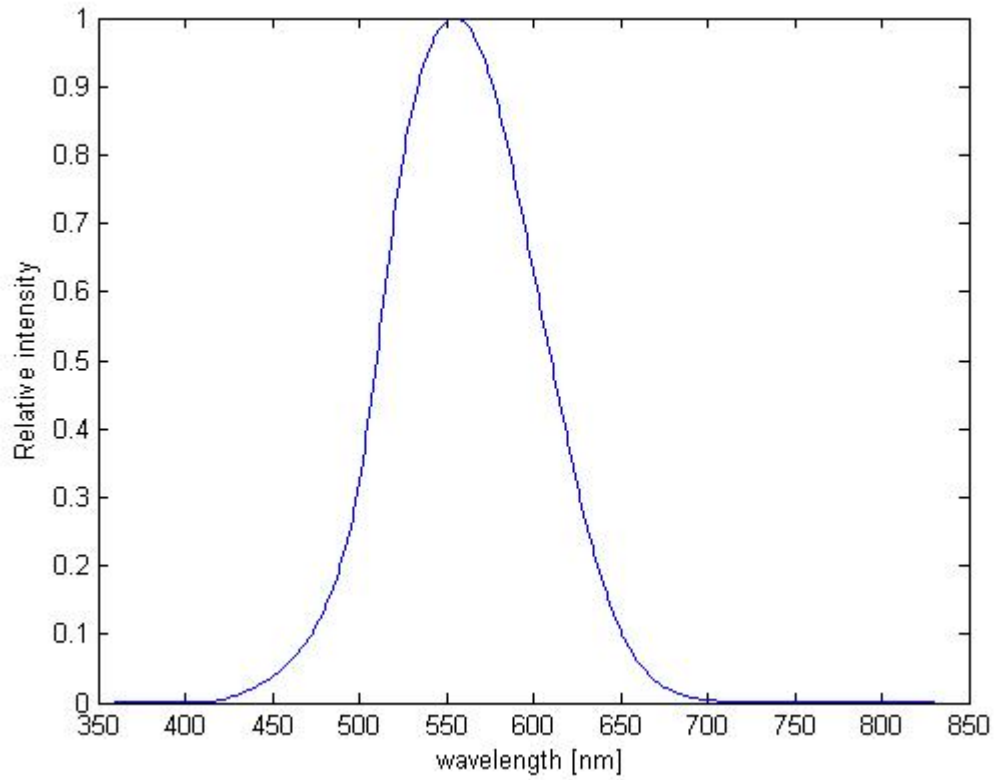
Each monochromatic light will determine one entry of the system vector, resulting in the *Scotopic Spectral Sensitivity function*.

Scotopic spectral sensitivity function

- In scotopic conditions, the eye is sensible only to *relative intensities* of the two lights. The spectral distribution is immaterial (in low illumination conditions, color is not "perceivable").
  - Physiological interpretation: the rhodopsin absorption coefficient depends on the wavelength, but the response is the same for any wavelength. Once a photon is absorbed, the information about its wavelength is lost. Hence, **the appearance of the stimulus is independent of its spectrum**.
  - **The shape of V`(λ) reflects the dependence of the absorption coefficients from the wavelength.**

- In order to measure the spectral sensitivity at each wavelength a set of equal energy spectral (monochromatic lights) are used as test lights
- Relative intensities are recorded ($I_{REF}/I_{test}=I_{REF}$ since I_{test}=const.=1) and the normalized (values between zero and one)
  - Prior to normalization, due to the linearity of the system, the system vector is unique up to a scale factor.
- *V`(λ) was adopted by CIE in 1951 in a field of 10 degrees (Crawford 1949) and eccentrically with more than 5 degrees (Wald 1945) with complete darkness adaptation.*

Scotopic spectral sensitivity function V’ (λ)



Data available at http://cvision.ucsd.edu/cie.htm

Photopic brightness matching

Bipartite field

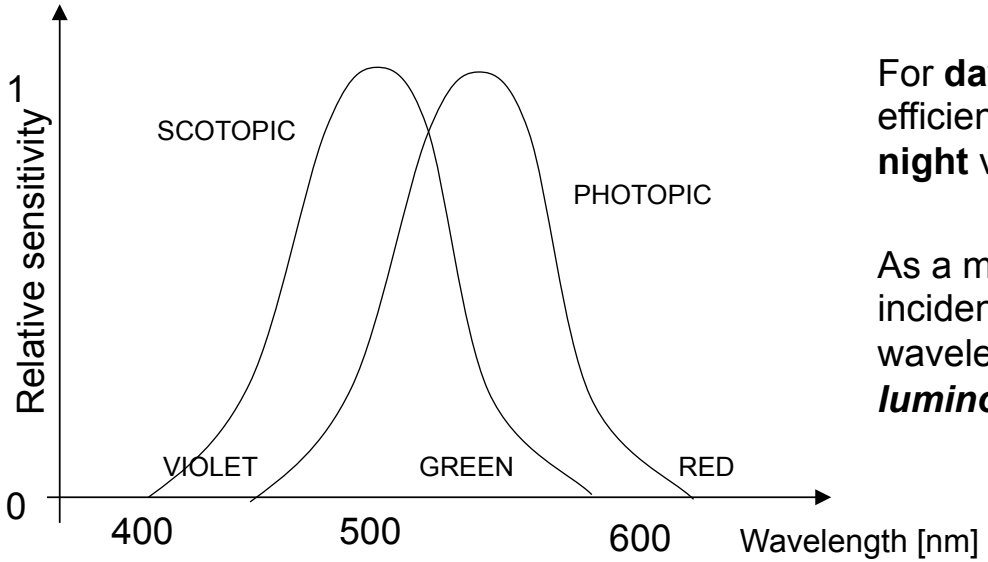
Primary light: only the *intensity* can vary

Test light

Task: Adjust the primary light intensity so that the primary and test lights appear indistinguishable, following a ad-hoc paradigm

Photopic curve V(λ)

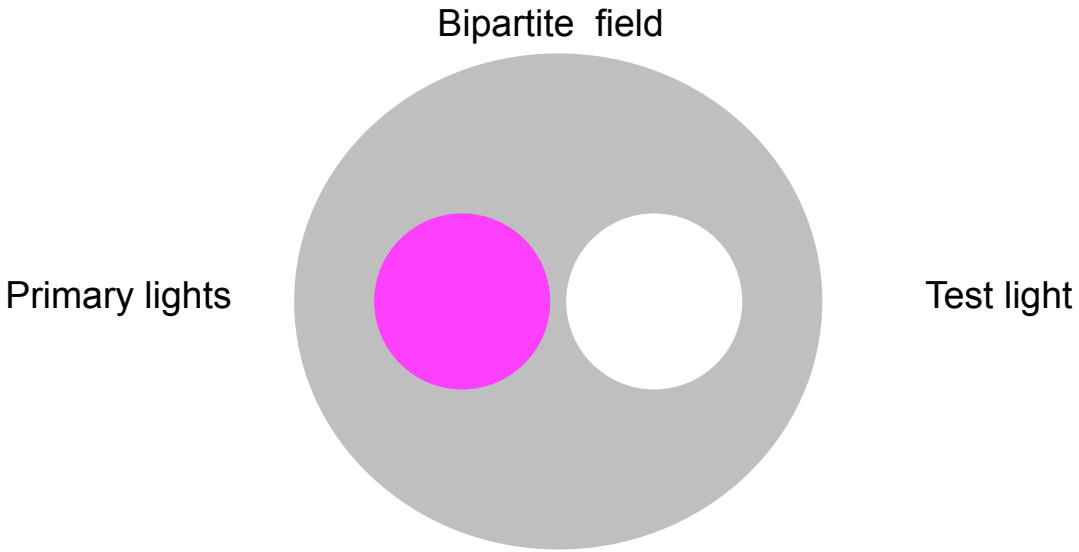
- High illumination levels
- Different paradigms
  - The direct comparison of the brightness leads to unreliable results due to the difference in color
    - Flickering method (Coblentz and Emerson, 1918)
    - Step-by-step method of heterochromatic fotometry (Hyde et al. 1918)
    - CIE adopted the (Gibson and Tindall, 1923)



For **daylight** vision the maximum efficiency is at **555 nm (yellow)** while for **night** vision it shifts to **505 nm (blue)**

As a measure of the sensitivity of the eye to incident monochromatic light at each wavelength, it corresponds to a measure of the ***luminous efficacy*** of the eyes.

Color matching

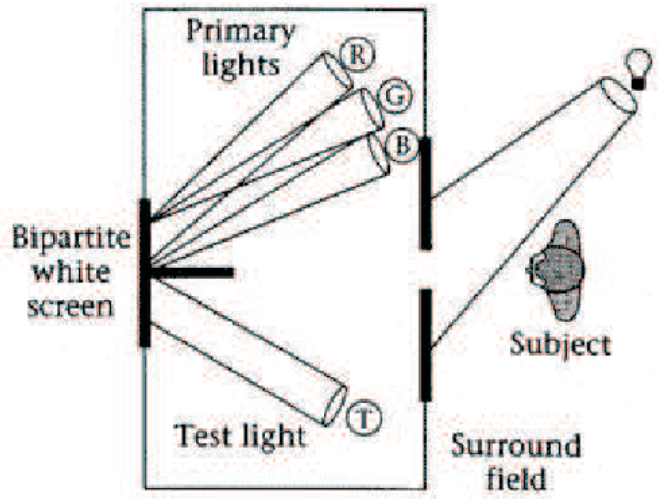


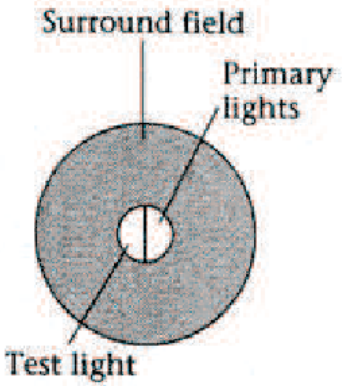
There are **three primary lights** with fixed relative spectral distribution and only the intensity can vary. These are chosen to be **monochromatic**

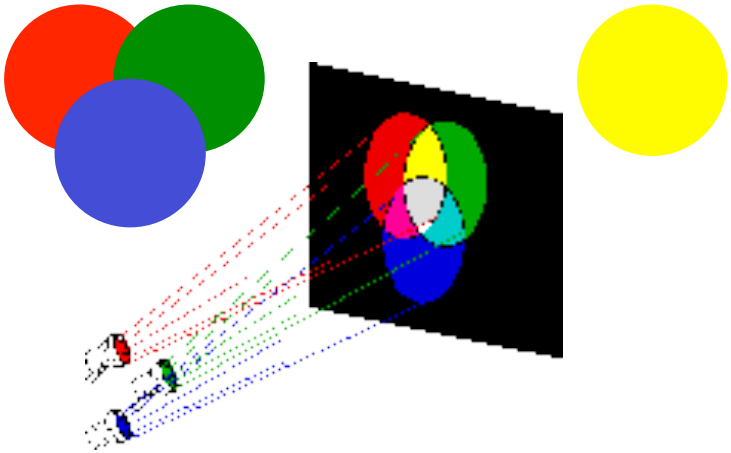
The test light can have any spectral distribution. It is common to chose a equal energy light and decompose it into the monochromatic components for testing the entire set of wavelengths.

