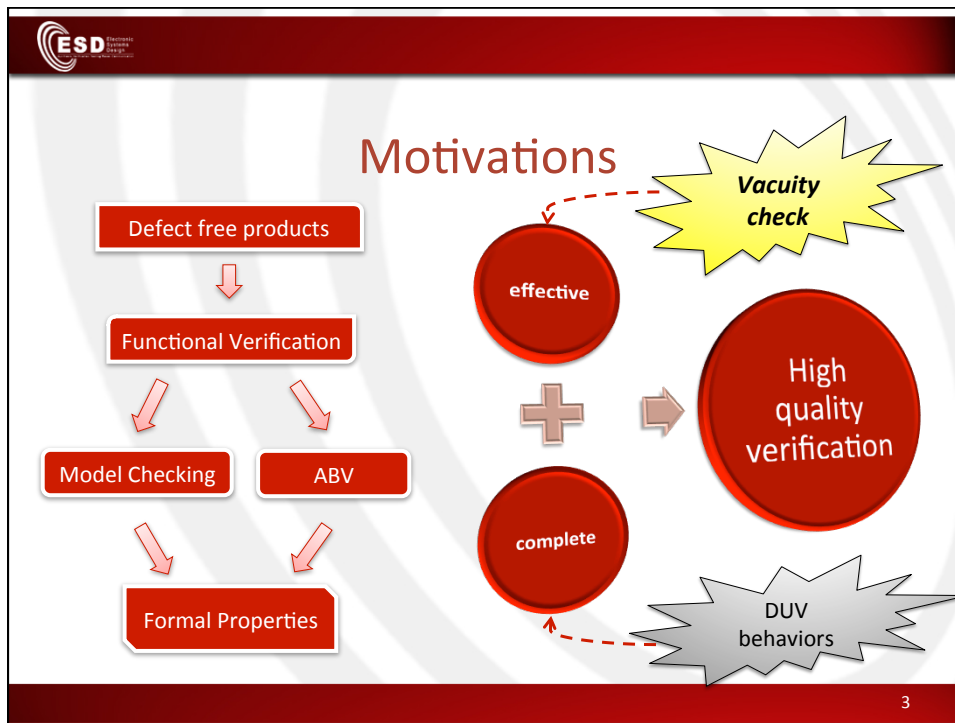


# Vacuity Analysis

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## Agenda

- Motivations
- State of the art
- Goal
- Methodology
- Pros and cons
- Conclusions



The diagram, titled "Preliminaries", contains a bulleted list:

- Vacuous pass:
  - i.e., formula that is trivially true;
    - e.g.,  $AG(req \rightarrow AX(ack))$ ;
    - true if "req" never happens;
    - then the truth value is independent from "ack" value.
  - useless for verification purpose;
  - leads verification engineers to a false sense of safety.

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## Preliminaries: Affect and Vacuity

- Definition 1 (Affect) [Beer01]
  - A sub-formula  $\psi$  of a formula  $\phi$  **affects**  $\phi$  in model  $M$  if there is a formula  $\psi'$  such that the truth values of  $\phi$  and  $\phi[\psi \leftarrow \psi']$  are different in  $M$ .
- Definition 2 (Vacuity) [Beer01]
  - A formula  $\phi$  **passes vacuously** in model  $M$  if  $M \models \phi$  and  $\phi$  includes a sub-formula  $\psi$  that does not affect  $\phi$  in  $M$ .
    - In this case, we say that  $\phi$  is  $\psi$ -vacuous in  $M$

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## Preliminaries: Affect and Vacuity

- Example
  - $AG( req \rightarrow AX(ack))$ 
    - The formula is true if  $req$  never happens independently from the value of  $AX(ack)$
    - $AX(ack)$  does not affect the formula, which then passes vacuously in a model where  $req$  never happens
- Previous definitions capture the intuitive notion of vacuity independently from the logic
  - But they are not practically useful

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## Preliminaries: Sub-formulas

