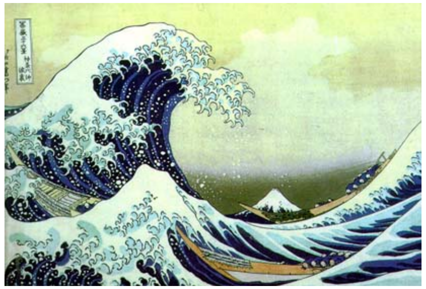


Multimedia communications

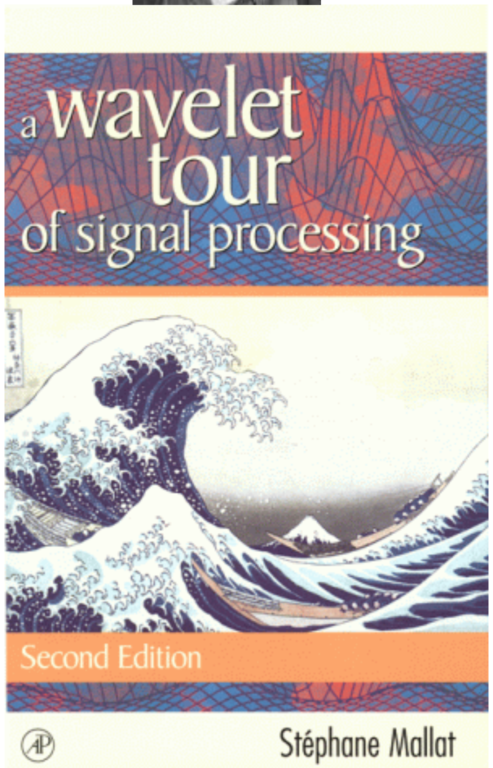
Comunicazione multimediale

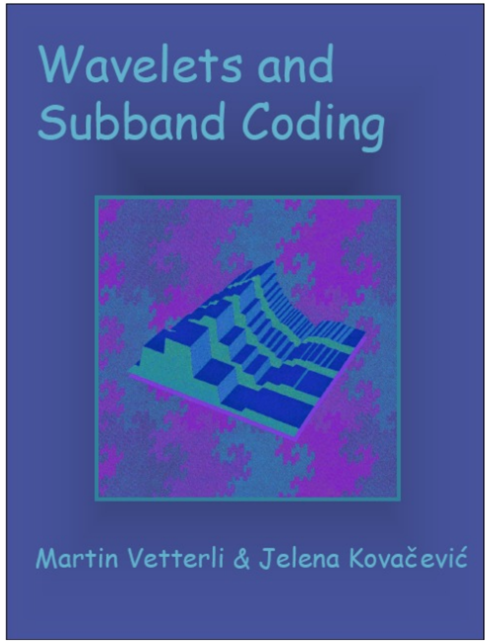
G. Menegaz

gloria.menegaz@univr.it

Course overview

- Goal
  - The course is about wavelets and multiresolution
    - Theory: 3 hours per week
      - Wed. 11.30-13.30, room B
      - Thu. 16.30-17.30, room I (+1h)
    - Laboratory
      - Wed. 14.30-16.30 (Lab. Alpha) LM32
      - Fri. 9.30-11.30 (Lab. Alpha) LM9
- Contents
  - Review of Fourier theory
  - Wavelets and multiresolution
  - Review of Information theoretic concepts
  - Applications
    - Image coding (JPEG2000)
    - Feature extraction and signal/image analysis
  - Wavelets in vision
- Exam
  - Oral form
  - Can be complemented by a project for the lab.ß





“Scale”

“Scale”

"Scale"

Telecommunications for Multimedia

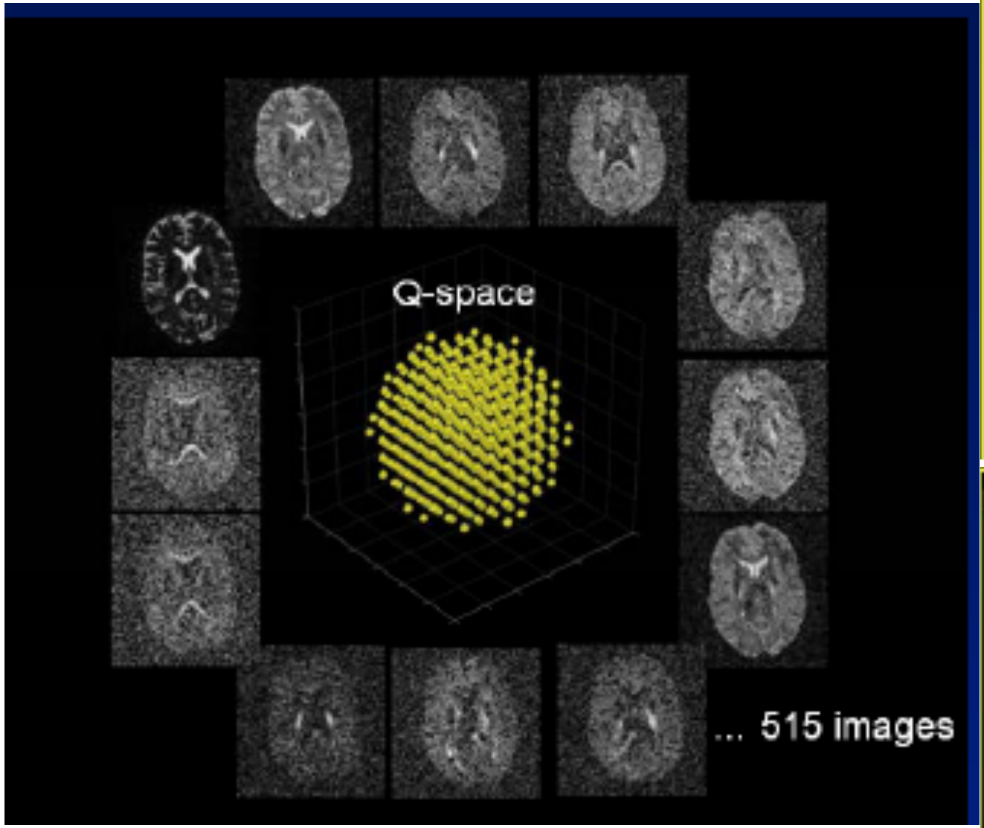
Good news

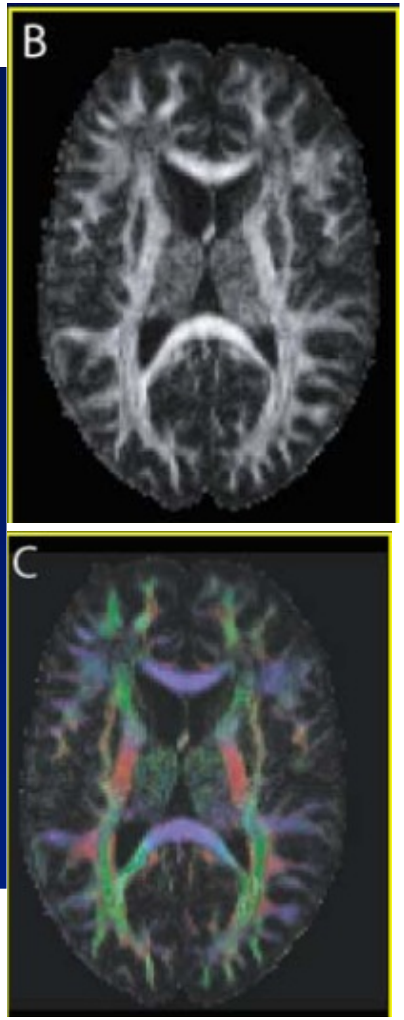
- It is fun!
- Get in touch with the state-of-the-art technology
- Convince yourself that the time spent on maths&stats was not wasted
- Learn how to map theories into applications
- Acquiring the tools for doing good research!

