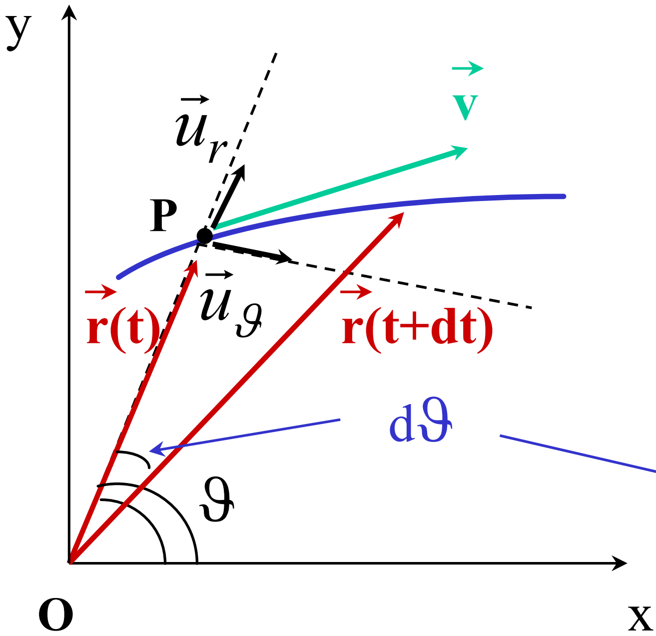
Componenti polari della velocità:

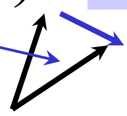


$$\vec{u}_r^{\,2} \equiv 1 = \text{ costante}$$

$$\Rightarrow \quad d\vec{u}_r^{\,2} = 2\vec{u}_r \cdot d\vec{u}_r = 0$$

$$\Rightarrow \quad \vec{u}_r \perp d\vec{u}_r$$

$$\boxed{d\vec{u}_r = d\vartheta \vec{u}_\vartheta}$$



$$\vec{r}(t) = r(t)\vec{u}_r(t)$$

$$= \frac{d\vartheta}{dt}\vec{u}_\vartheta$$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{dr(t)}{dt}\vec{u}_r + r(t)\underbrace{\frac{d\vec{u}_r(t)}{dt}}$$

$$\Rightarrow \quad \boxed{\vec{v}(t) = \underbrace{\frac{dr(t)}{dt}\vec{u}_r}_{\vec{v}_r} + \underbrace{r(t)\frac{d\vartheta(t)}{dt}\vec{u}_\vartheta}_{\vec{v}_\vartheta}}$$

"velocità radiale" "velocità trasversa"

$$\vec{v}(t) = \underbrace{\left(\frac{dr(t)}{dt}, r(t)\frac{d\vartheta(t)}{dt}\right)}_{\text{componenti polari}} = \underbrace{\left(\frac{dx(t)}{dt}, \frac{dy(t)}{dt}\right)}_{\text{componenti cartesiane}}$$

Componenti polari dell'accelerazione

$$\vec{a} \equiv \frac{d\vec{v}(t)}{dt} = \frac{d}{dt}\left(\frac{dr}{dt}\vec{u}_r + r\frac{d\vartheta}{dt}\vec{u}_\vartheta\right) =$$

$$= \frac{d^2 r}{dt^2}\vec{u}_r + \frac{dr}{dt}\frac{d\vec{u}_r}{dt} + \frac{dr}{dt}\frac{d\vartheta}{dt}\vec{u}_\vartheta + r\frac{d^2\vartheta}{dt^2}\vec{u}_\vartheta + r\frac{d\vartheta}{dt}\frac{d\vec{u}_\vartheta}{dt}$$

$$= \frac{d\vartheta}{dt}\vec{u}_\vartheta \qquad\qquad = -\frac{d\vartheta}{dt}\vec{u}_r$$

$$= 2\frac{dr}{dt}\frac{d\vartheta}{dt}\vec{u}_\vartheta$$

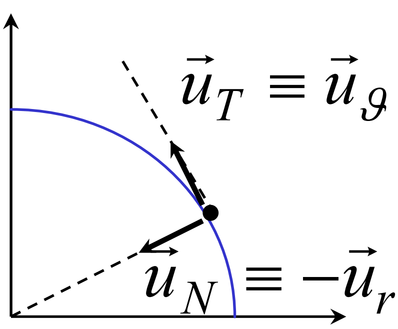
$$\Rightarrow \quad \boxed{\vec{a} = \underbrace{\left(\frac{d^2 r}{dt^2} - r\left(\frac{d\vartheta}{dt}\right)^2\right)\vec{u}_r}_{\vec{a}_r} + \underbrace{\left(2\frac{dr}{dt}\frac{d\vartheta}{dt} + r\frac{d^2\vartheta}{dt^2}\right)\vec{u}_\vartheta}_{\vec{a}_\vartheta}}$$

