# Computational Algebra 24/02/14

1. Determine the splitting field of
   (a) $x^3 - x^2 - x$ over $\mathbb{F}_3$
   (b) $(x^3 - x^2 - x)(x^4 - x^2 - 1)$ over $\mathbb{F}_3$

2. Let $K$ the smallest field of characteristic 2 containing a primitive 7-th root of unity.
   (a) Determine the number of elements of $K$.
   (b) Find a primitive element of $K$.
   (c) Determine all the primitive elements of $K$.

3. Decompose $x^8 - x$ in irreducible factors in $\mathbb{F}_2$.

4. (a) Find a primitive element of $\mathbb{F}_{13}$.
   (b) Construct a Reed-Solomon code $\mathcal{C}$ of dimensions $[12, 7]$ over $\mathbb{F}_{13}$.
   (c) Determine the minimal distance of $\mathcal{C}$.
   (d) Find a parity check matrix for $\mathcal{C}$.

5. Consider the primitive element $\alpha$ of $\mathbb{F}_{16}$ satisfing $\alpha^4 = 1 + \alpha$. The elements of $\mathbb{F}_{16}$ are listed in the table belove.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0000 | $0$ | 1000 | $\alpha^3$ | 1011 | $\alpha^7$ | 1110 | $\alpha^{11}$ |
| 0001 | $1$ | 0011 | $\alpha^4$ | 0101 | $\alpha^8$ | 1111 | $\alpha^{12}$ |
| 0010 | $\alpha$ | 0110 | $\alpha^5$ | 1010 | $\alpha^9$ | 1101 | $\alpha^{13}$ |
| 0100 | $\alpha^2$ | 1100 | $\alpha^6$ | 0111 | $\alpha^{10}$ | 1001 | $\alpha^{14}$ |

   Consider the BCH code of dimensions $[15, 5]$ over $\mathbb{F}_2[x]$ (with $b = 1$) with defining set $T = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12\}$. Using the primitive 15-root of unity $\alpha$ form the previous table, the generator polynomial of $\mathcal{C}$ is $g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$. Suppose $\mathcal{C}$ is used to transmit a codeword and $y(x)$ is received. Correct the received word using the Peterson-Gorenstein-Zierler Decoding Algorithm, in case $y(x) = x^4 + x^5 + x^7 + x^9 + x^{10} + x^{12}$. Verify that the correct word is actually a codeword. Correct the same $y(x)$ using the Sugiyama Decoding Algorithm.

6. (a) Give the definition of $\mathbb{Z}_4$-linear code.
   (b) What are the Hamming, Lee and Euclidean distances between the vectors $(30012221)$ and $(20202213)$ in $\mathbb{Z}_4^8$?