- To limit the analysis to a small subset of sub-formulae
  - Definition 3 (**Minimal sub-formulas**) [Beer01]
    - Let  $S$  be a set of sub-formulas. The minimal sub-formulas of  $S$  is defined as:
 
$$\min(S) = \{ \psi \in S \mid \nexists \psi' \in S \text{ such that } \psi' \leq \psi \}$$
 where  $\psi' \leq \psi$  means that  $\psi'$  is a sub-formula of  $\psi$
    - Assumption:
      - each sub-formula is unique

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## Preliminaries: Sub-formulas

- Example
  - $\phi = AG((\alpha \wedge \beta) \rightarrow AX(\gamma \vee \neg \alpha))$
  - Subformulae of  $\phi$ 
    - $S = \{\alpha_1 \wedge \beta_1, \alpha_1, \beta_1, \gamma_1 \vee \neg \alpha_2, \gamma_1, \neg \alpha_2, \alpha_2\}$
  - Minimal sub-formulae
    - $\min(s) = \{\alpha_1, \beta_1, \gamma_1, \alpha_2\}$

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## Preliminaries: vacuity

- Theorem 1 [Beer01]
  - In a logic with polarity, for a formula  $\varphi$ , and a set  $S$  of sub-formulas of  $\varphi$ , for every model  $M$ ,  $\varphi$  is **S-vacuous** in  $M$  iff
    - there is  $\psi \in \min(S)$  such that
    - $M \models \varphi[\psi \leftarrow X]$ , where  $X = \text{false}$  if  $M \models \varphi$  and  $\psi$  is of positive polarity, otherwise,  $X = \text{true}$ .
- minimal sub-formulae are substituted with either true or false, thus reducing the number of checks
  - $\phi[\psi \leftarrow X]$  is a witness formula

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## Preliminaries: vacuity

- From Theorem 1:
  - $\varphi$  is not  $\psi$ -vacuous if  $M \not\models \varphi[\psi \leftarrow X]$
  - In this case
    - The counterexample is an interesting witness proving the non vacuity of  $\phi$  with respect to  $\psi$

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## State of the Art

- Vacuity checking:
  - *witness formulas* obtained by the substitutions of sub-formulas with **true** or **false** according to their polarity;
  - *witness formulas* failure highlight how substituted sub-formula affects the original formula;
  - Theorem 1 allows to focus only on model checking  $\varphi$  where the minimal sub-formulas are substituted;
- Limits:
  - model checking;
  - verification of new set of formulas.



Increasing  
verification time

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## State of the Art

- [Beer97]
  - identification of w-CTL formulas for which vacuity detection can be done efficiently
    - A w-CTL formula is an CTL formula in which for all binary operators at least one of the operand is a propositional formula
- [Vardi99]
  - vacuity detection for CTL\* formulas consists of replacing **each sub-formula**  $\psi$  of a formula  $\phi$  by true and false
  - vacuity with respect to sub-formulas occurrences
- [Beer01]
  - vacuity detection for CTL\* formulas consists of replacing **only minimal sub-formulas** by either true or false according to their polarity
  - vacuity with respect to sub-formulas occurrences