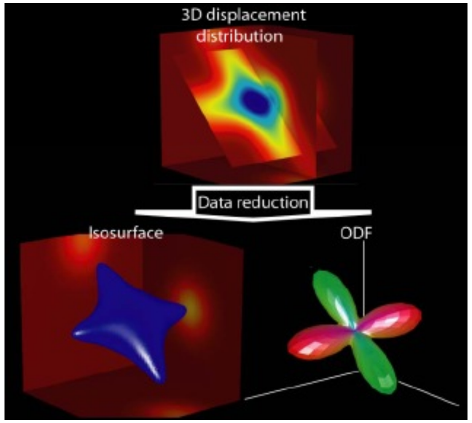
Bad news

- Some theoretical background is unavoidable
  - Mathematics
    - Fourier transform
    - Linear operators
    - Digital filters
    - Wavelet transform
  - (some) Information theory

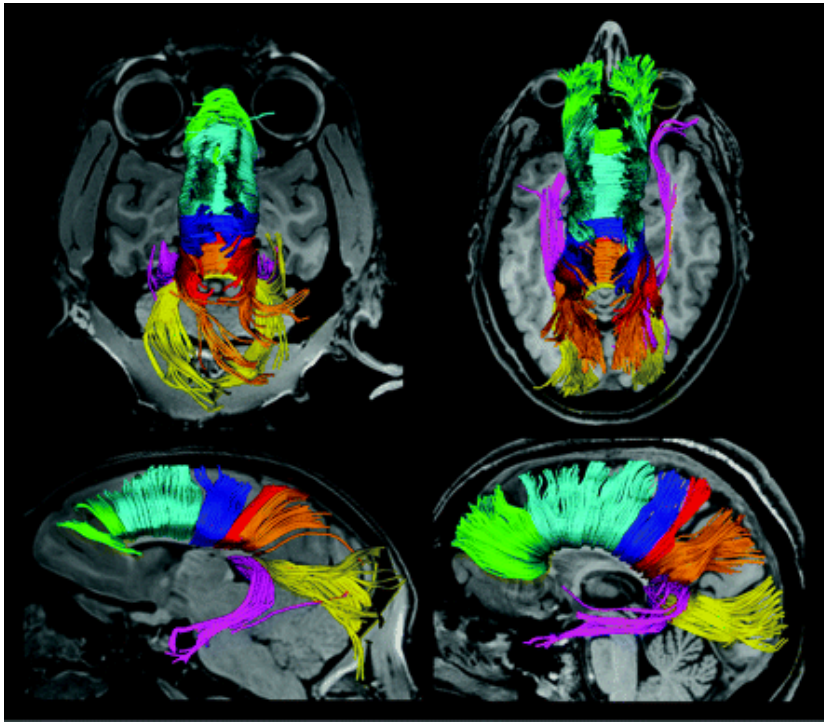
A glance on applications

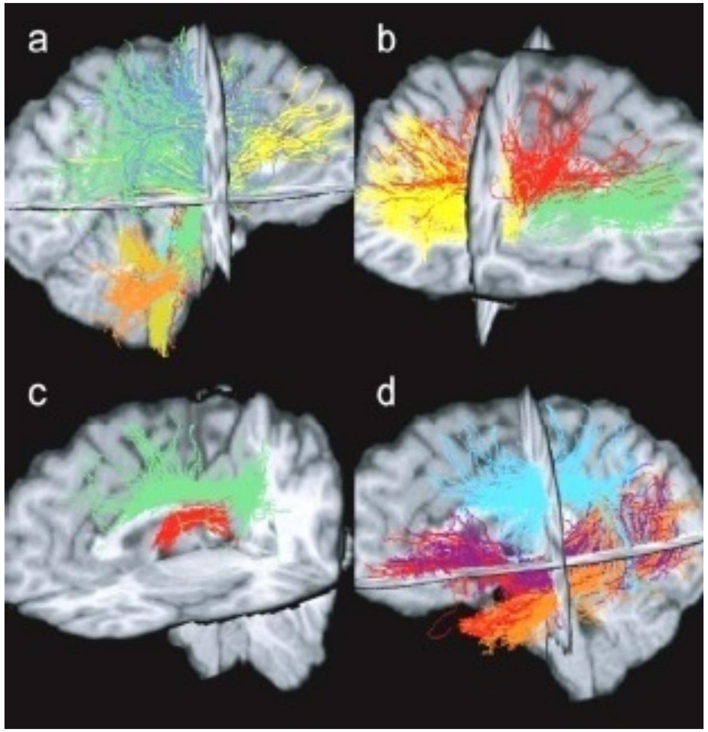




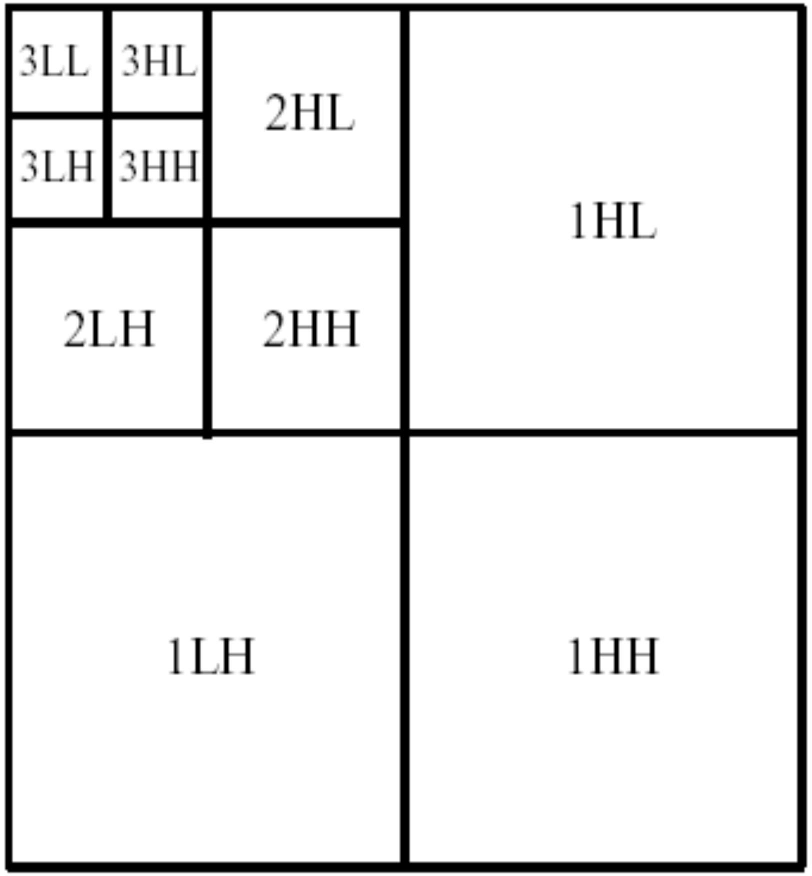


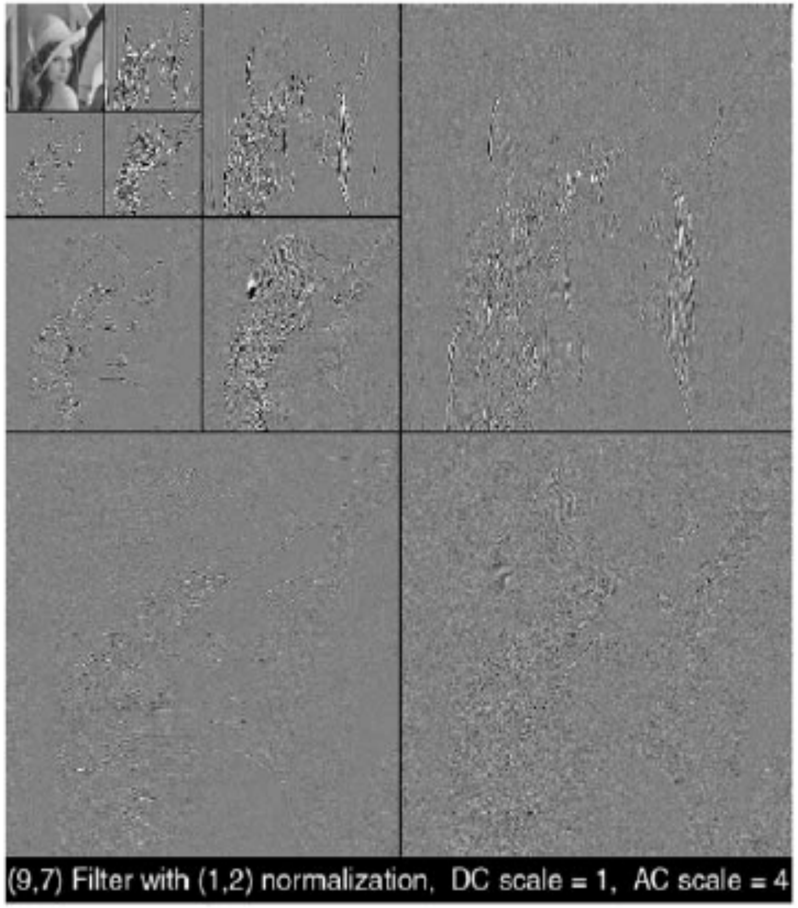
A glance on applications





JPEG2000


3LL
3HL
3LH
3HH
2HL
2LH
2HH
1HL
1LH
1HH


(9,7) Filter with (1,2) normalization, DC scale = 1, AC scale = 4

Mathematical tools

Introduction

- Sparse representations: few coefficients reveal the information we are looking for.
  - Such representations can be constructed by decomposing signals over elementary waveforms chosen in a family called a *dictionary*.
  - An orthogonal basis is a dictionary of minimum size that can yield a sparse representation if designed to concentrate the signal energy over a set of few vectors. This set gives a *geometric* signal description.
    - Signal compression and noise reduction
  - Dictionaries of vectors that are larger than bases are needed to build sparse representations of complex signals. But choosing is difficult and requires more complex algorithms.
    - Sparse representations in redundant dictionaries can improve pattern recognition, compression and noise reduction
- Basic ingredients: Fourier and Wavelet transforms
  - They decompose signals over oscillatory waveforms that reveal many signal properties and provide a path to sparse representations