Task: Adjust the intensities of the primary lights so that the primary and test lights appear indistinguishable

Color matching







Measuring the CMFs

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

R : system matrix (transfer function). Each line represents the *Color Matching Function* (CMF) for the corresponding primary light

t : spectral distribution of the test light

e : response of the observer

Assuming a equal-energy test light

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix} = t \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

since we are measuring relative intensities we can choose $t=1$

Color Matching Functions (CMFs)

Assuming that the symmetry, transitivity and homogeneity hold (*Grassmann's laws of additive color mixtures*), the system matrix can be measured by feeding it with n_λ monochromatic lights

$$\vec{e} = \begin{bmatrix} r_1^1 & r_2^1 & \dots & r_{n_\lambda}^1 \\ r_1^2 & r_2^2 & \dots & r_{n_\lambda}^2 \\ r_1^3 & r_2^3 & \dots & r_{n_\lambda}^3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} r_1^1 \\ r_1^2 \\ r_1^3 \end{bmatrix}$$

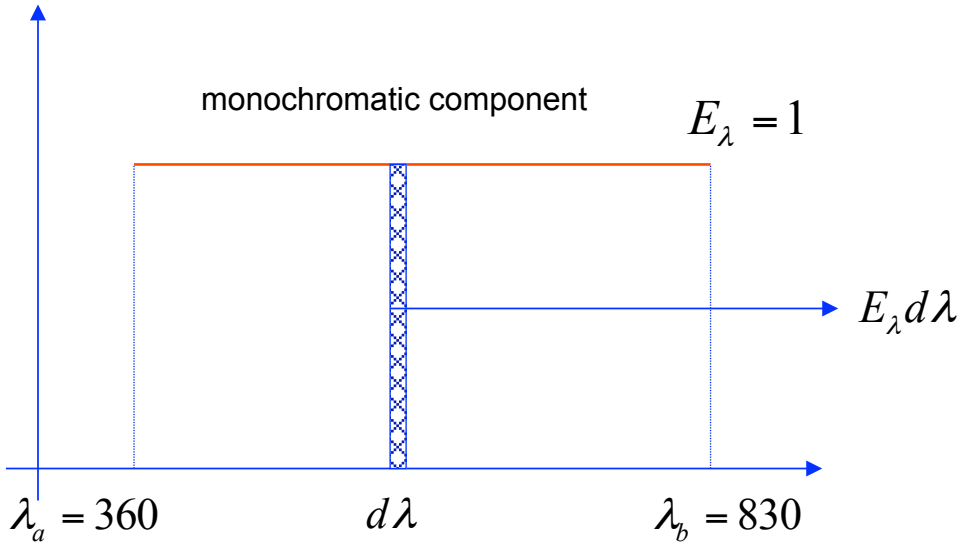
The response to each monochromatic light will determine one *column* of the system matrix, so one entry of each CMF.

It can be shown that the system matrix is not unique. Using **different sets of primaries** leads to different CMFs. Though, different sets of CMFs are related by a **linear transformation**

→ **Need to choose one set of primaries**

Color Matching Functions

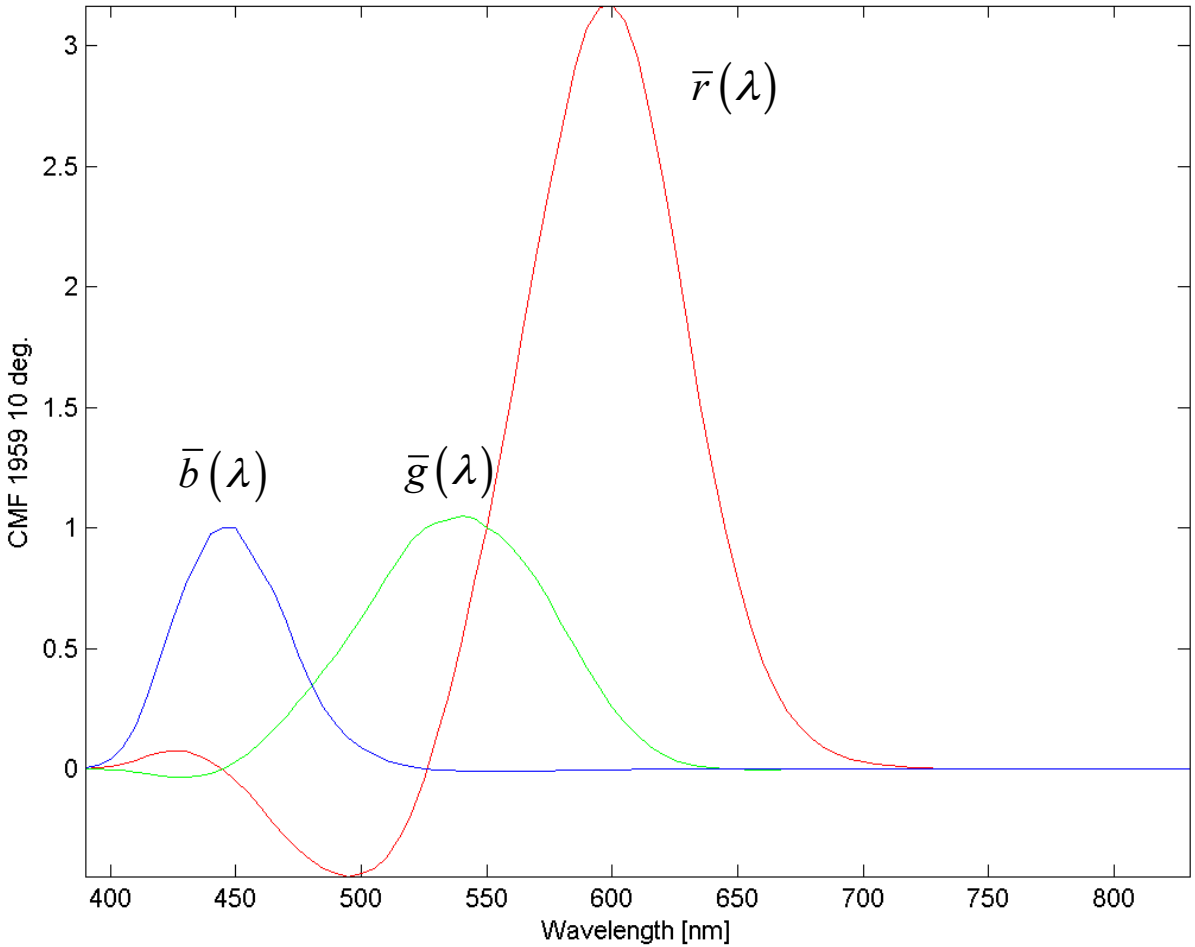
- In other words, the CMFs are the *spectral* tristimulus values of the *equal energy* stimulus E (*reference white*)



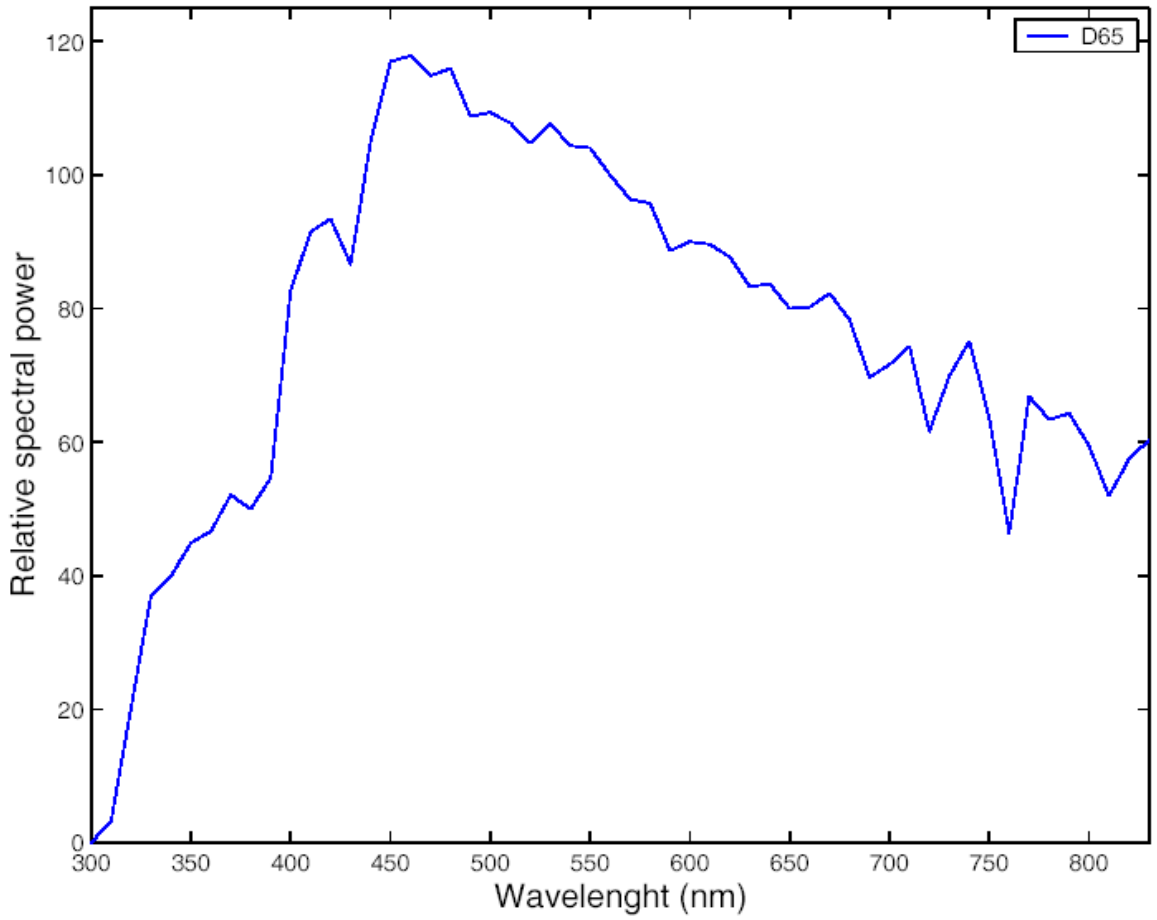
$$E_\lambda = \bar{r}(\lambda)R + \bar{g}(\lambda)G + \bar{b}(\lambda)B$$
$$\lambda_R = 700nm$$
$$\lambda_G = 546.1nm$$
$$\lambda_B = 435.8nm$$