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## State of the art

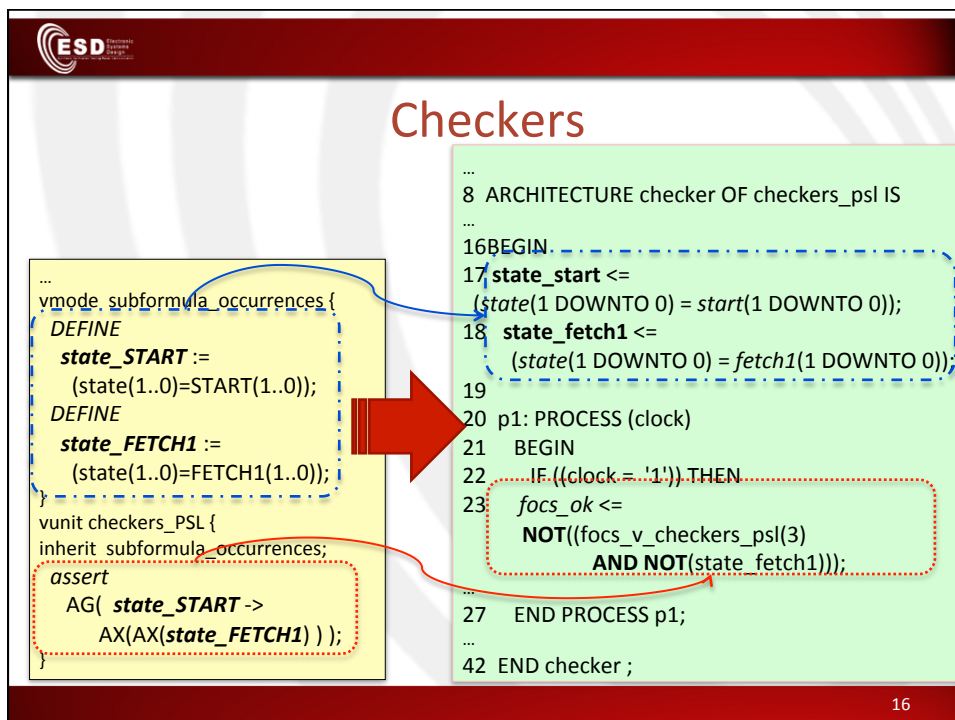
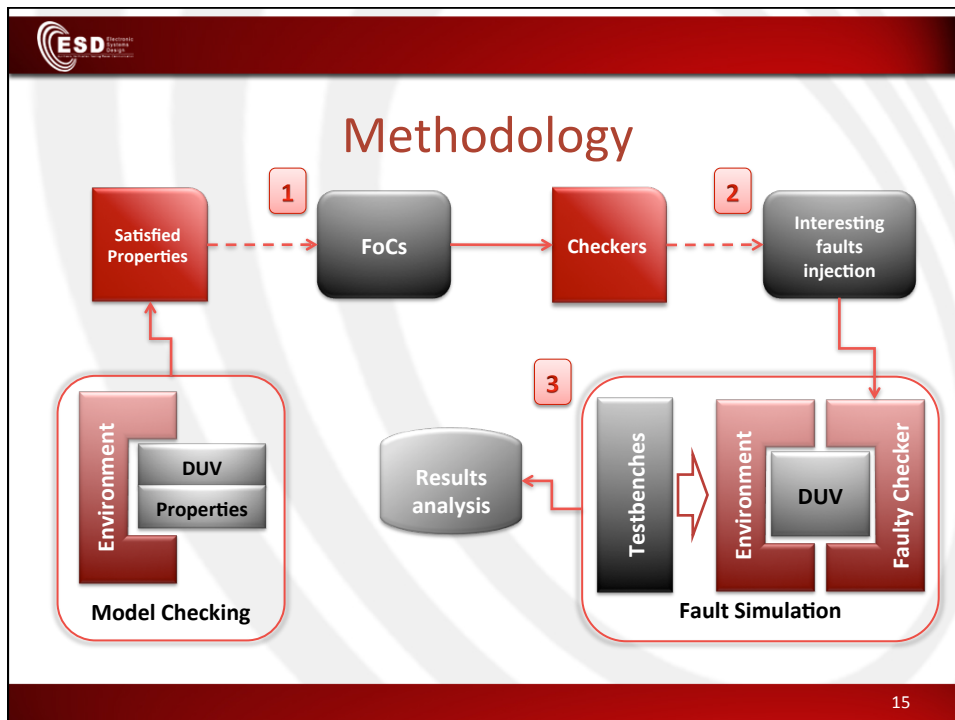
- [Armoni03]
  - vacuity detection with respect to both sub-formulas and sub-formulas occurrences but
  - required manual approach for analyzing vacuity alert
- [Gurfinkel07]
  - the method is based on a naive algorithm based on model checking formulas obtained by replacing atomic sub-formulas with unconstrained Boolean variables
  - the approach detects vacuity as defined in [Beer97], [Beer01]

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## Goals

- New vacuity analysis approach:
  - without model checking
  - without *witness formulas* definition
  - effective as current approaches
  - more efficient
  - based on fault simulation
- It requires:
  - checker generation
  - interesting faults injection
  - results analysis