Signals as functions

- CT analogue signals (real valued functions of continuous independent variables)
  - 1D: $f=f(t)$
  - 2D: $f=f(x,y)$ x,y
  - Real world signals (audio, ECG, pictures taken with an analog camera)
- DT analogue signals (real valued functions of discrete variables)
  - 1D: $f=f[k]$
  - 2D: $f=f[i,j]$
  - *Sampled* signals
- Digital signals (discrete valued functions of DT variables)
  - 1D: $y=y[k]$
  - 2D: $y=y[i,j]$
  - *Sampled and discretized* signals

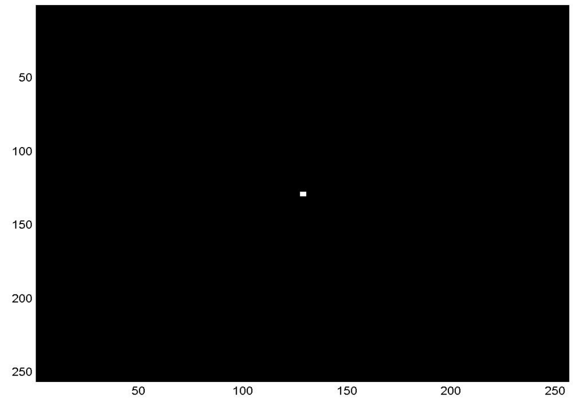
Images as functions

- Gray scale images: 2D functions
  - Domain of the functions: set of (x,y) values for which $f(x,y)$ is defined : 2D lattice $[i,j]$ defining the pixel locations
  - Set of values taken by the function : gray levels

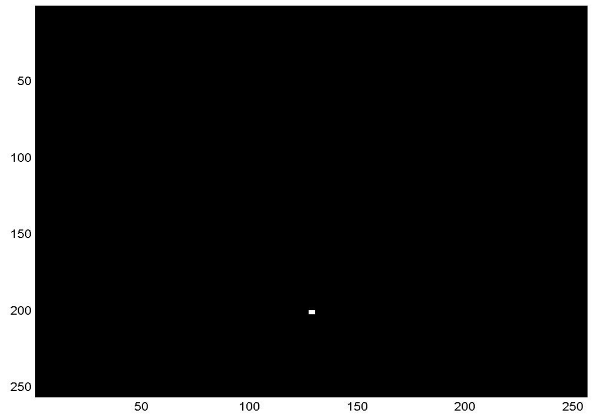
- Digital images can be seen as functions defined over a discrete domain $\{i,j: 0<i<I, 0<j<J\}$
  - I,J: number of rows (columns) of the matrix corresponding to the image
  - $f=f[i,j]$: gray level in position $[i,j]$

Example 1: δ function

$$\delta[i,j] = \begin{cases} 1 & i = j = 0 \\ 0 & i, j \neq 0; i \neq j \end{cases}$$



$$\delta[i,j-J] = \begin{cases} 1 & i = 0; j = J \\ 0 & otherwise \end{cases}$$



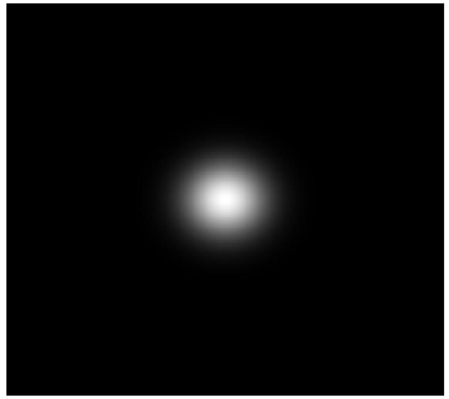
Example 2: Gaussian

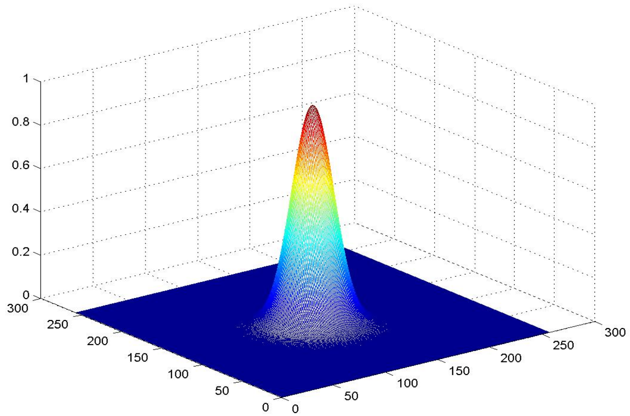
Continuous function

$$f(x,y) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{x^2+y^2}{2\sigma^2}}$$

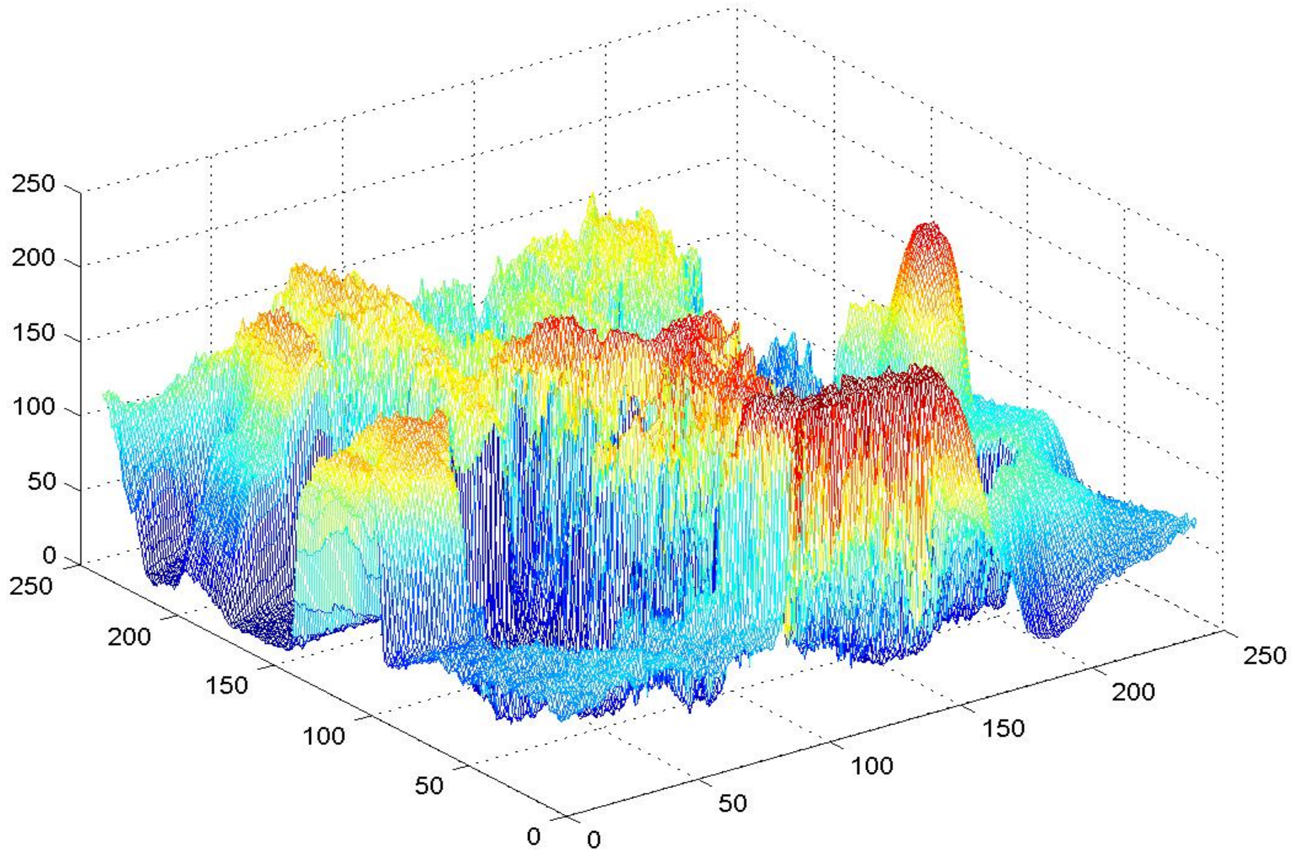
Discrete version

$$f[i,j] = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{i^2+j^2}{2\sigma^2}}$$