$\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ are called color matching functions

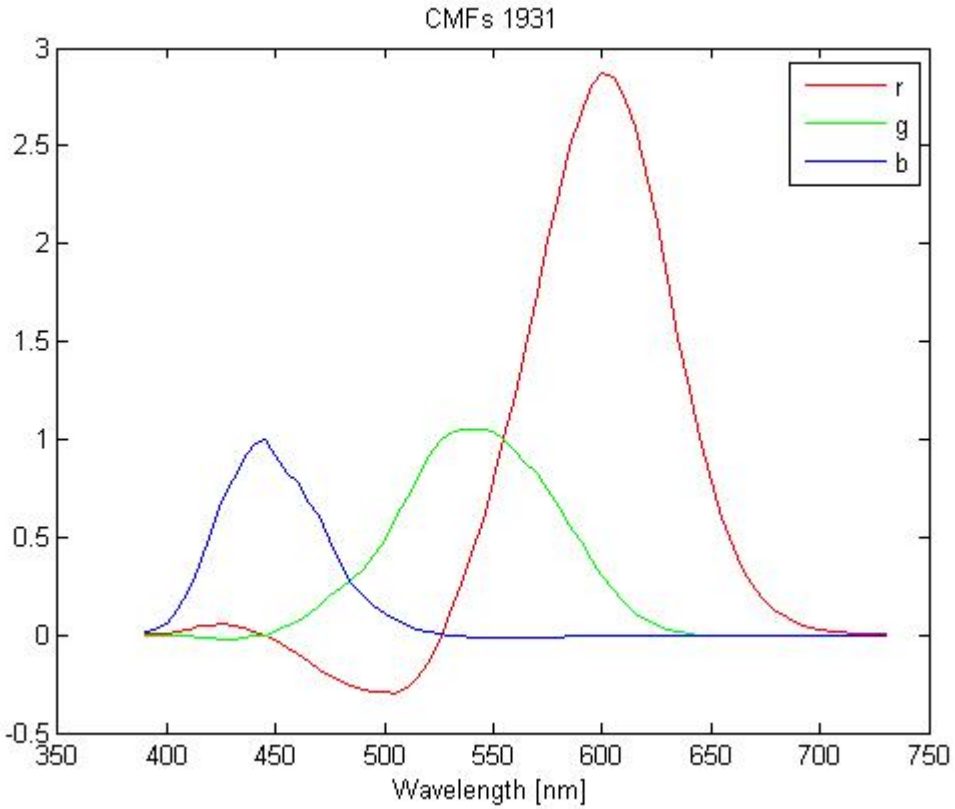
CMFs


$\bar{r}(\lambda)$
$\bar{g}(\lambda)$
$\bar{b}(\lambda)$
CMF 1959 10 deg.
Wavelength [nm]
3
2.5
2
1.5
1
0.5
0
400
450
500
550
600
650
700
750
800

D65


D65
Relative spectral power
Wavelenght (nm)
0
20
40
60
80
100
120
300
350
400
450
500
550
600
650
700
750
800

CMF rgb 1931


CMFs 1931
r
g
b
3
2.5
2
1.5
1
0.5
0
-0.5
350
400
450
500
550
600
650
700
750
Wavelength [nm]

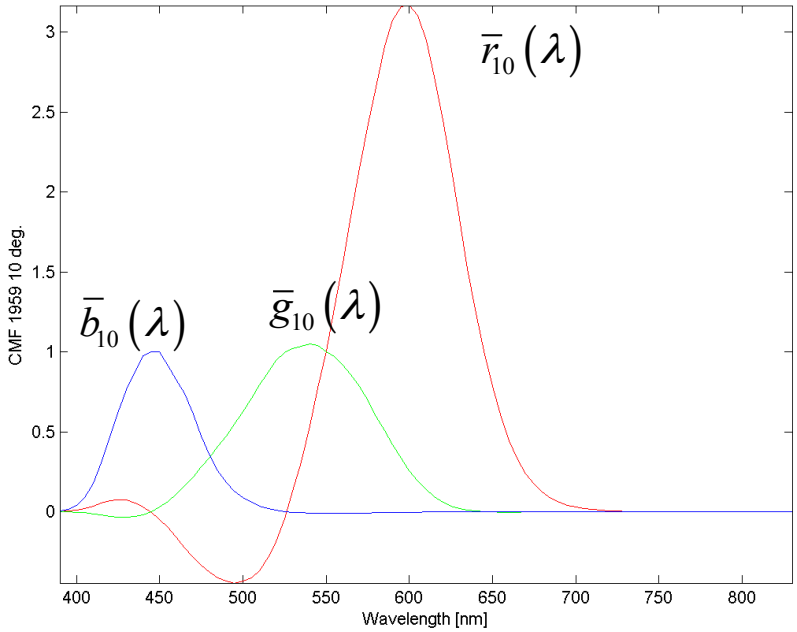
Stiles and Burch 10deg (1959)

Primary lights: monochromatic

λ_R=645.16 nm
λ_G= 526.32 nm
λ_B= 444.44 nm

$\bar{r}_{10}(\lambda), \bar{g}_{10}(\lambda), \bar{b}_{10}(\lambda)$ CMFs

$$t(\lambda) = R \cdot \bar{r}_{10}(\lambda) + G \cdot \bar{g}_{10}(\lambda) + B \cdot \bar{b}_{10}(\lambda)$$



t : monochromatic test light

(R,G,B) : ***tristimulus values*** of ***t***

A **10 degrees bipartite field** was used

Negative values for the tristimulus value mean that the corresponding primary was added to the test light in order to match the color appearance.

This outlines that not every test color can be matched by an additive mixture of the three primaries.

The presence of negative values could be impractical, so another color coordinate system was chosen as the reference by the *Commisione Internationale d'Eclairage (CIE)* in 1931.

Cone photopigments and CMF

- How well do the spectral sensitivities of the cone photopigments predict performance on the photopic color matching experiment?

Biological measurements

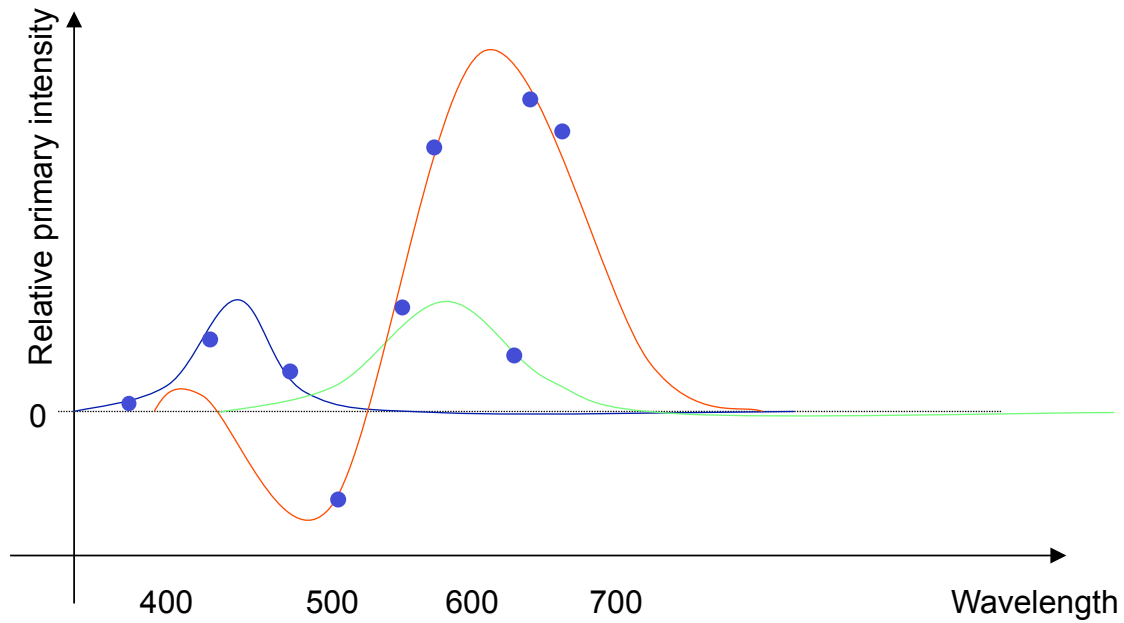
$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} \text{Spectral sensitivity of L photopigments} \\ \text{Spectral sensitivity of M photopigments} \\ \text{Spectral sensitivity of S photopigments} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