"accelerazione radiale" "accelerazione trasversa"

In un moto circolare (r = costante) :

$$a_r = -r\left(\frac{d\vartheta}{dt}\right)^2 = -r\omega^2 \equiv -a_N$$

$$a_\vartheta = r\frac{d^2\vartheta}{dt^2} = r\frac{d\omega}{dt} = r\alpha \equiv a_T$$



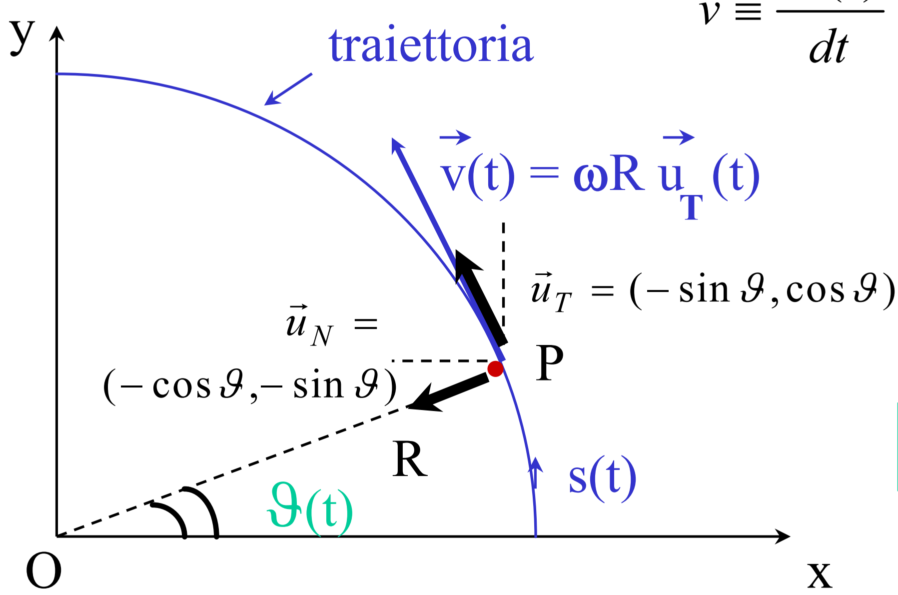
Moto circolare uniforme:

velocità con modulo costante:

coordinata curvilinea

$s(t)=R\vartheta(t)$

$$v \equiv \frac{ds(t)}{dt} = R\frac{d\vartheta(t)}{dt} = \omega R$$



"velocità angolare"

$$\boxed{\omega \equiv \frac{d\vartheta(t)}{dt}}$$

$$\boxed{\vartheta(t) = \vartheta_0 + \omega t}$$

$$x(t) = R\cos\vartheta(t) \qquad y(t) = R\sin\vartheta(t)$$

$$\Rightarrow \quad v_x(t) = \frac{dx(t)}{dt} = -R\sin\vartheta(t)\frac{d\vartheta}{dt} \equiv -R\omega\sin\vartheta(t)$$

$$v_y(t) = \frac{dy(t)}{dt} = R\cos\vartheta(t)\frac{d\vartheta}{dt} \equiv R\omega\cos\vartheta(t)$$

$$\Rightarrow \quad \vec{v}(t) = (v_x(t), v_y(t)) = R\omega\underbrace{(-\sin\vartheta(t), \cos\vartheta(t))}_{\vec{u}_T}$$

$$\Rightarrow \quad \boxed{\vec{v}(t) = R\omega\vec{u}_T(t) = v\vec{u}_T(t)}$$

$$a_x(t) = \frac{dv_x(t)}{dt} = -R\omega\cos\vartheta(t)\frac{d\vartheta}{dt} = -R\omega^2\cos\vartheta(t)$$

$$a_y(t) = \frac{dv_y(t)}{dt} = -R\omega\sin\vartheta(t)\frac{d\vartheta}{dt} = -R\omega^2\sin\vartheta(t)$$

$$\Rightarrow \quad \vec{a}(t) = (a_x(t), a_y(t)) = R\omega^2\underbrace{(-\cos\vartheta(t), -\sin\vartheta(t))}_{\vec{u}_N}$$

$$\Rightarrow \quad \boxed{\vec{a}(t) = R\omega^2\vec{u}_N(t) = \frac{v^2}{R}\vec{u}_N(t)}$$