Example 3: Natural image



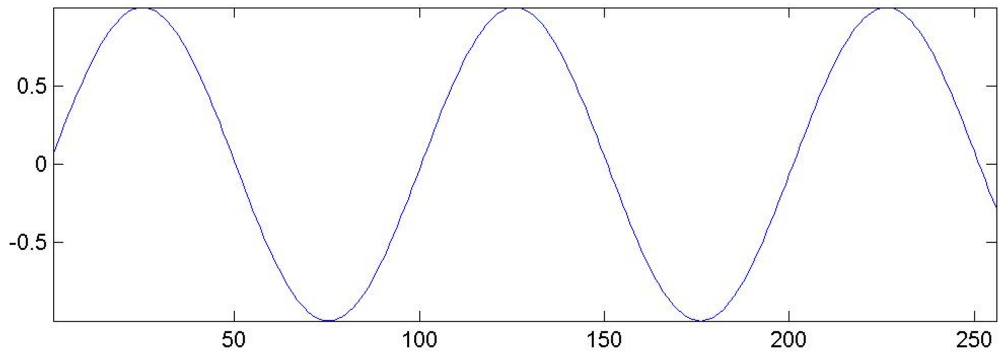
Example 3: Natural image

The Fourier kingdom

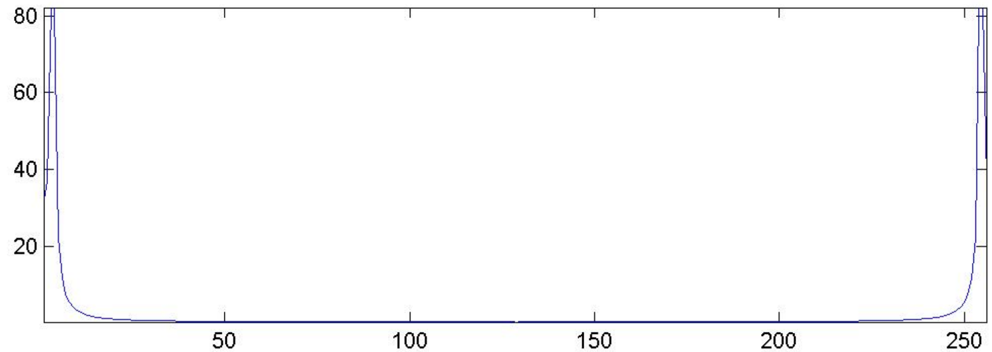
- Frequency domain characterization of signals