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## Interesting Faults

- They perturb only one minimal sub-formula in the checker
  - Sub-formulae are grouped by scope of the temporal operators to address tautology
    - *Contemporaneous*: occurrences of the same subformula that are in the scope of the same temporal operators
    - *Opposite*: contemporaneous occurrences of the same subformula with opposite polarity
- its value is related to sub-formula polarity
- if faults are detected, formula does not pass vacuously

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## Interesting Faults

- Definition 4
  - Let
    - $\phi$  a formula in a logic with polarity
    - $C$  the corresponding checker,
    - $S$  the set of minimal sub-formulas of  $\phi$ , and
    - $A$  the set of concurrent assignments of  $C$  storing the value of sub-formulas in  $S$  during simulation.
  - The set of interesting faults for  $C$  is defined as follows:
    - $F = \{ \text{stuck-at } X \text{ on } a \mid a \in A \}$   
 where  $X = \text{false}$  if the sub-formula associated to assignment  $a$  has positive polarity,  $X = \text{true}$  otherwise

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## Interesting Faults

- Example:
  - considering the property
    - $AG( state\_START \rightarrow AX(AX(state\_FETCH1) ) );$
  - state\_START is of negative polarity:
    - corresponding interesting fault is stuck-at-1
    - $AG( true \rightarrow AX(AX(state\_FETCH1) ) );$
  - state\_FETCH1 is of positive polarity:
    - corresponding interesting fault is stuck-at-0
    - $AG( state\_START \rightarrow AX(AX( false ) ) );$

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## Interesting Faults

- Theorem 2 (Detectable faults vs. vacuity)
  - Let
    - $\phi$  a formula in a logic with polarity,
    - $C$  the corresponding checker,
    - $S$  the set of minimal sub-formulas of  $\phi$ , and
    - $F$  the set of interesting faults of  $C$  associated to  $S$
  - The fault  $f \in F$ , such that  $f$  is associated to sub-formula  $\psi \in S$  as defined in Def. 4, **is detectable**, **iff  $\phi$  is not  $\psi$ -vacuous**

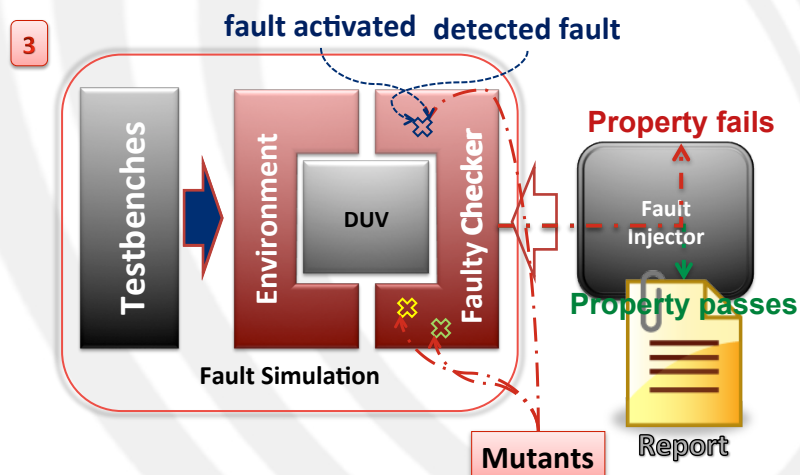
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## Interesting Faults

- Previous theorem allows:
  - To reason about vacuity by fault simulating interesting faults in the checker instead of model checking witness formulae
    - A checker failure due to the effect of an interesting fault  $f$  corresponds to prove that the sub-formula  $\psi$  perturbed by  $f$  affects the truth value of  $\phi$
- Consequently, the sequence of values generated by the testbench that causes the checker failure (i.e., the test sequence of  $f$ ) is an *interesting witness* proving that  $\phi$  is not  $\psi$ -vacuous

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## Simulation Environment



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## Pros and Cons

- Testbenches, drawbacks:
  - inefficiency of testbenches used during fault simulation may prevent the detection of interesting faults
  - from practical point of view:
    - formula does not pass vacuously if interesting faults are detected
    - on the contrary, it is not possible to conclude that a formula is vacuous