Psychophyiscal measurements

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \text{CMF of primary 1} \\ \text{CMF of primary 2} \\ \text{CMF of primary 3} \end{bmatrix} \cdot \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

- There should be a linear transformation that maps the cone absorption curves to the system matrix of the color matching experiment
- *Linking hypothesis*

Cone photopigments and CMFs



From the agreement between these two datasets one can conclude that the photopigment spectral responsivities provide a satisfactory biological basis to explain the photopic color matching experiments

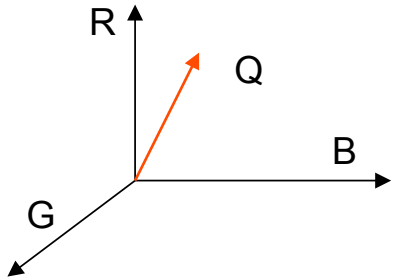
Tristimulus values for complex stimuli

- Color stimuli are represented by vectors in a three-dimensional space, called the *tristimulus* space
  - Let Q be an arbitrary **monochromatic** color stimulus and **R,G and B the fixed primary stimuli** chosen for the color matching experiment

$$\vec{Q} = R_Q\vec{R} + G_Q\vec{G} + B_Q\vec{B}$$

  - R_Q,G_Q,B_Q : *tristimulus values* of Q
  - The scalar multipliers R_Q,G_Q,B_Q are measured in terms of the *assigned respective units of the corresponding primaries*
  - It is customary to choose these units such that when additively mixed yield a complete color match with a specified *achromatic* stimulus, usually one with an *equal-energy spectrum* on a wavelength basis

The units of these primaries was chosen in the radiant power ratio of 72.1:1.4:1.0, which places the chromaticity coordinates of the equal energy stimulus E at the center of the (r,g) chromaticity diagram
→ $R_W=G_W=B_W=1$ for the reference white

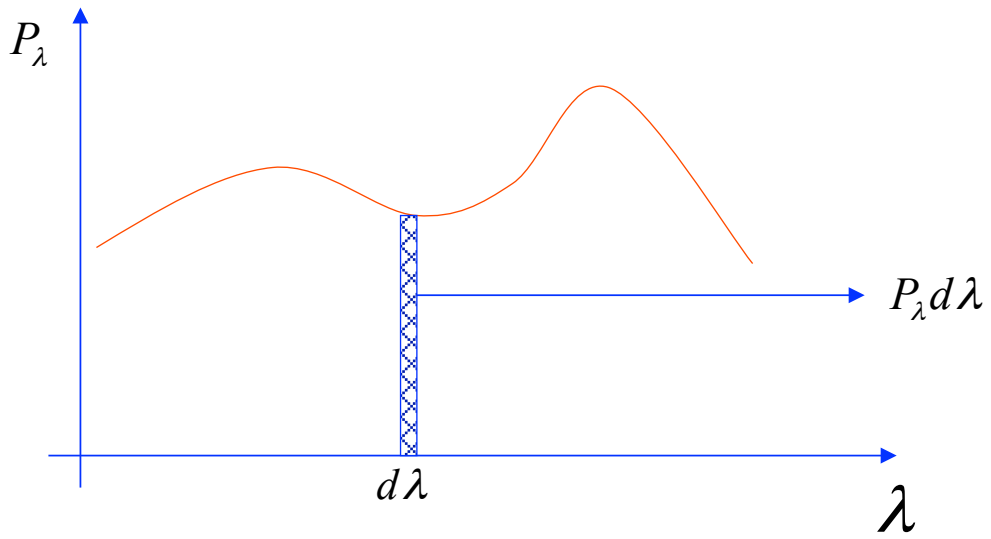


Complex stimuli

- A given complex stimuli Q with spectral power density (SPD) $\{P_\lambda d\lambda\}_Q$ can be seen as an *additive mixture* of a set of monochromatic stimuli Q_i with SPD $\{P_\lambda\, d\lambda\}_{Qi}$
  - For each monochromatic stimulus

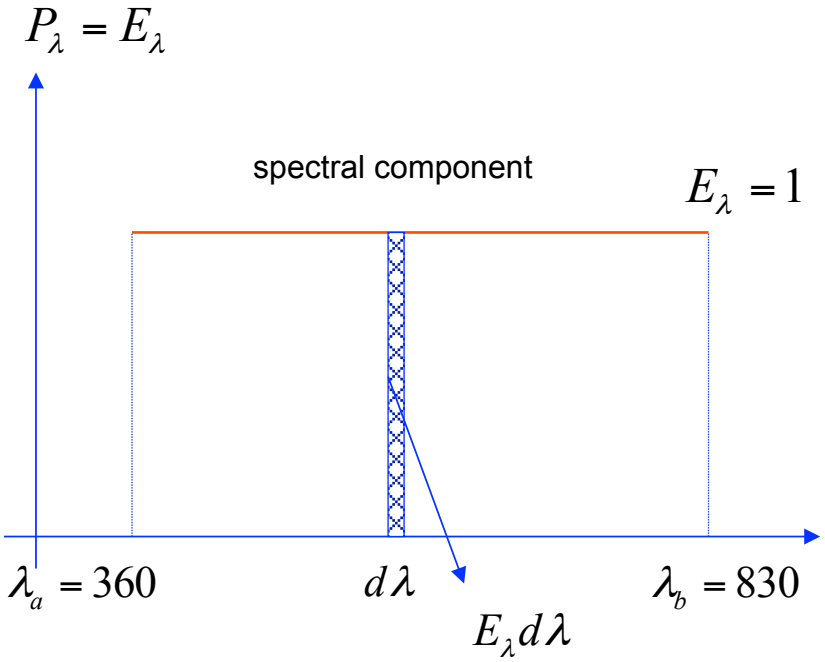
$$\vec{P}_\lambda = R_\lambda \vec{R} + G_\lambda \vec{G} + B_\lambda \vec{B}$$

$R_\lambda, G_\lambda, B_\lambda$ spectral tristimulus values



Special case: reference white

- The *reference white* is used to express the complex spectrum in a different form



The Color Matching Functions are normalized such that the tristimulus values of the reference white are equal.

$$\vec{E}_\lambda = \bar{r}(\lambda)\vec{R} + \bar{g}(\lambda)\vec{G} + \bar{b}(\lambda)\vec{B} \Rightarrow$$

$$\int_\lambda \vec{E}_\lambda \, d\lambda = \int_\lambda \left(\bar{r}(\lambda)\vec{R} + \bar{g}(\lambda)\vec{G} + \bar{b}(\lambda)\vec{B}\right) d\lambda =$$

$$\int_\lambda \left(\bar{r}(\lambda)\vec{R}\right) d\lambda + \int_\lambda \left(+\bar{g}(\lambda)\vec{G}\right) d\lambda + \int_\lambda \left(\bar{b}(\lambda)\vec{B}\right) d\lambda =$$

$$\int_\lambda \bar{r}(\lambda) d\lambda \times \vec{R} + \int_\lambda \bar{g}(\lambda) d\lambda \times \vec{G} + \int_\lambda \bar{b}(\lambda) d\lambda \times \vec{B}$$

Normalization conditions

$$E_R = \int_{-\infty}^{+\infty} \bar{r}(\lambda) d\lambda = 1$$

$$E_G = \int_{-\infty}^{+\infty} \bar{g}(\lambda) d\lambda = 1$$

$$E_B = \int_{-\infty}^{+\infty} \bar{b}(\lambda) d\lambda = 1$$

Then

$$\vec{E} = 1\vec{R} + 1\vec{G} + 1\vec{B}$$

Tristimulus values of a complex stimulus

$$Q_\lambda = \left(P_\lambda d\lambda\right)E_\lambda = \left(P_\lambda d\lambda\right)\bar{r}\left(\lambda\right)\vec{R} + \left(P_\lambda d\lambda\right)\bar{g}\left(\lambda\right)\vec{G} + \left(P_\lambda d\lambda\right)\bar{b}\left(\lambda\right)\vec{B} \rightarrow$$

$$R_Q = \int_\lambda \left(P_\lambda d\lambda\right)\bar{r}\left(\lambda\right) = \int_\lambda P_\lambda \bar{r}\left(\lambda\right)d\lambda$$

$$G_Q = \int_\lambda \left(P_\lambda d\lambda\right)\bar{g}\left(\lambda\right) = \int_\lambda P_\lambda \bar{g}\left(\lambda\right)d\lambda$$

$$B_Q = \int_\lambda \left(P_\lambda d\lambda\right)\bar{b}\left(\lambda\right) = \int_\lambda P_\lambda \bar{b}\left(\lambda\right)d\lambda$$

Metameric stimuli: different SPD, same color appearance

$$R_Q = \int P^1_\lambda \bar{r}(\lambda)d\lambda = \int P^2_\lambda \bar{r}(\lambda)d\lambda$$

$$G_Q = \int P^1_\lambda \bar{g}(\lambda)d\lambda = \int P^2_\lambda \bar{g}(\lambda)d\lambda$$

$$B_Q = \int P^1_\lambda \bar{b}(\lambda)d\lambda = \int P^2_\lambda \bar{b}(\lambda)d\lambda$$