$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} dt$$

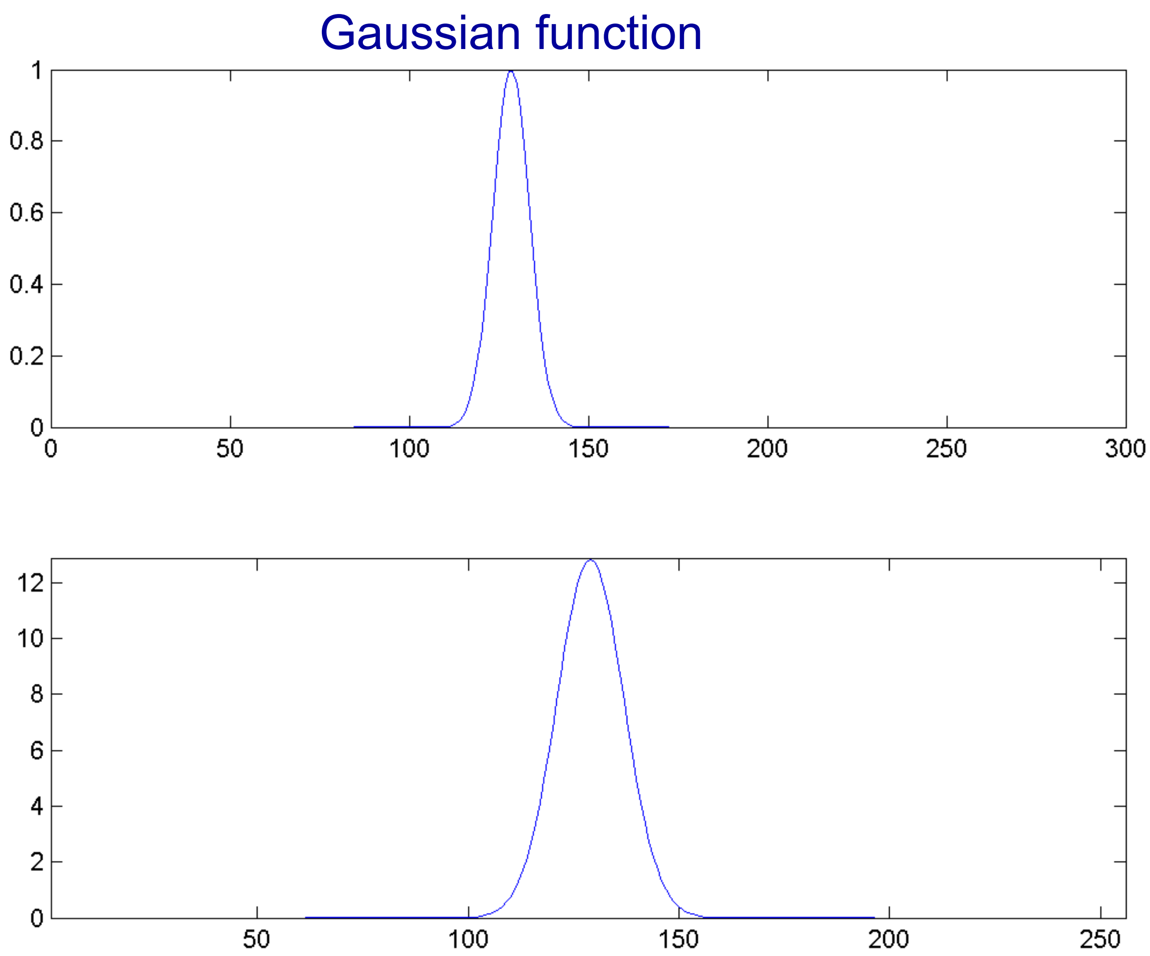


Signal domain



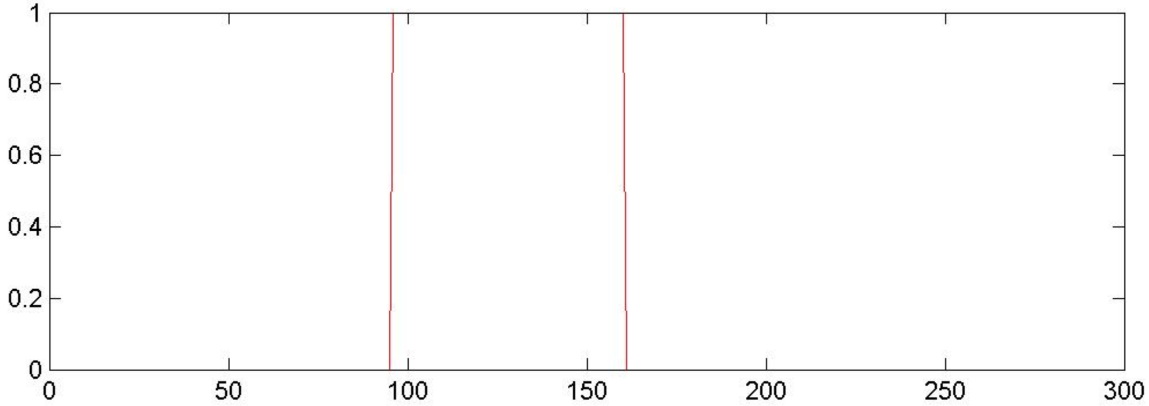
Frequency domain

The Fourier kingdom

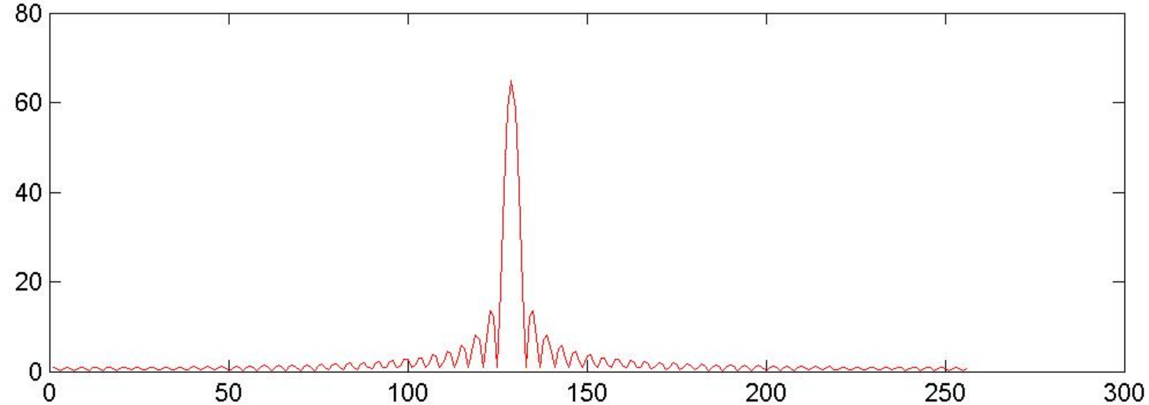


The Fourier kingdom

rect function



sinc function



2D Fourier transform

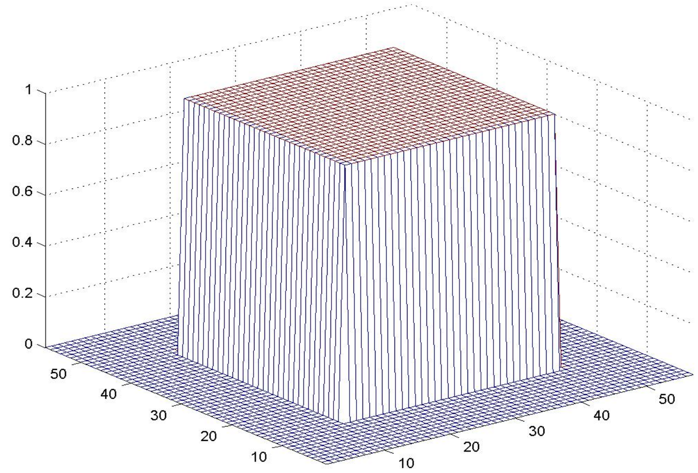
$$\hat{f}(\omega_x,\omega_y) = \int_{-\infty}^{+\infty} f(x,y)\, e^{-j(\omega_x x+\omega_y y)}\, dxdy$$

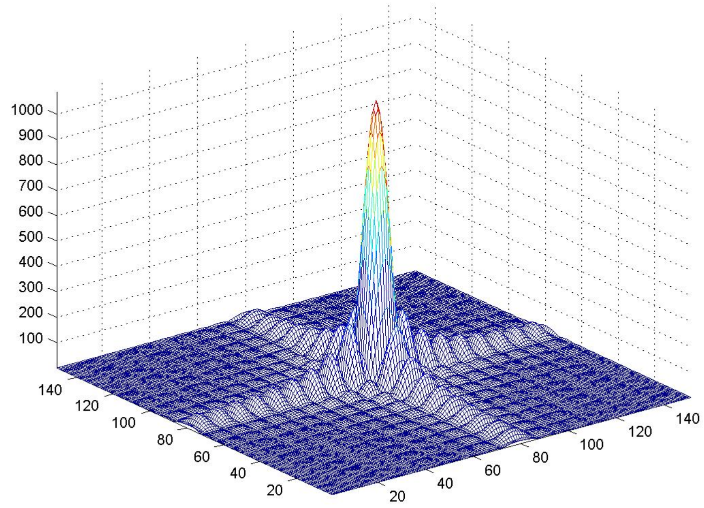
$$f(x,y) = \frac{1}{4\pi^2}\int_{-\infty}^{+\infty} \hat{f}(\omega_x,\omega_y)\, e^{j(\omega_x x+\omega_y y)}\, d\omega_x d\omega_y$$

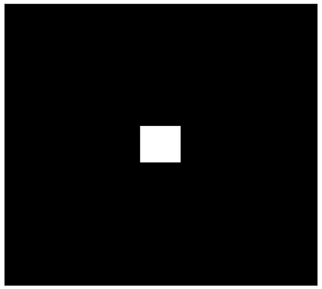
$$\iint f(x,y)\, g^*(x,y)\, dxdy = \frac{1}{4\pi^2}\iint \hat{f}(\omega_x,\omega_y)\hat{g}^*(\omega_x,\omega_y)\, d\omega_x d\omega_y$$ Parseval formula

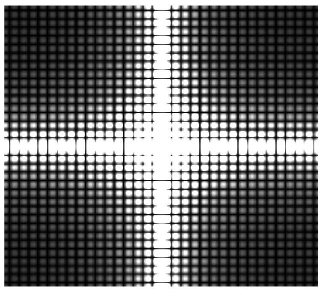
$$f = g \rightarrow \iint |f(x,y)|^2\, dxdy = \frac{1}{4\pi^2}\iint |\hat{f}(\omega_x,\omega_y)|^2\, d\omega_x d\omega_y$$ Plancherel equality

