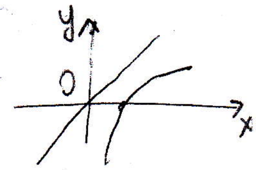
EX2) Studiare il grafico di $f(x) = -x^{-1}\ln x + 1 = \dfrac{x - \ln x}{x}$

Ris

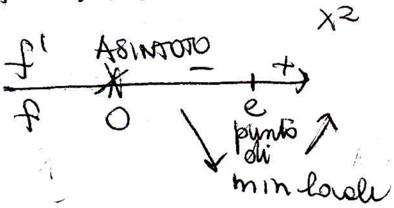
- dominio $A = \{x \in \mathbb{R} : x > 0\}$
- segno $-\dfrac{\ln x}{x} + 1 > 0$ sse $\dfrac{\ln x}{x} < 1$ sse $\ln x < x$ sse $\forall x > 0$



$\Rightarrow f(x) > 0 \quad \forall x > 0$

- nessuna intersezione con gli assi
- $f \in C^\infty(A)$ $\quad f'(x) = \left(-\dfrac{\ln x}{x} + 1\right)' = \dfrac{\ln x - 1}{x^2}$

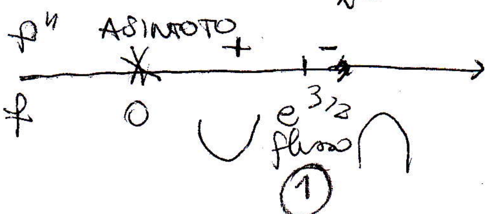
$f'(x) > 0$ sse $\dfrac{\ln x - 1}{x^2} > 0$ sse $\ln x > 1$ sse $x > e$



$f(e) = -\dfrac{\ln e}{e} + 1 = -\dfrac{1}{e} + 1$ minimo locale

- $f''(x) = \dfrac{\frac{1}{x}x^2 - (\ln x - 1)\,2x}{x^4} = \dfrac{x + 2x - 2x\ln x}{x^4} = \dfrac{x(3 - 2\ln x)}{x^4} = \dfrac{3 - 2\ln x}{x^3}$

$f''(x) > 0$ sse $\dfrac{3 - 2\ln x}{x^3} > 0$ sse $3 - 2\ln x > 0$ sse $\ln x < \dfrac{3}{2}$ sse $0 < x < e^{3/2}$



$f(e^{3/2}) = -\dfrac{\ln e^{3/2}}{e^{3/2}} + 1 = -\dfrac{3}{2}e^{-\frac{3}{2}} + 1$

$y = 1$ asintoto orizzontale

$\lim_{x \to +\infty} -\dfrac{\ln x}{x} \quad \left[\dfrac{\infty}{\infty}\right]$

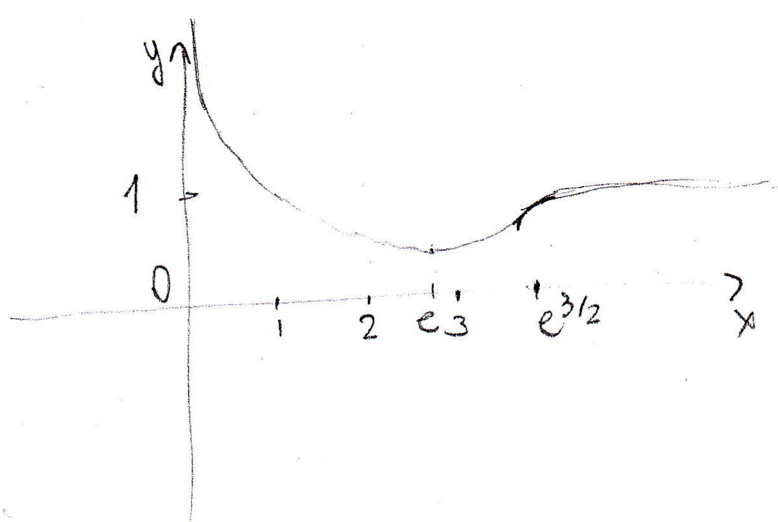
$\overset{H}{=} \lim_{x \to +\infty} \dfrac{-1/x}{1} = \lim_{x \to +\infty} -\dfrac{1}{x} = 0 \Rightarrow \lim_{x \to +\infty} -\dfrac{\ln x}{x} + 1 = +1$

$\lim_{x \to 0^+} -\dfrac{\overbrace{\ln x}^{\to -\infty}}{\underbrace{x}_{\to 0^+}} + 1 = +\infty \Rightarrow f$ non è superiormente limitata

$x = 0$ asintoto verticale

$\Rightarrow x_0 = e$ punto di minimo assoluto

Grafico



EX2 fila C