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## Pros and Cons

- Testbenches, advantages:
  - in ABV testbenches must be as effective as possible
  - from practical point of view:
    - it is mandatory refining testbenches that allow a formula to pass vacuously
  - in this context, the proposed approach is satisfactory

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## Pros and Cons

- Checkers, drawbacks:
  - it is not possible to conservatively generate checkers for all kind of temporal formulas
  - FoCs is able to generate checkers for formulas defined according to the *Simple Subset* of PSL
  - restrictions guarantees that formulas can be simulated

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## Pros and Cons

- Checkers, advantages:
  - proposed methodology is independent from the tool adopted for checkers generation
  - the idea of storing (and perturbing) the values of sub-formulas in explicit signals driven by concurrent statements can be exploited whatever the structure of the generated checker is

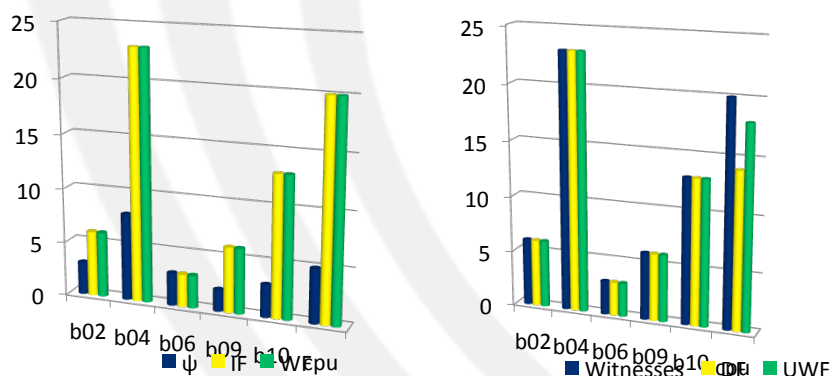
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## Experimental results

- Benchmarks:
  - ITC-99 benchmarks;
  - control unit of an 8-bit CPU with an instruction set architecture composed of 13 instruction;
- Properties:
  - for ITC-99 benchmarks have been taken from the *vis-verilog-models-1.0* archive;
  - for cpu design have been defined by analyzing its specification;
- Checkers:
  - generated by using FoCs;
- Interesting faults injection and testbench generation:
  - using the features of the Laerte++ automatic test pattern generator;
- Compared with:
  - formal vacuity analysis performed according to the approach proposed in [Beer01] by using VIS to model check witness formulae.

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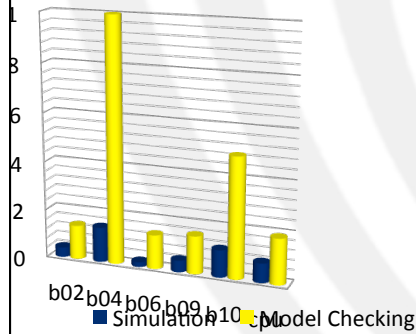
## Experimental Results



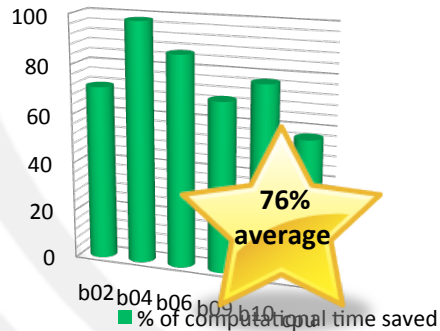
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## Verification Time

Time Comparison



Saved Time



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## Tautologies: Problem

- Not marked as vacuous formulas when two *separate occurrences* of the same sub-formula are considered to be different sub-formulas
  - e.g.,  $(a \vee \neg a)$ :
    - $(\text{false} \vee \neg a) = (\neg a)$
    - $(a \vee \neg \text{true}) = (\neg a)$
- marked as vacuous by considering *sub-formulas instead of occurrences*
  - e.g.,  $(a \vee \neg a)$ :
    - $(\text{true} \vee \neg \text{true}) = (a \vee \neg a)$
    - $(\text{false} \vee \neg \text{false}) = (a \vee \neg a)$