The Fourier kingdom

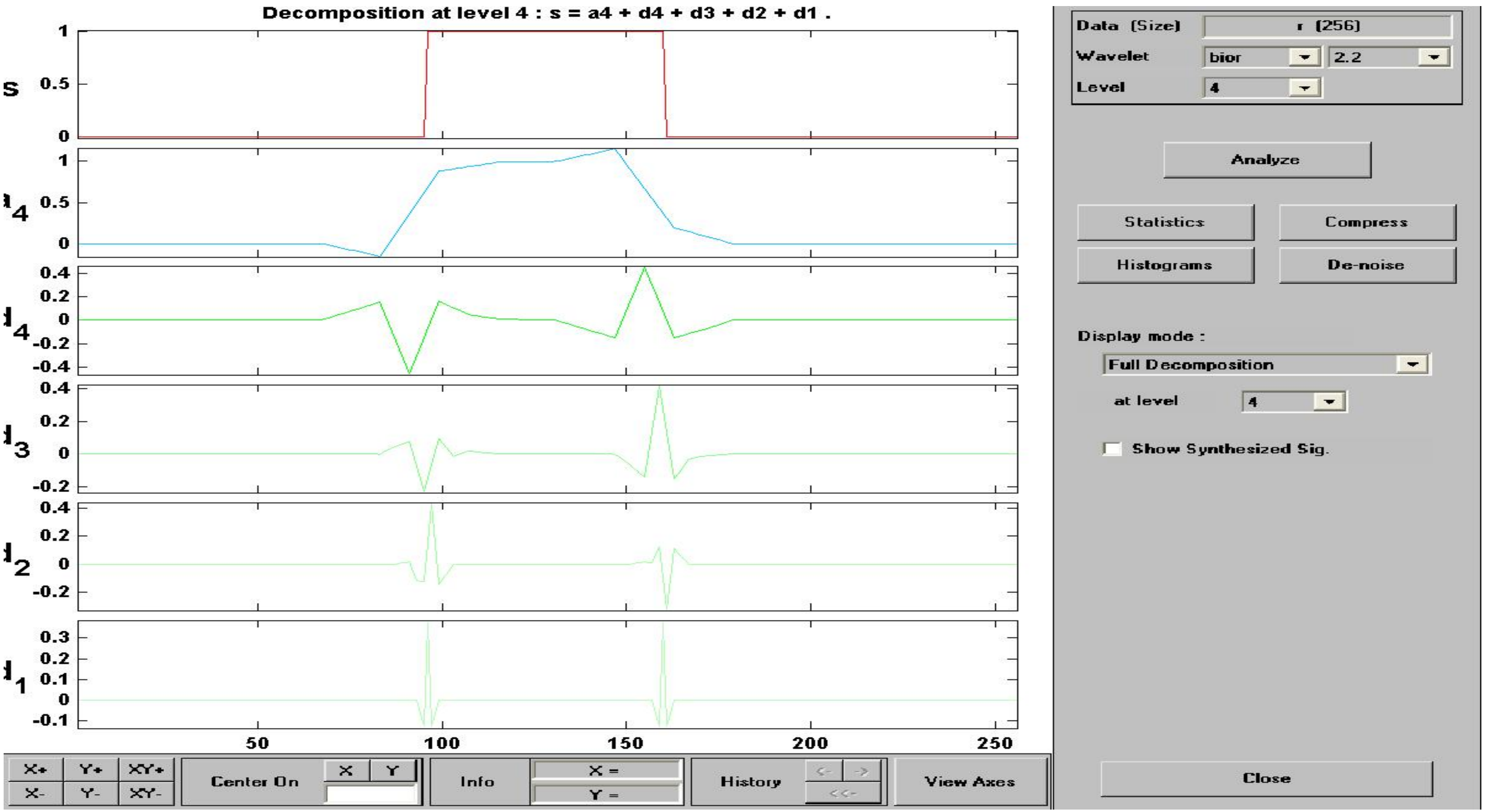




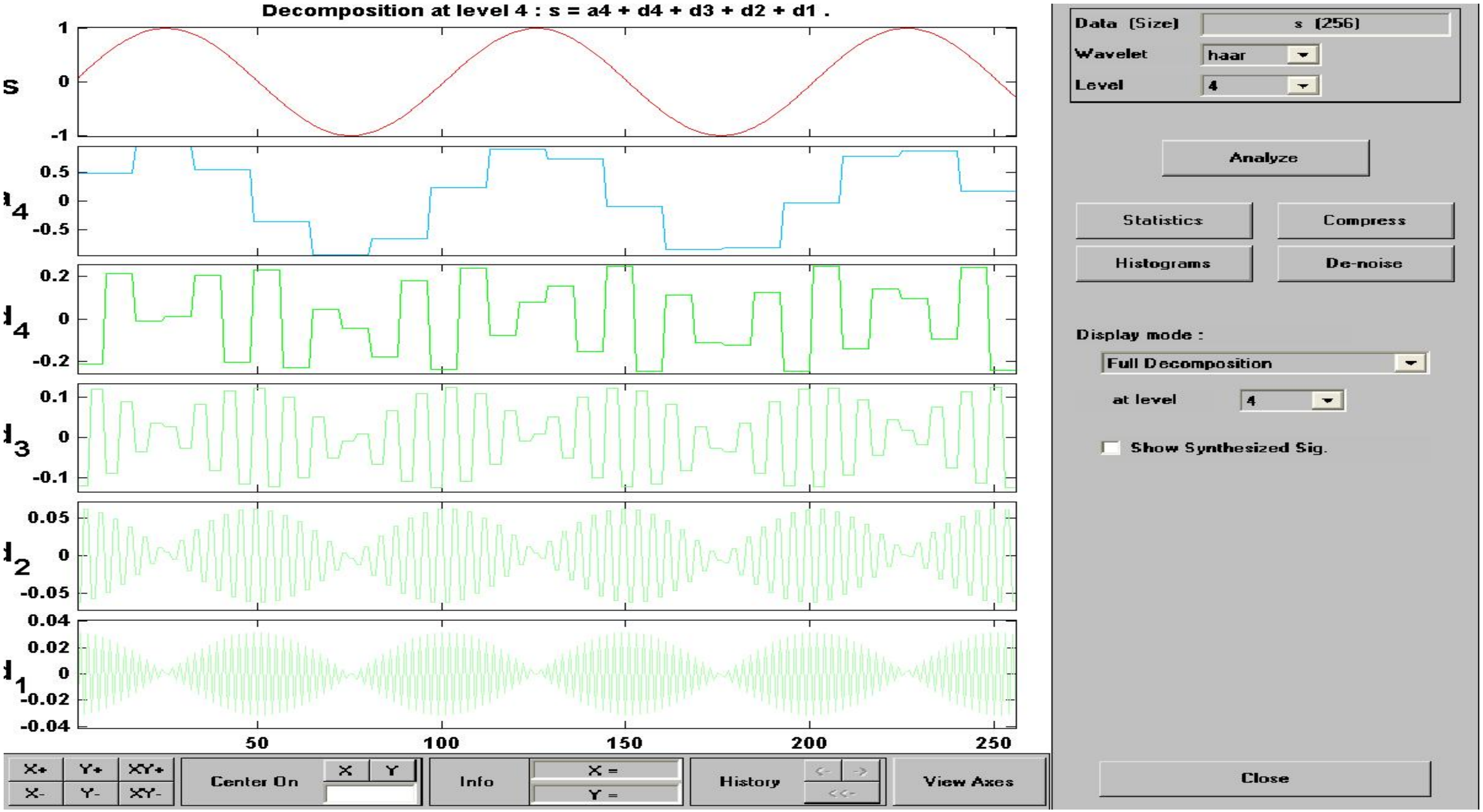




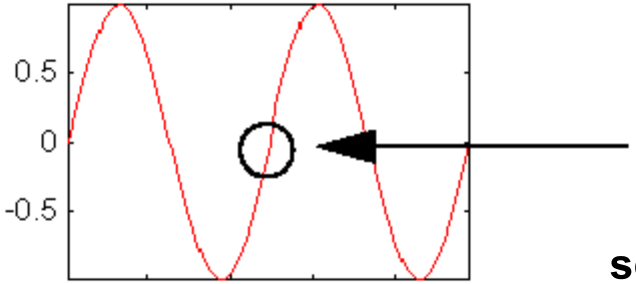
Wavelet representation



Wavelet representation

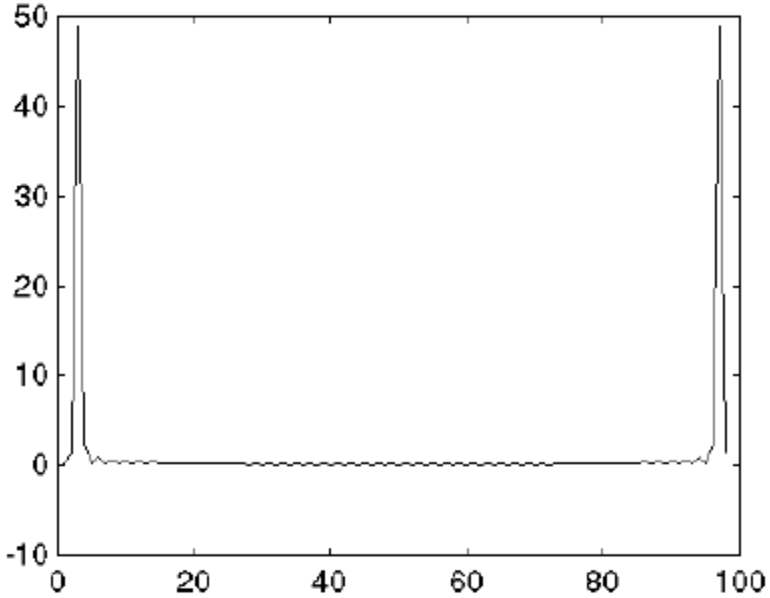


Wavelets are good for transients

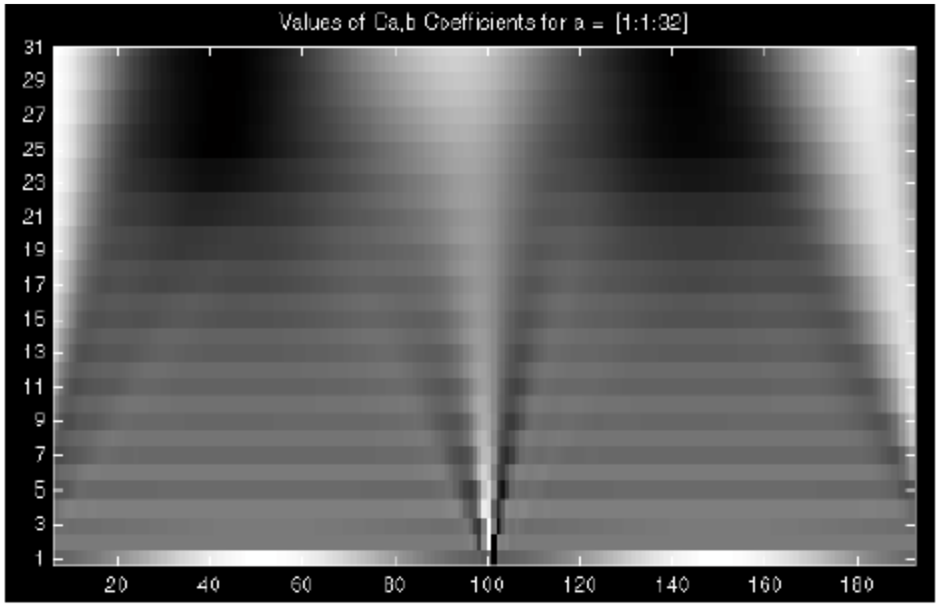


Sinusoid with a small discontinuity

scalogram

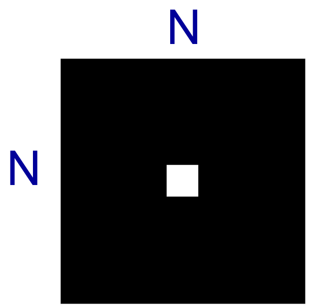


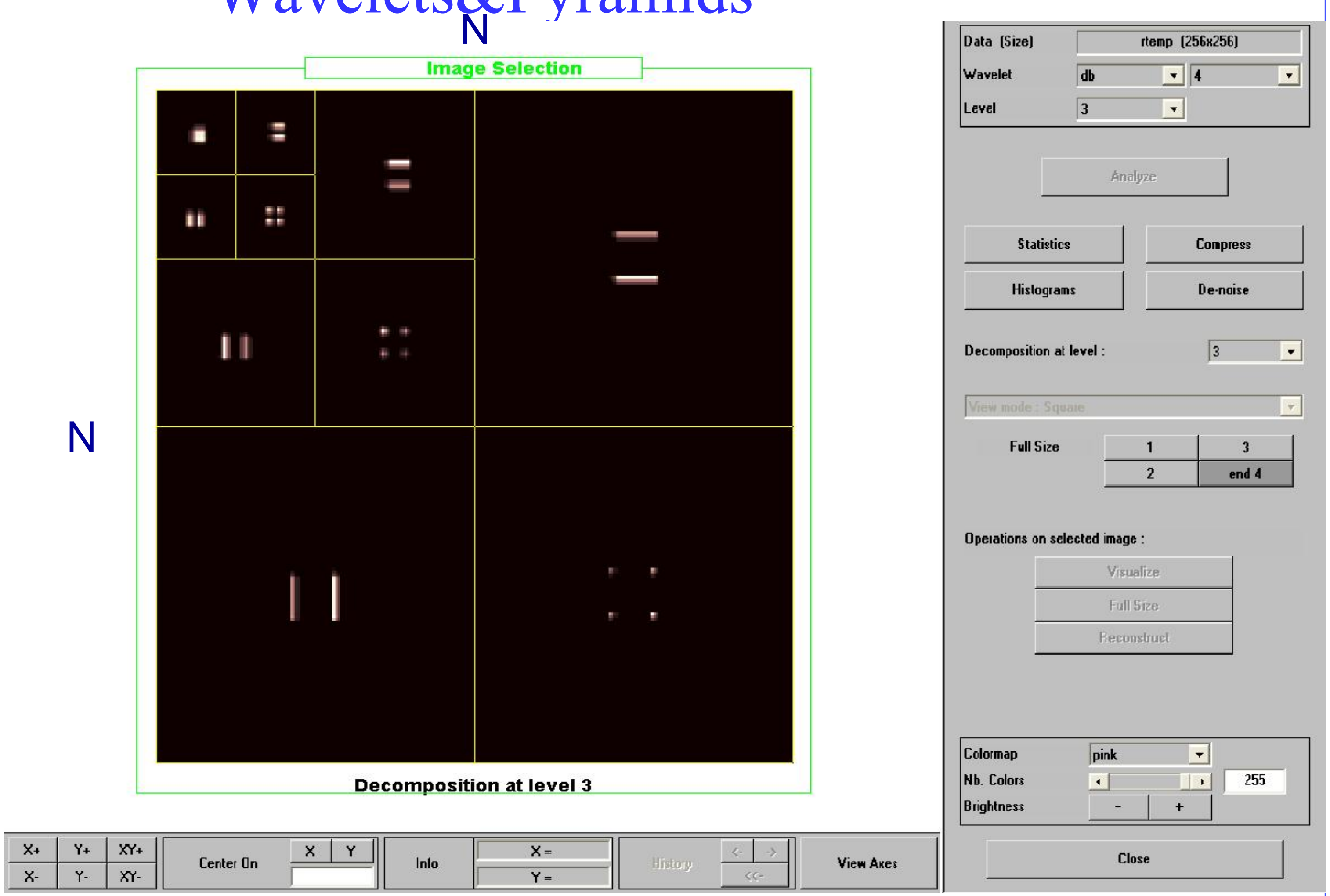
Fourier Coefficients



Wavelet Coefficients

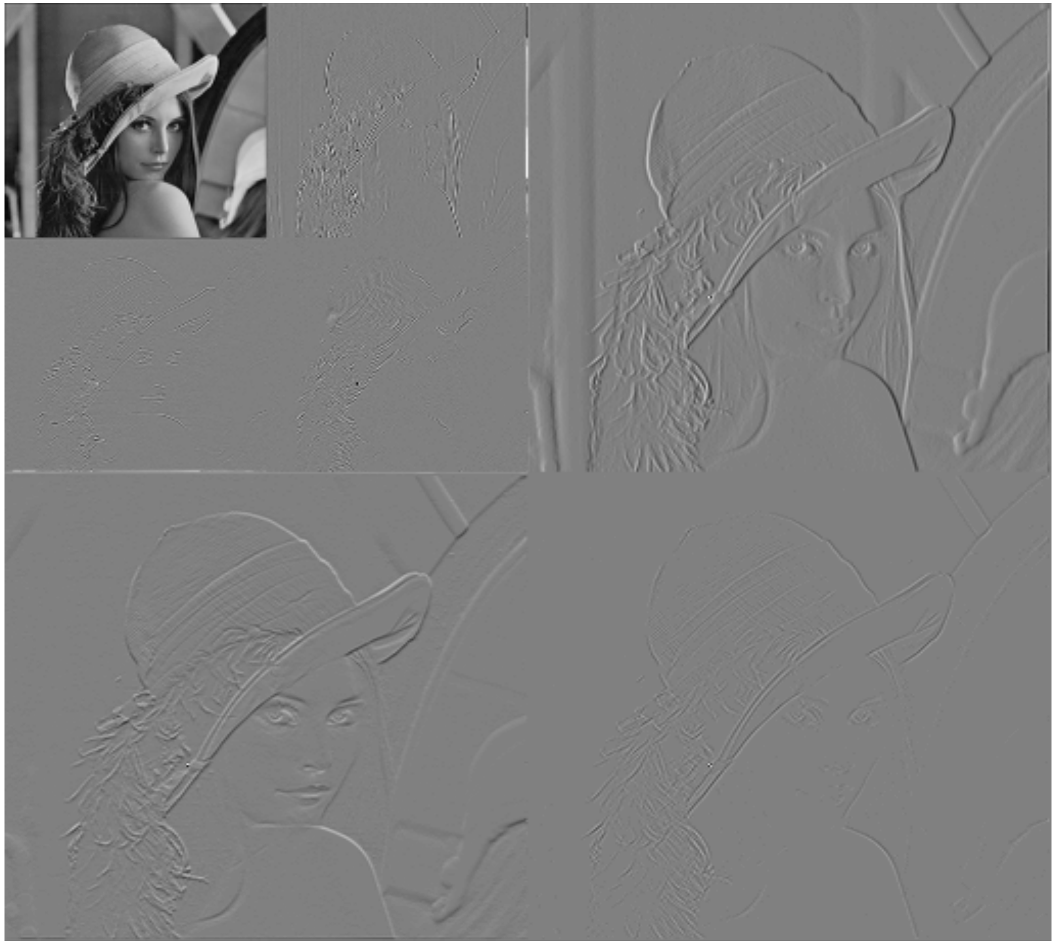
Wavelets&Pyramids



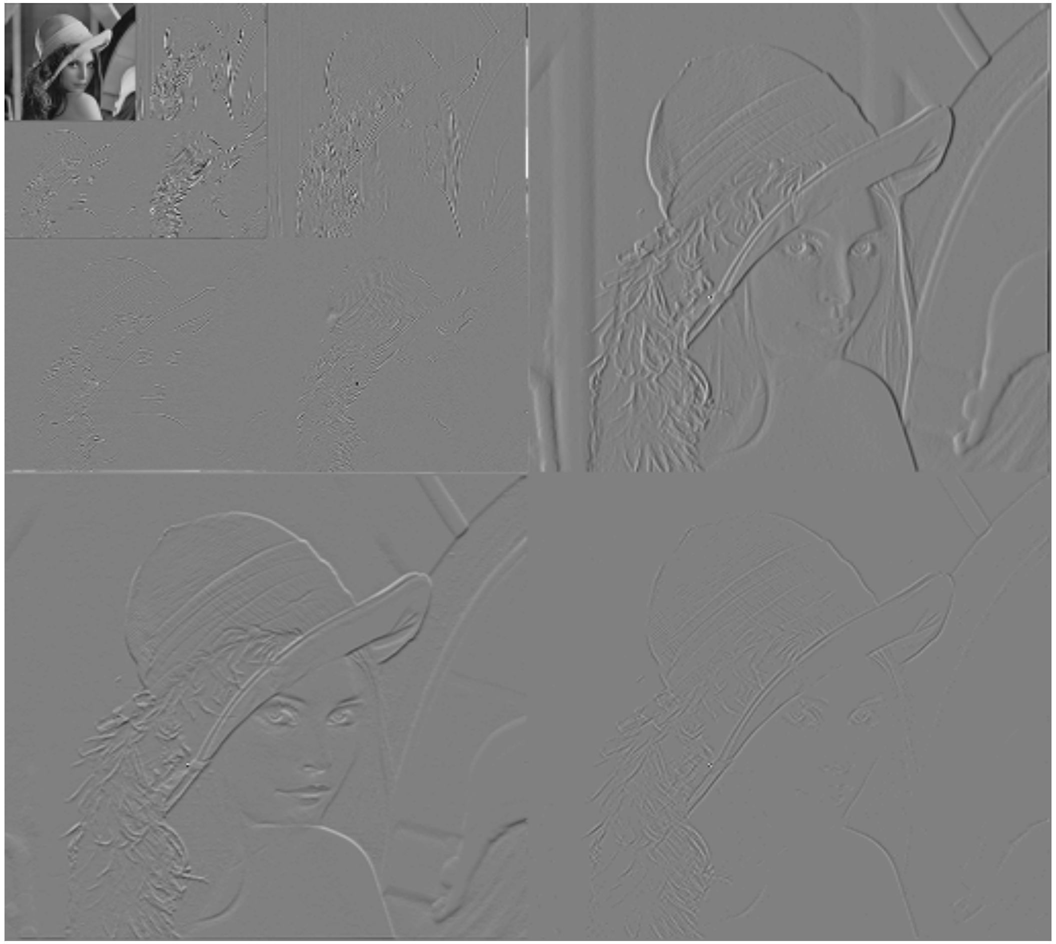


Wavelets&Pyramids

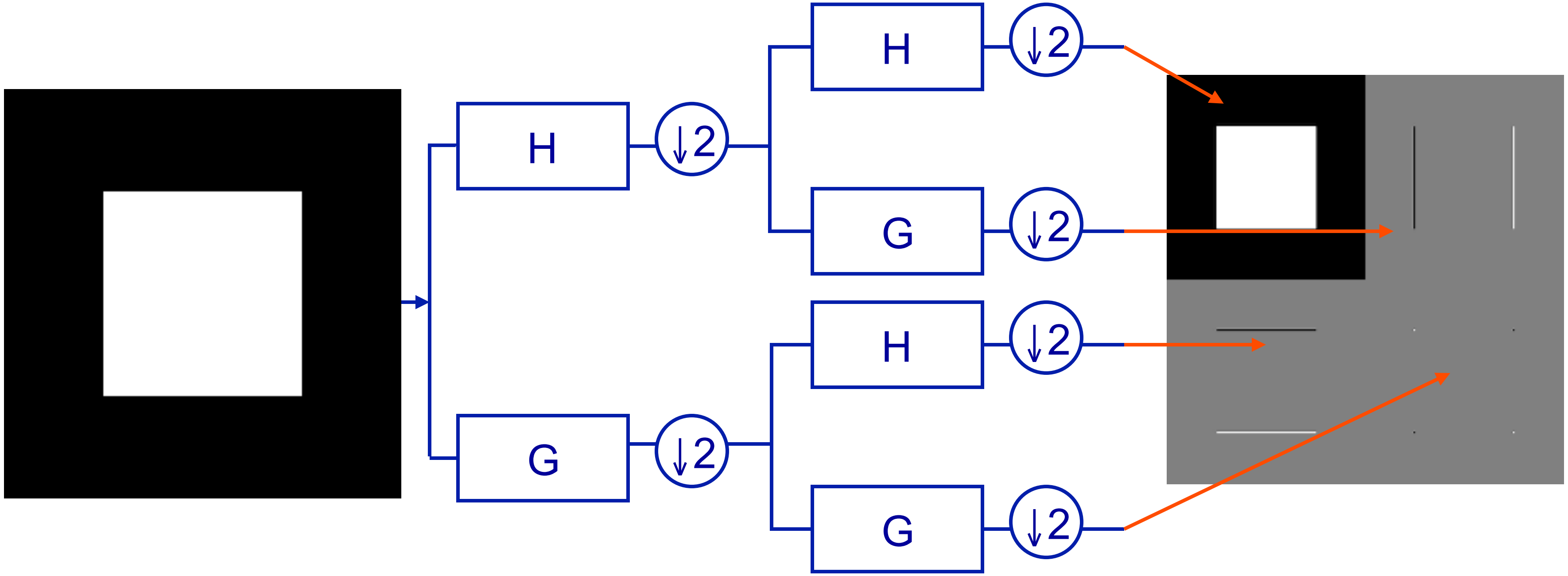
Wavelets&Pyramids



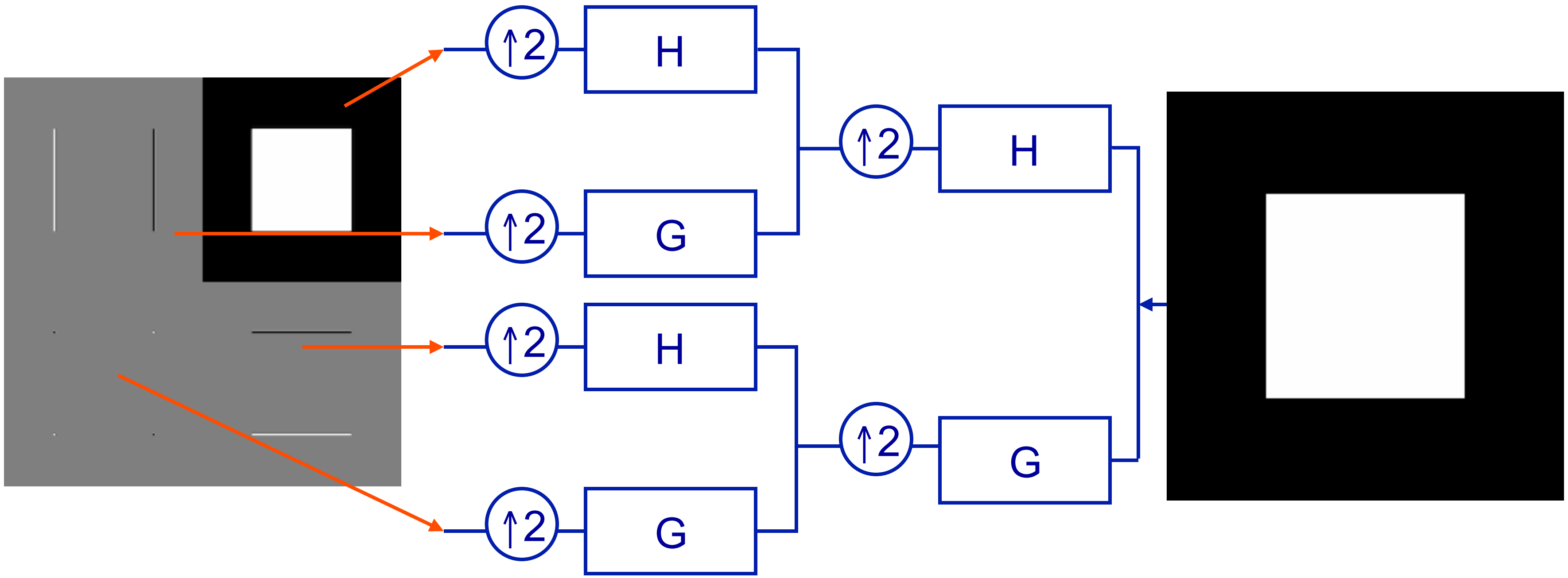
Waveletsy&Pyramids



