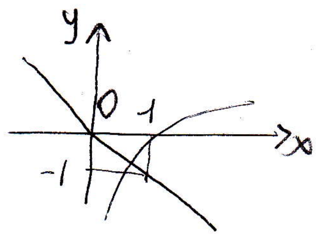
EX2) Studiare il grafico di $f(x) = \frac{\ln x}{x} + 1 = x^{-1}\ln x + 1 = \frac{\ln x + x}{x}$

Ris

- dominio $A = \{x \in \mathbb{R} : x > 0\}$
- segno $\frac{\ln x + x}{x} > 0$ sse $\ln x + x > 0$ sse $\ln x > -x$



Sia $\bar{x} \in (0,1)$ t.c. $\ln \bar{x} = -\bar{x}$

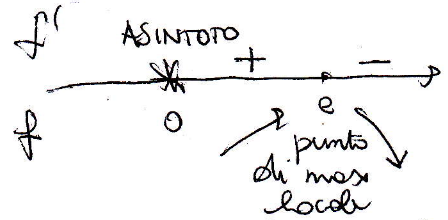
$f(x) > 0$ se $x > \bar{x}$

$f(x) = 0$ se $x = \bar{x}$

$f(x) < 0$ se $0 < x < \bar{x}$

- intersezioni con gli assi: $(\bar{x}, 0)$, nessuna intersezione con l'asse y
- $f \in C^\infty(A)$ $\quad f'(x) = \left(\frac{\ln x}{x} + 1\right)' = \frac{1 - \ln x}{x^2}$

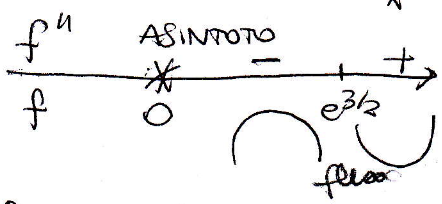
$f'(x) > 0$ sse $\frac{1 - \ln x}{x^2} > 0$ sse $1 - \ln x > 0$ sse $\ln x < 1$ sse $0 < x < e$



$f(e) = \frac{\ln e}{e} + 1 = \frac{1}{e} + 1$ max locale

- $f''(x) = \frac{-\frac{1}{x}x^2 - (1 - \ln x)2x}{x^4} = \frac{-x - 2x + 2x\ln x}{x^4} = \frac{x(-3 + 2\ln x)}{x^4} = \frac{2\ln x - 3}{x^3}$

$f''(x) > 0$ sse $\frac{2\ln x - 3}{x^3} > 0$ sse $2\ln x - 3 > 0$ sse $\ln x > \frac{3}{2}$ sse $x > e^{3/2}$



$f(e^{3/2}) = \frac{\ln(e^{3/2})}{e^{3/2}} + 1 = \frac{3}{2} e^{-\frac{3}{2}} + 1$

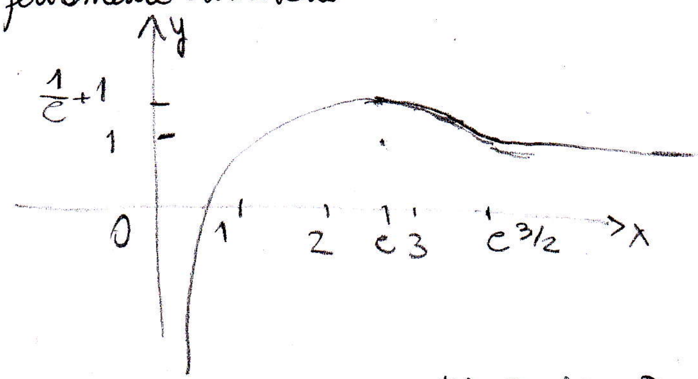
$\lim_{x \to +\infty} \frac{\ln x}{x} \quad \left[\frac{\infty}{\infty}\right]$

$\overset{H}{=} \lim_{x \to +\infty} \frac{\frac{1}{x}}{1} = \lim_{x \to +\infty} \frac{1}{x} = 0 \Rightarrow \lim_{x \to +\infty} \frac{\ln x}{x} + 1 = +1$

$\lim_{x \to 0^+} \frac{\ln x}{x} + 1 = -\infty$ ($\ln x \to -\infty$, $x \to 0^+$) $\Rightarrow$ f non è inferiormente limitata

$\Rightarrow x_0 = e$ punto di max assoluto

Grafico



EX2 fila D