Es 2 foglio A Analisi Matematica 1

Studiare la funzione

$$f(x) = \frac{1+x^3}{x^2}$$

Ris $A = (-\infty, 0) \cup (0, +\infty)$

segno $f(x) > 0$ sse $1+x^3 > 0$ sse $x^3 > -1$ sse $x > -1$ in A

$f(x) = 0$ sse $x = -1$

$f(x) < 0$ sse $x < -1$

intersezione assi $(-1, 0)$

$\lim_{x \to 0^\pm} \frac{1+x^3}{x^2} = +\infty \Rightarrow x = 0$ asintoto verticale

$\lim_{x \to \pm\infty} \frac{1+x^3}{x^2} = \lim_{x \to \pm\infty} \frac{x^3\left(\frac{1}{x^3}+1\right)}{x^2} = \pm\infty \Rightarrow$ $\nexists$ asintoti orizzontali

$\lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{1+x^3}{x^3} = 1 = m$

$\lim_{x \to \pm\infty} f(x) - mx = \lim_{x \to \pm\infty} \left(\frac{1+x^3}{x^2} - x\right) = \lim_{x \to \pm\infty} \frac{1+x^3-x^3}{x^2} = \lim_{x \to \pm\infty} \frac{1}{x^2} = 0 = q$

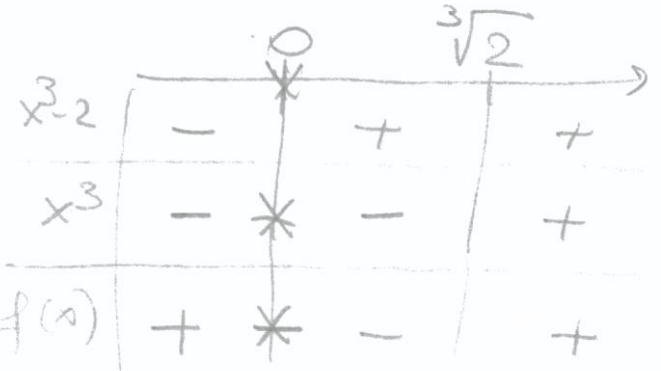
$y = x$ asintoto obliquo

$$f'(x) = \frac{3x^2 \cdot x^2 - 2x(1+x^3)}{x^4} = \frac{3x^4 - 2x - 2x^4}{x^4} = \frac{x^4 - 2x}{x^4} = \frac{x^3 - 2}{x^3}$$

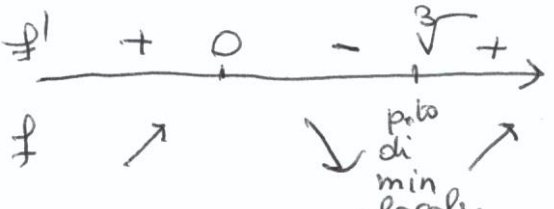
$f'(x) > 0$ sse $\frac{x^3-2}{x^3} > 0$

$x^3 - 2 > 0$ sse $x^3 > 2$ sse $x > \sqrt[3]{2}$

$x^3 > 0$ sse $x > 0$



$f(\sqrt[3]{2}) = \frac{3\sqrt{2}}{2}$



$$f'(x) = \frac{x^3-2}{x^3} = 1 - \frac{2}{x^3} = 1 - 2x^{-3}$$

$f''(x) = 6x^{-4} > 0 \quad \forall x \in A$

$\Rightarrow f$ è convessa in $(-\infty, 0) \cup (0, +\infty)$