Wavelets&Filterbanks



Waveletts&Filterbanks



Very efficient implementation by recursive filtering

Fourier versus Wavelets

Fourier

- Basis functions are sinusoids
  - More in general, complex exponentials
- Switching from signal domain *t* to frequency domain *f*
  - Either spatial or temporal
- Good localization either in time or in frequency
  - Transformed domain: Information on the sharpness of the transient but not on its position
- Good for stationary signals but unsuitable for transient phenomena

Wavelets

- Different families of basis functions are possible
  - Haar, Daubechies', biorthogonal
- Switching from the signal domain to a *multiresolution* representation
- *Good localization in time and frequency*
  - Information on *both* the *sharpness* of the transient and the *point* where it happens
- Good for any type of signal

Applications

- Compression and coding
  - Critically sampled representations (discrete wavelet transforms, DWT)
- Feature extraction for signal analysis
  - Overcomplete bases (continuous wavelet transform, wavelet frames)
- Image modeling
  - Modeling the human visual system: Objective metrics for visual quality assessment
  - Texture synthesis
- Image enhancement
  - Denoising by wavelet thresholding, deblurring, hole filling
- Image processing on manyfolds
  - Wavelet transform on the sphere: applications in diffusion MRI

Wavelet-based coding

Original image

Wavelet transform (DWT) → (Perceptual) quantization → Entropy coding → Encoded image