$$|\mathbb{R}| = |[0,1)_{\mathbb{R}}|$$

$$|[0,1)_{\mathbb{R}}| \neq \aleph_0$$

$\{b \mid b : \mathbb{N} \to \{0,1\}\}$ NON È NUMERABILE

DIMOSTRAZIONE PER ASSURDO

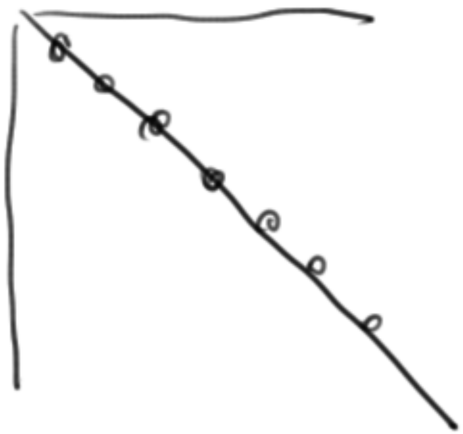
$\{0,1\}^{\mathbb{N}}$ $\qquad b \in \{0,1\}^{\mathbb{N}}$ $\qquad b[i]$

$$|\{0,1\}^{\mathbb{N}}| = \aleph_0$$

$\exists f : \mathbb{N} \to \{0,1\}^{\mathbb{N}}$ biiettiva.

$$\{ b_0, \; b_1, \; \dots\dots \} = \{0,1\}^{\mathbb{N}}$$

$$\{0,1\}^{\mathbb{N}} \left\{ \begin{array}{l} b_0 \\ b_1 \\ \vdots \\ b_k \\ \vdots \end{array} \right.$$



$$\bar b \qquad \forall i \qquad \bar b \neq b_i$$

$$\bar b[j] = \begin{cases} 1 & \text{se } b_j[j] = 0 \\ 0 & \text{se } b_j[j] = 1 \end{cases}$$

$$\exists k \quad \bar b = b_k$$

$$\underbrace{\bar b[k]}_{=0} = b_k[k]$$

$$= 1 \iff b_k[k] = 0$$

ASSURDO

OGNI INSIEME INFINITO CONTIENE UN INSIEME NUMERABILE

FUNZIONE DI SCELTA X

$$\left.\begin{aligned} &\psi : \mathcal{P}(X) \to X \\ &\quad \psi(A) \in A \end{aligned}\right] \qquad A \in \mathcal{P}(X)$$

X $\qquad \{e_i\}_{i \in \mathbb{N}}$ $\qquad e : \mathbb{N} \to X$

iniettiva

$e(i)$ $\qquad e_i$

$\mathrm{imm}\,(e) \subseteq X$

$$e_0 = \Psi(X)$$

$$e_1 = \Psi(X - \{e_0\})$$

$$e_2 = \Psi(X - \{e_0, e_1\})$$

$$\vdots$$

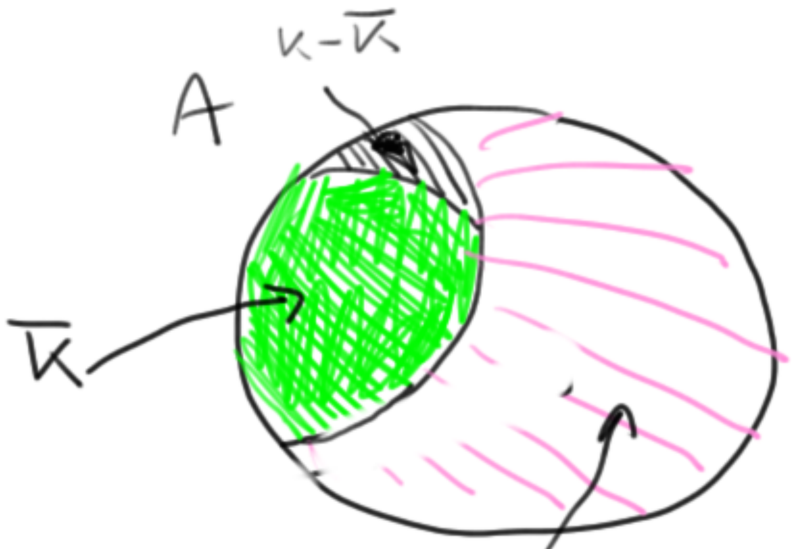
$$e_{k+1} = \Psi(X - \{e_0, \ldots, e_k\})$$