Chromatic coordinates

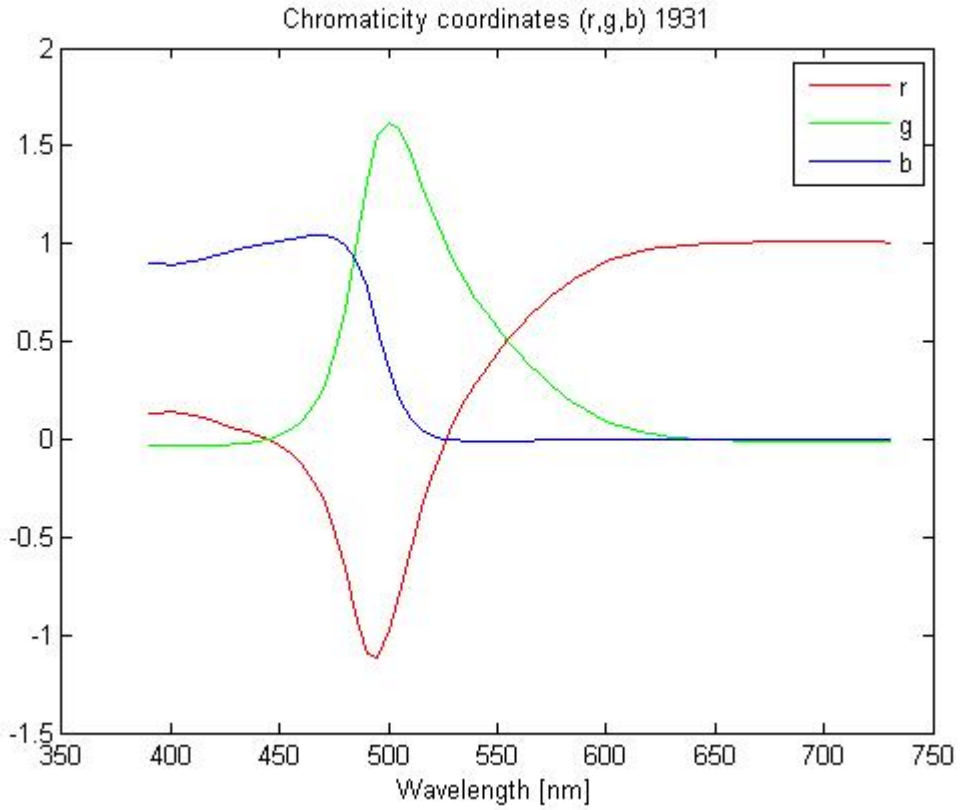
- Spectral chromaticity coordinates

$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$r(\lambda) + g(\lambda) + b(\lambda) = 1$$

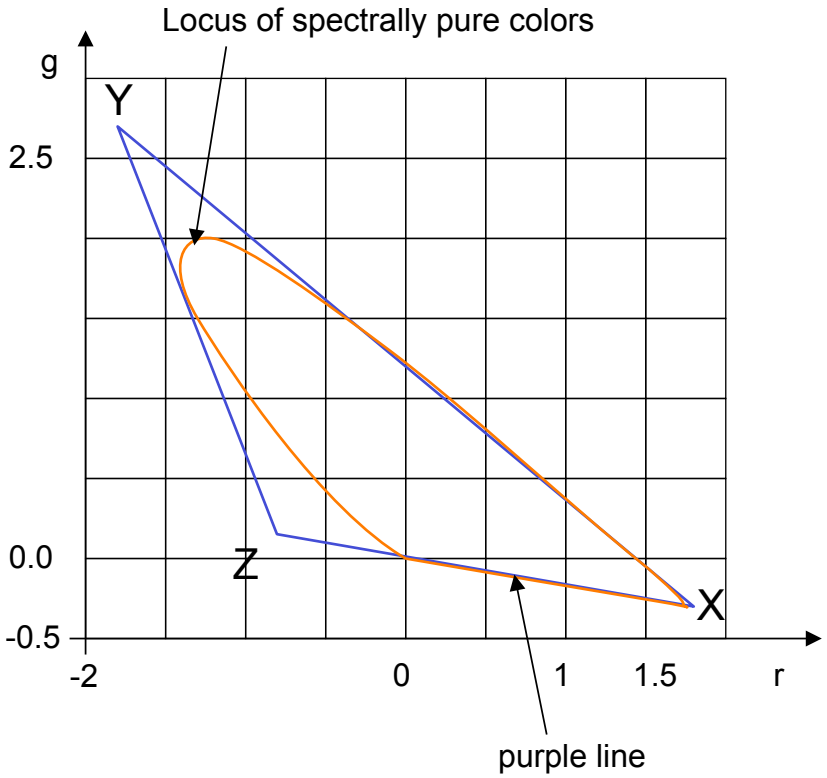


(r,g) chromaticity diagram

$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$



r,g,b : chromaticity coordinates

$\bar{r},\bar{g},\bar{b}$ color matching functions (tristimulus values of the reference white)

Chromaticity coordinates

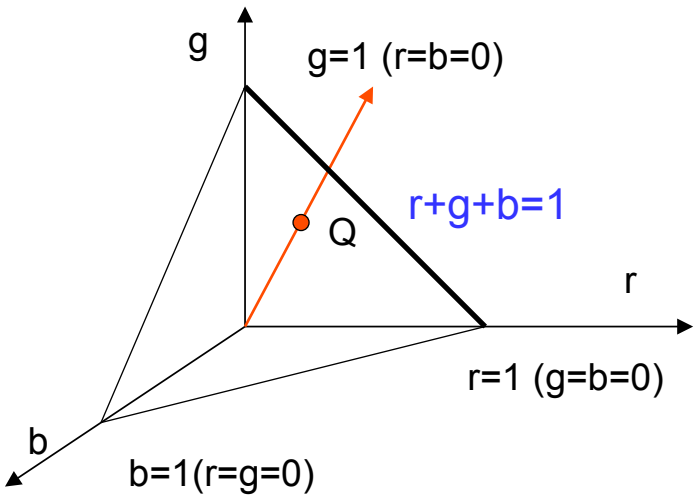
Chromaticity coordinates

$$r = \frac{R}{R+G+B}$$

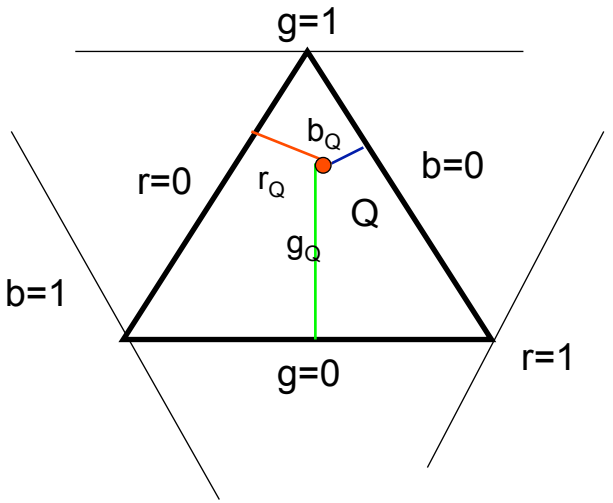
$$g = \frac{G}{R+G+B} \quad \Rightarrow \quad r+g+b=1$$

$$b = \frac{B}{R+G+B}$$

R,G,B: tristimulus value of the generic color



Maxwell color triangle



(r,g) specify the *hue and saturation* of the color while the information about the luminance is lost

A step in colorimetry…

CIE 1931 Standard Observer

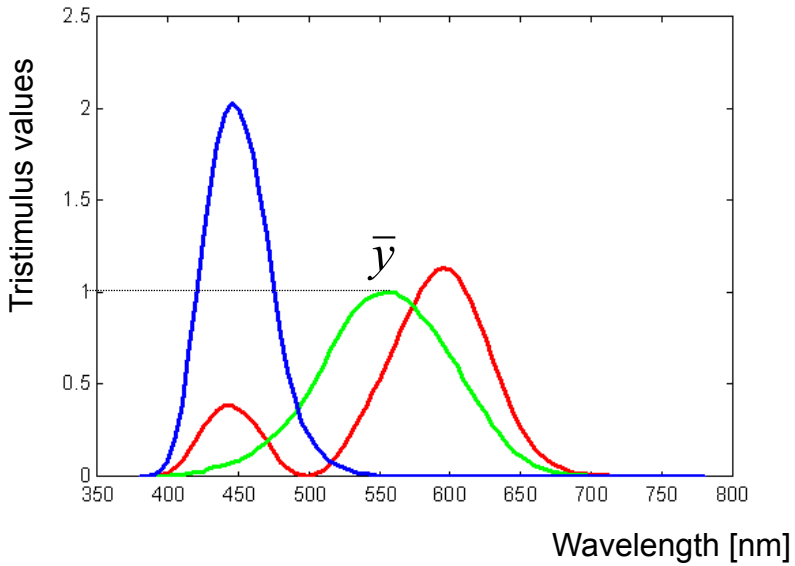
- In colorimetric practice, the main objective is to obtain results valid for the group of normal trichromats. To this end, the color matching properties of an *ideal trichromatic observer* are defined by specifying three independent functions of λ which are identified with the ideal observer CMFs.
- The CIE 1931 SO also embodies the additivity law for brightness (V(λ) photopic luminous efficiency function)
  - For an observer who makes brightness matches that *conform to the additivity law for brightness*, and who also *makes color matches* that are trichromatic in the stronger sense, it can be shown that *V(λ) is a combination of the CMFs*, provided all the pairs of metameric stimuli are also in brightness match.

    For such an observer, it is possible to select from the infinitely many equivalent sets of CMFs one set for which one of the three CMFs, usually taken to be the central one $\bar{y}$ coincides with V(λ).

    In this way, the CIE 1931 SO combines both color matching and heterochromatic brightness matching properties in a single quantitative scheme.

CIE 1931 Standard Colorimetric Observer

- Standard system for color representation: X,Y,Z tristimulus coordinate system
- Color matching functions $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$



Features

- λ=380 to 780 nm, Δλ=5nm
- Measured at 2 degrees
- Always non negative
- $\bar{y}$ is a rough approximation of the *brightness* of monochromatic lights of equal size and duration (*Standard photopic luminosity function V(λ)*)
- They cannot be measured by color matching experiments
- Quite inaccurate at low wavelengths

Improvements

- In 1959 a new set of CIE XYZ coordinates was derived based on the CMFs measured by Stiles&Burch at 10 degrees (CIE 1964 Supplementary Standard Colorimetric Observer).

Guidelines for the derivation of CIE 1931 SO

- Projective transformation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r_x & r_y & r_z \\ g_x & g_y & g_z \\ b_x & b_y & b_z \end{bmatrix}^{-1} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

(r_x,g_x,b_x) : coordinates of (1,0,0) as measured in the {r,g,b} system, so the CMFs

....