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## Tautologies: Problem

- But if we use sub-formulae instead of occurrences for
  - $\alpha \sqcap (G (\neg \beta \vee \alpha))$ 
    - $\text{false} \wedge (G (\neg \beta \vee \text{false}))$  is false
    - $\alpha \wedge (G (\neg \text{true} \vee \alpha))$  is  $\alpha \wedge (G \alpha)$  which can be falsified
  - is marked as non-vacuous, even if it is vacuous when  $\beta$  never occurs

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## Tautologies: Solution

- Modify the formula to remove temporal overlapping among operators
- **Opposite (contemporaneous)** formulae are substituted simultaneously
- **Non opposite (contemporaneous)** formulae are substituted individually

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## Tautologies

- Example
  - $G(\neg \alpha \vee F(\alpha))$ 
    - Is equivalent to
    - $G(\neg \alpha \vee \alpha \vee X(F(\alpha)))$
  - Sub-formulae  $S = \{\neg\alpha_1, \alpha_1, \alpha_2, \alpha_3\}$
  - $\text{Min}(S) = \{\alpha_1, \alpha_2, \alpha_3\}$ 
    - $\alpha_1, \alpha_2$  are opposite
    - $\alpha_3$  has a different temporal scope

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## Tautologies

- $\varphi_1 = \alpha \vee \neg\alpha$ 
  - $\text{Min}(S) = \{\alpha_1, \alpha_2\}$
  - $\alpha_1, \alpha_2$  are opposite,  $\rightarrow$  simultaneous substitution
    - $\text{false} \vee \neg \text{false} = \text{true} \rightarrow \text{vacuous}$
    - $\text{true} \vee \neg \text{true} = \text{true} \rightarrow \text{vacuous}$

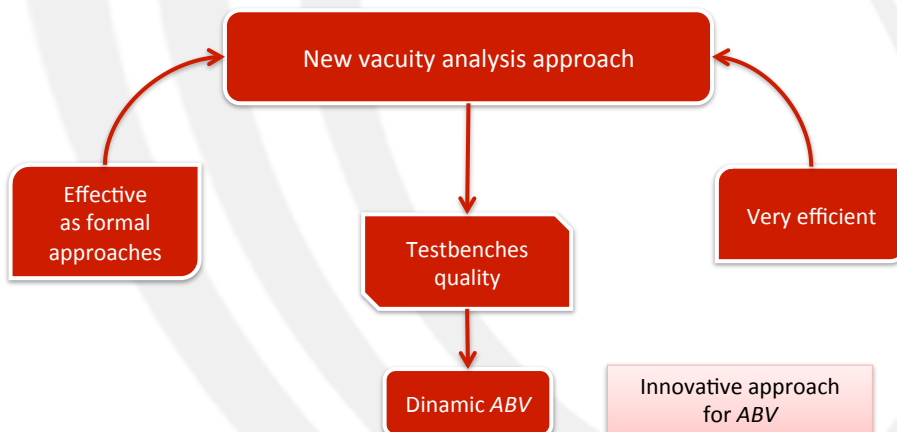
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## Tautologies

- $\varphi_2 = \alpha \wedge (G (\neg \beta \vee \alpha))$ 
  - Let us consider that  $\beta$  never occurs
  - $\text{Min}(S) = \{\alpha_1, \alpha_2, \beta_1\}$  are not opposite
  - $\varphi_2[\alpha_1 \leftarrow \text{false}] = \text{false}$
  - $\varphi_2[\beta_1 \leftarrow \text{true}] = \alpha \wedge G(\alpha)$ 
    - Possibly false
  - $\varphi_2[\alpha_2 \leftarrow \text{false}] = \alpha \wedge G(\neg \beta)$ 
    - Since  $\beta$  never occurs and  $\varphi_2$  is satisfied by the model,  $\varphi_2[\alpha_2 \leftarrow \text{false}] = \alpha = \text{true} \rightarrow \text{vacuous}$

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## Conclusions



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