$$\vdots$$

$$A = \{e_n \mid n \in \mathbb{N}\}$$

A è infinito allora

$$\exists\, B \subsetneq A \qquad A \approx B$$



$$S = \overline{K} \cup (A - K)$$

$$|K| = \aleph_0$$

$$K \approx \overline{K} \subsetneq K$$

$$f : K \to \overline{K}$$

bijettiva

$$\varphi : A \to S$$

$$\varphi_{|A-K} = id \qquad \varphi_{|K} = f$$

A ē infinito A é in corr. biun. con una sua parte propria

$$\bigcup_{i=1,n} A_i \quad \text{t.c.} \quad |A_i| = \aleph_0$$

$$\Rightarrow \left| \bigcup_{i=1,n} A_i \right| = \aleph_0$$

- $\mathcal{G}$ famiglia di insiemi

t.c. $\forall X, Y \in \mathcal{G} \quad X \cap Y = \emptyset$

$\forall X \in \mathcal{G} \quad X \neq \emptyset$

$\forall X \in \mathcal{G} \quad X$ è finito

$|\mathcal{G}| = \aleph_0$

$$\left| \bigcup_{X \in \mathcal{G}} X \right| = \aleph_0$$

X $\quad P$ è una partizione di X

se $|X| = \aleph_0 \Rightarrow P$ è finito o num.

$\mathbb{N} \quad |\{\mathbb{N}\}| = 1$

$\mathcal{N} = \{\{x\} \mid x \in \mathbb{N}\} \quad |\mathcal{N}| = \aleph_0$

$\mathbb{N}$ $\qquad |\{\mathbb{N}\}| = 1$

$\mathcal{N} = \{ \{x\} \mid x \in \mathbb{N} \}$ $\qquad \underbrace{|\mathcal{N}| = \aleph_0}_{?}$

$f : \mathbb{N} \to \mathcal{N}$

$f(n) = \{n\}$

$|A|$

$c(A)$

$f : A \to B$ $\quad$ f suriettiva

$X \subseteq B \quad f^{-1}[X] = \{a \mid a \in A \ f(a) \in X\}$

$P = \{ f^{-1}[\{b\}] \mid b \in B \}$

$\boxed{1)\ P \text{ è una partizione}}$ $\boxed{2)\ |P| = |B|}$

a) $\forall X \in P \quad X \neq \emptyset$ $\quad$ $X \in P \ \exists b \in B$ t.c. $f^{-1}[\{b\}] = X$

f è suriett. $\exists a \in A \ f(a) = b$

$a \in f^{-1}[\{b\}]$

b) $\forall X, Y \in P \quad X \neq Y \Rightarrow X \cap Y = \emptyset$

$X \neq Y \quad \exists b_1, b_2 \ b_1 \neq b_2 \ X = f^{-1}[\{b_1\}] \ Y = f^{-1}[\{b_2\}]$

PER ASSURDO CHE

$\exists a \in X \cap Y \Rightarrow f(a) = b_1 \ \& \ f(a) = b_2$ ASSURDO

c) $\bigcup_{X \in P} X = A \quad A \subseteq \bigcup_{X \in P} X$ $\quad$ $\underline{a \in A \Rightarrow \exists X \in P \ a \in X}$

$f^{-1}[\{f(a)\}] = X$

$f : A \to B$ $\quad$ f suriettiva

$X \subseteq B$ $\quad$ $f^{-1}[X] = \{a \mid a \in A \ f(a) \in X\}$

$P = \{ f^{-1}[\{b\}] \mid b \in B \}$

$\boxed{\text{1) } P \text{ è una partizione}}$ $\quad$ $\boxed{\text{2) } |P| = |B|}$

$g : B \to P$

$g(b) = f^{-1}[\{b\}]$

g è SURIETTIVA? $\quad$ $\forall X \in P \ \exists \ b \in B \ \ g(b) = X$

$X \in P \quad X = f^{-1}[\{b\}] \quad g(b) = X$

g è iniettiva $\quad$ $b_1 \neq b_2 \quad g(b_1) \neq g(b_2)$?

$g(b_1) = f^{-1}[\{b_1\}] \qquad g(b_2) = f^{-1}[\{b_2\}]$

$g(b_1) \cap g(b_2) = \emptyset \quad \Rightarrow \quad g(b_1) \neq g(b_2)$

$f : A \to B$ suriettiva

$$P_f = \{ f^{-1}[\{b\}] \mid b \in B \}$$

$$a \sim_f b \iff f(a) = f(b)$$

$$P_f = A/_{\sim_f}$$