  - Need to determine the matrix A of the transformation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & \cdots & \\ & & a_{3,3} \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & \cdots & \\ & & a_{3,3} \end{bmatrix} = \begin{bmatrix} r_x & r_y & r_z \\ g_x & g_y & g_z \\ b_x & b_y & b_z \end{bmatrix}^{-1}$$

  - This is accomplished by imposing some conditions

Guidelines for the derivation of CIE 1931 SO

1. The function $\bar{y}(\lambda)$ must be equal to the luminosity function of the eye V(λ)

   $\bar{y}(\lambda) = V(\lambda)$

   this sets a relation among 3 coefficients

   $a_{11} + a_{12} + a_{13} = const.$

2. The constant spectrum of white, E(λ)=1, should have equal tristimulus values

$$\sum_{i=1}^{N} \bar{x}(\lambda_i) = \sum_{i=1}^{N} \bar{y}(\lambda_i) = \sum_{i=1}^{N} \bar{z}(\lambda_i)$$

$$\sum_{i=1}^{N} \bar{x}(\lambda_i) = a_{11} \sum_{i=1}^{N} \bar{r}(\lambda_i) + a_{12} \sum_{i=1}^{N} \bar{g}(\lambda_i) + a_{13} \sum_{i=1}^{N} \bar{b}(\lambda_i)$$

but

$$\sum_{i=1}^{N} \bar{r}(\lambda_i) = \sum_{i=1}^{N} \bar{g}(\lambda_i) = \sum_{i=1}^{N} \bar{b}(\lambda_i) = S$$

thus

$$\sum_{i=1}^{N} \bar{x}(\lambda_i) = (a_{11} + a_{12} + a_{13})S$$

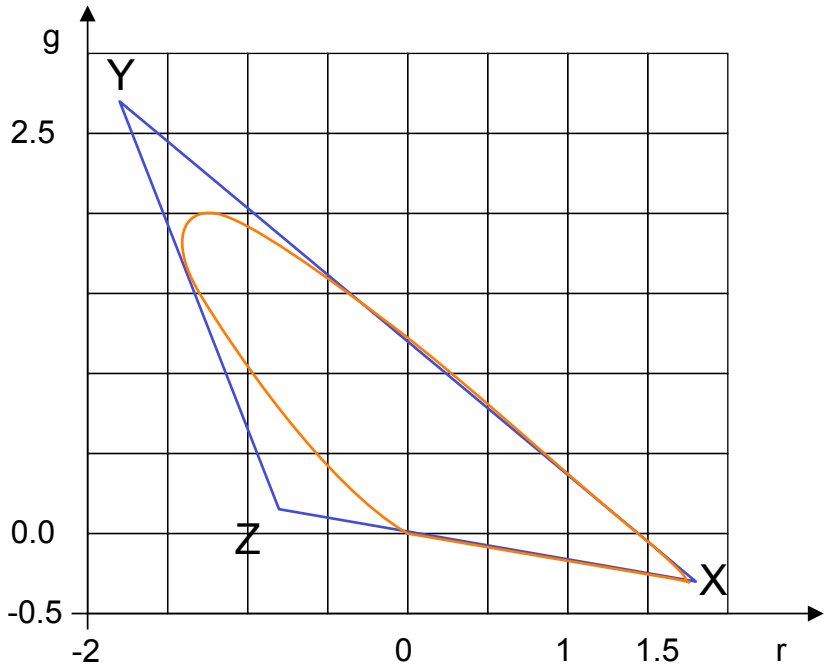
.....

$$\Rightarrow \quad (a_{11} + a_{12} + a_{13}) = (a_{21} + a_{22} + a_{23}) = (a_{31} + a_{32} + a_{33}) = \sigma$$

[Ref: Color vision and colorimetry, D. Malacara]

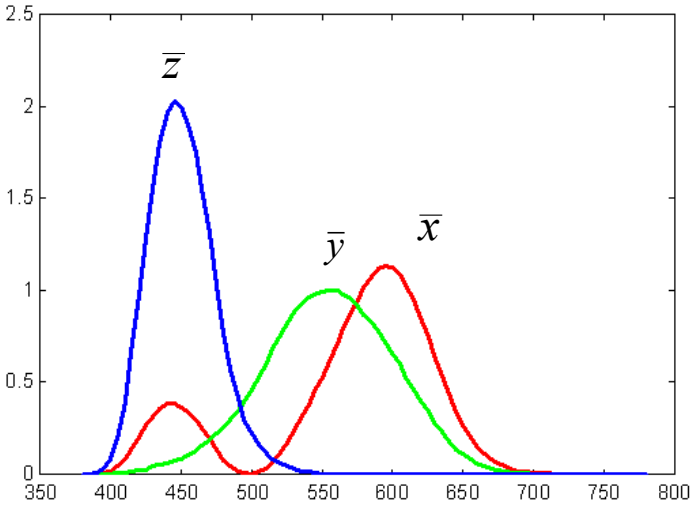
Guidelines for the derivation of CIE 1931 SO

3. The line joining X and Y be tangent to the curve on the red side
   - In this way, a linear combination of X and Y is sufficient to describe those colors without any Z
   - This introduces other conditions on a_{31}, a_{32}, a_{33} and sets their values
4. No values of $\bar{x}(\lambda)$ is negative
   - This adds a condition relating a11, a12 and a13, whose sum must be equal to known constant σ. This leaves one degree of freedom that is used to set the area of the XYZ triangle at its minimum



From rgb to xyz

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} 0.41846 & -0.1586 & -0.08283 \\ -0.09117 & 0.25243 & 0.01571 \\ 0.00092 & -0.00255 & 0.17860 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



rgb2xyz

- Chromaticity coordinates

$$x = \frac{0.49r + 0.31g + 0.2b}{0.66697r + 1.1324g + 1.20063b}$$
$$y = \frac{0.17697r + 0.81240g + 0.01063b}{0.66697r + 1.1324g + 1.20063b}$$
$$z = \frac{0.0r + 0.01g + 0.99b}{0.66697r + 1.1324g + 1.20063b}$$

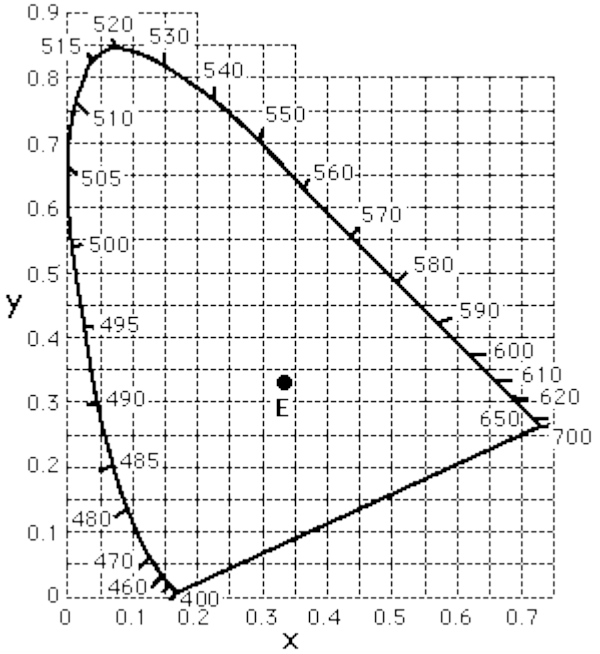
- Tristimulus values

$$X = \frac{x}{y}V \quad Y = V \quad Z = \frac{z}{y}V$$

- CMF

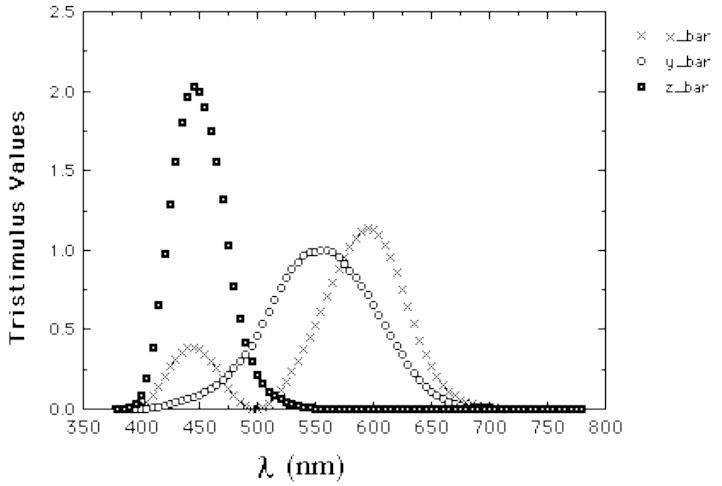
$$\bar{x}(\lambda) = \frac{x(\lambda)}{y(\lambda)}V(\lambda)$$
$$\bar{y}(\lambda) = V(\lambda)$$
$$\bar{z}(\lambda) = \frac{z(\lambda)}{y(\lambda)}V(\lambda)$$

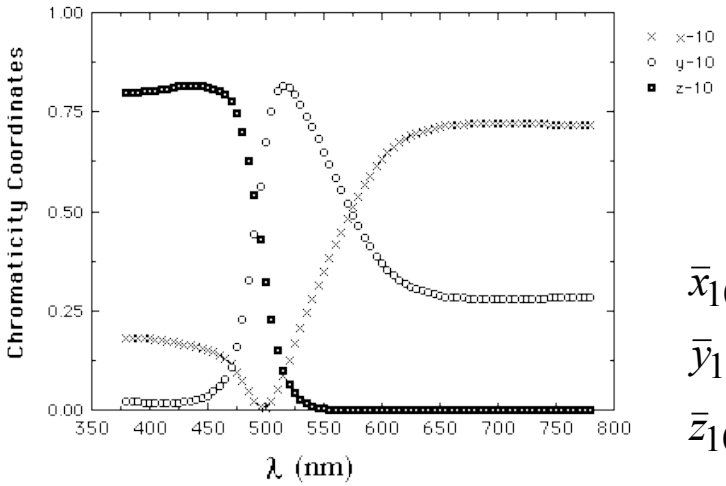
(x,y) chromaticity diagram



$$x_E = y_E = \frac{1}{3}$$

CIE 1964 SO





Features

- 10 degrees field
- Extended set of wavelengths (390 to 830 nm)
- r,g,b CMFs obtained directly from the observations
  - Measures of the radiant power of each monochromatic test stimulus
- High illumination intensity
  - To minimize rods intrusion
- Data extrapolated at 1nm resolution

$$\bar{x}_{10}(\lambda) = 0.341080\bar{r}_{10}(\lambda) + 0.189145\bar{g}_{10}(\lambda) + 0.387529\bar{b}_{10}(\lambda)$$

$$\bar{y}_{10}(\lambda) = 0.139058\bar{r}_{10}(\lambda) + 0.837460\bar{g}_{10}(\lambda) + 0.073316\bar{b}_{10}(\lambda)$$

$$\bar{z}_{10}(\lambda) = 0.0\bar{r}_{10}(\lambda) + 0.039553\bar{g}_{10}(\lambda) + 2.026200\bar{b}_{10}(\lambda)$$

CIE Chromaticity Coordinates

- (X,Y,Z) tristimulus values

$$X = \int P_\lambda \bar{x}(\lambda) d\lambda$$

$$Y = \int P_\lambda \bar{y}(\lambda) d\lambda$$

$$Z = \int P_\lambda \bar{z}(\lambda) d\lambda$$

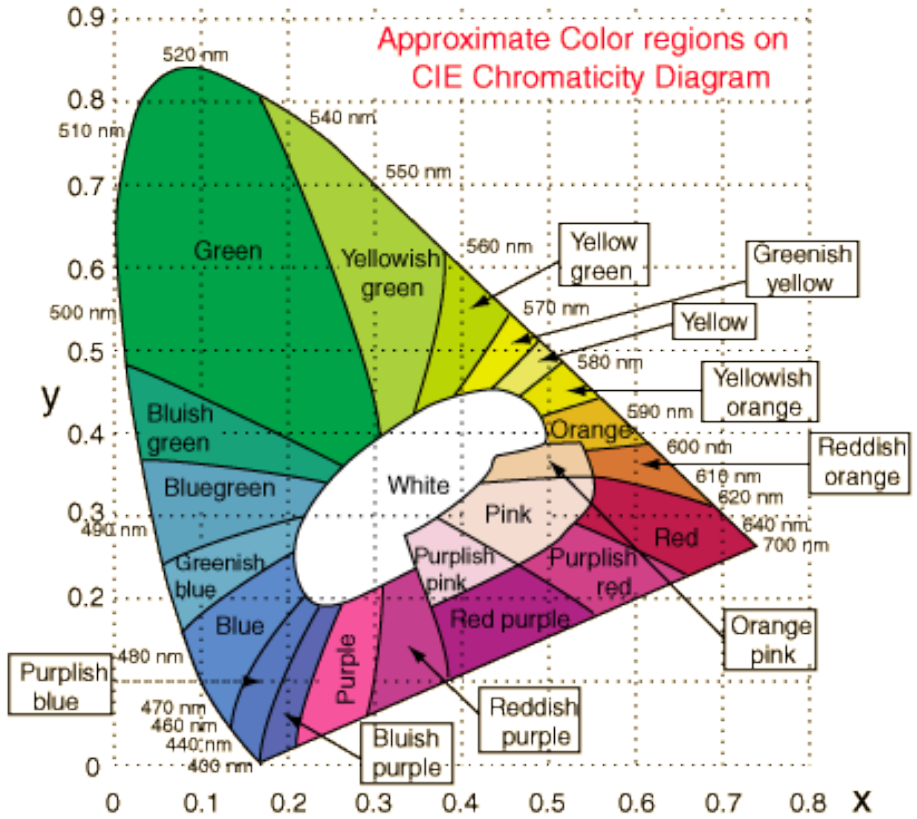
- Chromaticity coordinates

$$x(\lambda) = \frac{\bar{x}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

$$y(\lambda) = \frac{\bar{y}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

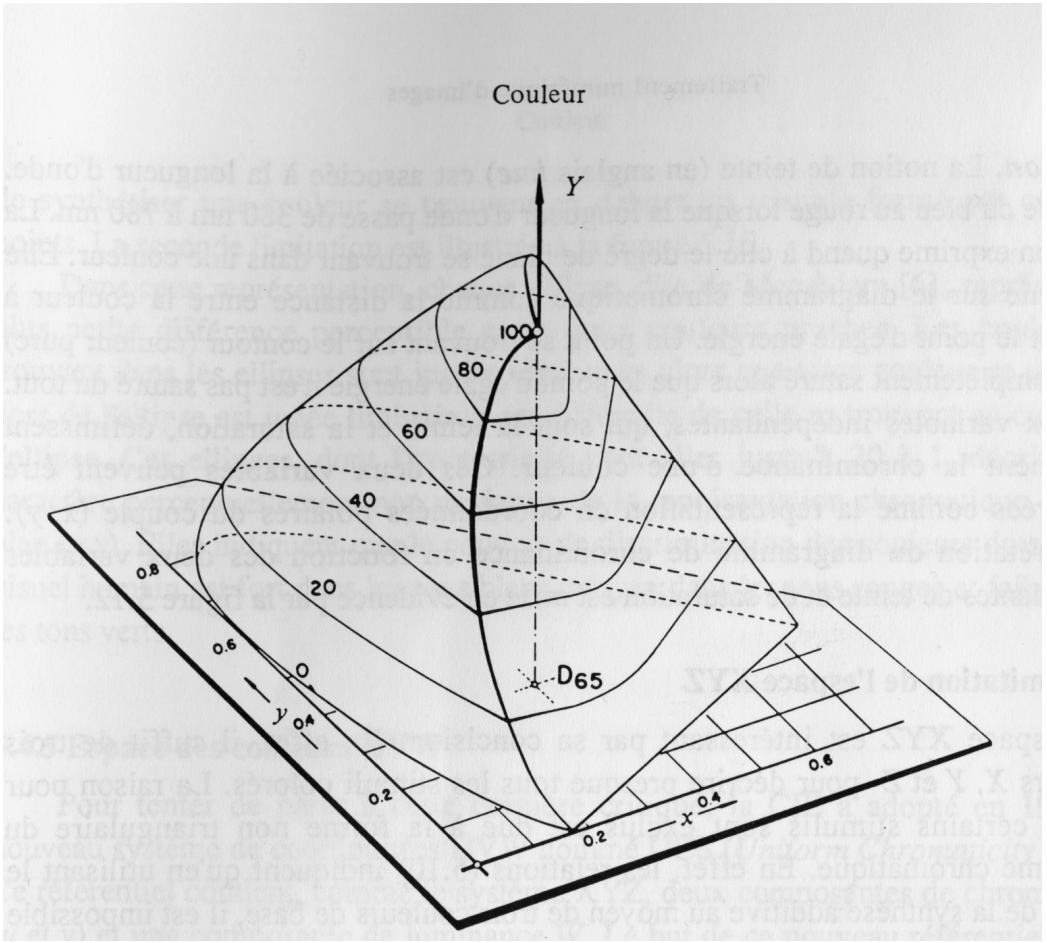
$$z(\lambda) = \frac{\bar{z}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

$$x(\lambda) + y(\lambda) + z(\lambda) = 1$$

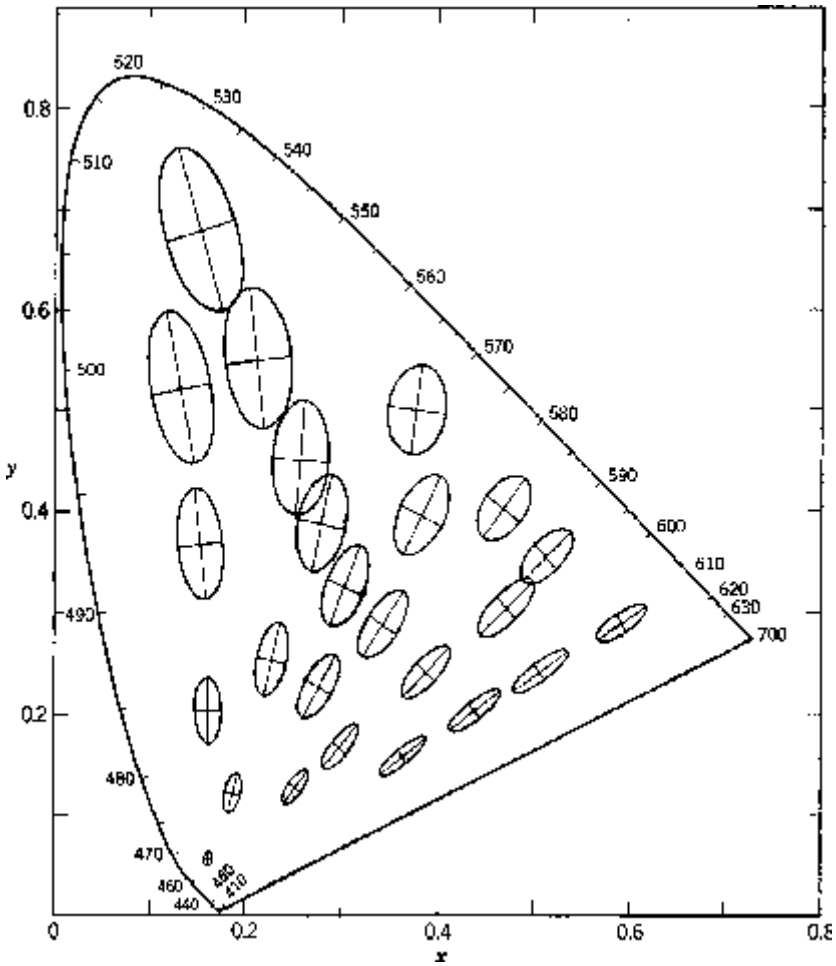


x - y chromaticity diagram

xyY



Mac Adams' ellipses



The ellipses represent a **constant perceptual color stimulus**, at a constant luminance, at various positions and in various directions, in the x,y diagram.

The areas of the ellipses vary greatly.

This means that the XYZ colorspace (as the RGB color space) is *not perceptually uniform*.

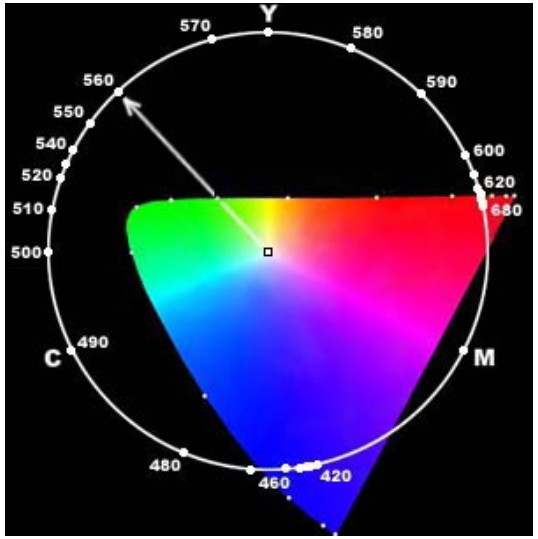
To avoid this nonuniformity, CIE recommended a new CIE 1964 UCS (Uniform-Chromaticity Scale) diagram, to be used with constant luminance levels.

In perceptually uniform colorspaces, the size of the MacAdams' ellipses are more uniform and the eccentricity is lower.

Perceptually uniform Colorspaces

- CIE 1960 Luv colorspace
  - reversible transformation

$$u = \frac{4X}{X+15Y+3Z} = \frac{4x}{-2x+12y+3}$$
$$v = \frac{6Y}{X+15Y+3Z} = \frac{6x}{-2x+12y+3}$$



- CIE 1976 L*u*v* (CIELUV)

$$u' = u$$
$$v' = 1.5v$$

L*: *perceived* lightness

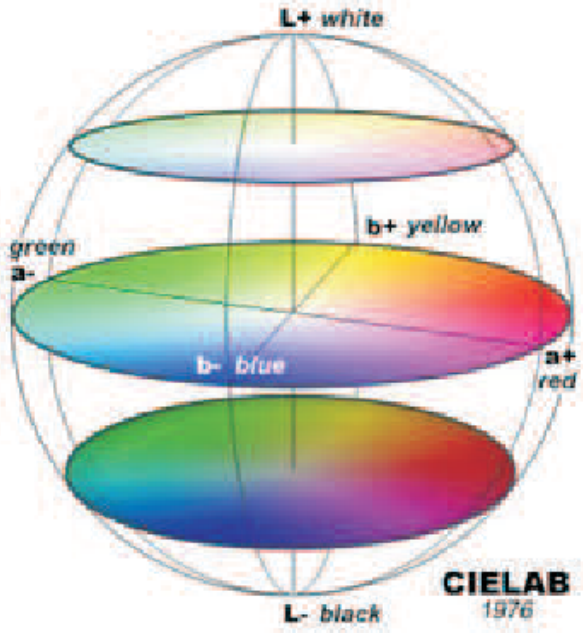
$$L^* = 116\left(\frac{Y}{Y_n}\right)^{1/3} - 16$$
$$u^* = 13L^*(u' - u'_n)$$
$$v^* = 13L^*(v' - v'_n)$$

$$u' = \frac{4X}{X+15Y+3Z} = \frac{4x}{-2+12y+3}$$
$$v' = \frac{9Y}{X+15Y+3Z} = \frac{9y}{-2x+12y+3}$$

u'$_n$,v'$_n$: reference white

Perceptually uniform Color models

CIE 1976 L*a*b* (CIELAB)



For: $\frac{Y}{Y_n}, \frac{X}{X_n}, \frac{Z}{Z_n} \geq 0.01$

$$
\begin{aligned}
L^* &= 116(Y/Y_n)^{1/3} - 16 \\
a^* &= 500[(X/X_n)^{1/3} - (Y/Y_n)^{1/3}] \\
b^* &= 200[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}]
\end{aligned}
$$

otherwise

$$
\begin{aligned}
L^* &= 116\left[f\left(\frac{Y}{Y_n}\right) - \frac{16}{116}\right] \\
a^* &= 500\left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right] \\
b^* &= 200\left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right]
\end{aligned}
$$

$$
f\left(\frac{Y}{Y_n}\right) = \begin{cases} \left(\frac{Y}{Y_n}\right)^{1/3} & for \quad \frac{Y}{Y_n} > 0.008856 \\ 7.787\frac{Y}{Y_n} + \frac{16}{116} & for \quad \frac{Y}{Y_n} \leq 0.008856 \end{cases}
$$

X_n, Y_n, Z_n : *reference white*

Tristimulus values for a nominally white object-color stimulus. Usually, it corresponds to the spectral radiance power od one of the CIE standard illuminants (as D65 or A), reflected into the observer's eye by a perfect reflecting diffuser. Under these conditions, X_n, Y_n, Z_n are the tristimulus values of the standard illuminant with Y_n=100.

Hint: the diffuse light depends on both the physical properties of the surface and the illuminant

Perceptual correlates

- Color difference formula

$$\Delta E^*_{u,v} = \left[\left(\Delta L^*\right)^2 + \left(\Delta u^*\right)^2 + \left(\Delta v^*\right)^2 \right]^{1/2}$$

- Perceptual correlates

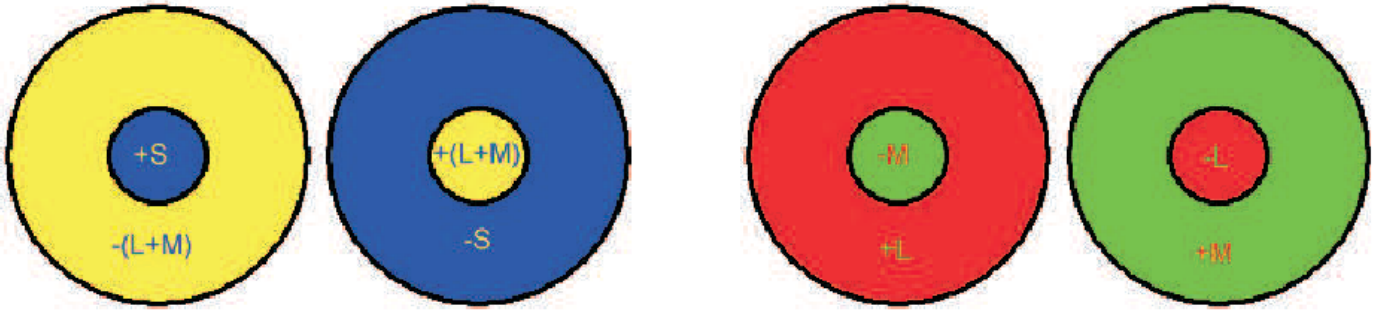
L^*: lightness

$C^*_{u,v} = \left[(u^*)^2 + (v^*)^2 \right]^{1/2}$: chroma

$s^*_{u,v} = \dfrac{C^*_{u,v}}{L^*}$: saturation

Opponent color models

- Underlying model: *opponent channels*



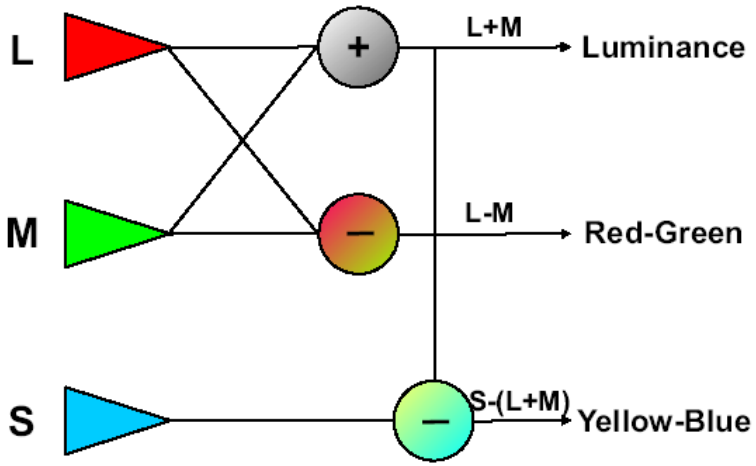
(a) (b)

Example of typical center-surround antagonistic receptive fields: (a) on-center yellow-blue receptive fields; (b) on-center red-green receptive fields.

Because of the fact that the L, M and S cones have different spectral sensitivities, are in different numbers and have different spatial distributions across the retina, the respective receptive fields have quite different properties.

Experimental evidence: color after-image, non existence of colors like “greenish-read” or “yellowish-blue”

Opponent color channels



Cone interconnections in the retina leading to opponent color channels

As a convenient simplification, the existence of three types of color receptive fields is assumed, which are called *opponent channels*.

The black-white or *achromatic* channel results from the sum of the signals coming from L and M cones (L + M). It has the highest spatial resolution.

The *red-green* channel is mainly the result of the M cones signals being subtracted from those of the L cones (L - M). Its spatial resolution is slightly lower than that of the achromatic channel (L + M - S).

Finally the *yellow-blue* channel results from the addition of L and M and subtraction of S cone signals. It has the lowest spatial resolution.

Additive Color Mixing with CIE

The result of adding two colors of light can be worked out as a weighted average of the CIE chromaticity coordinates for the two colors. The weighting factors involve the brightness parameter Y. If the coordinates of the two colors are

(x1,y1) with brightness Y1

(x2,y2) with brightness Y2

then the additive mixture color coordinates are

$$x_3 = \frac{Y_1}{Y_1 + Y_2} x_1 + \frac{Y_2}{Y_1 + Y_2} x_2$$

$$y_3 = \frac{Y_1}{Y_1 + Y_2} y_1 + \frac{Y_2}{Y_1 + Y_2} y_2$$

This linear procedure is valid only if the colors are relatively close to each other in value.