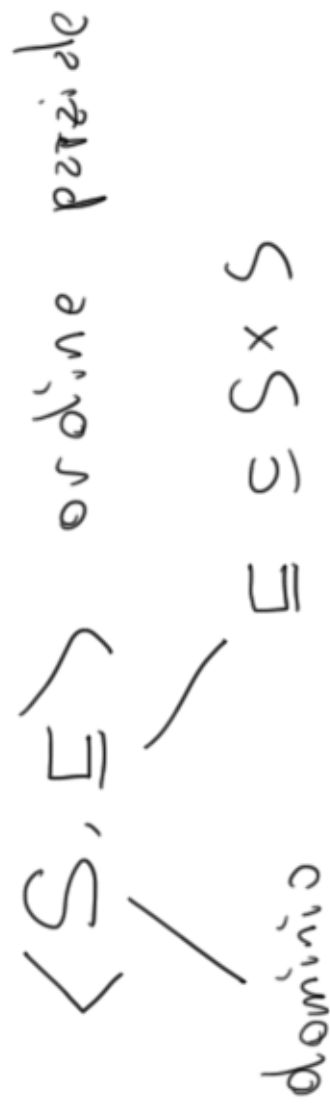
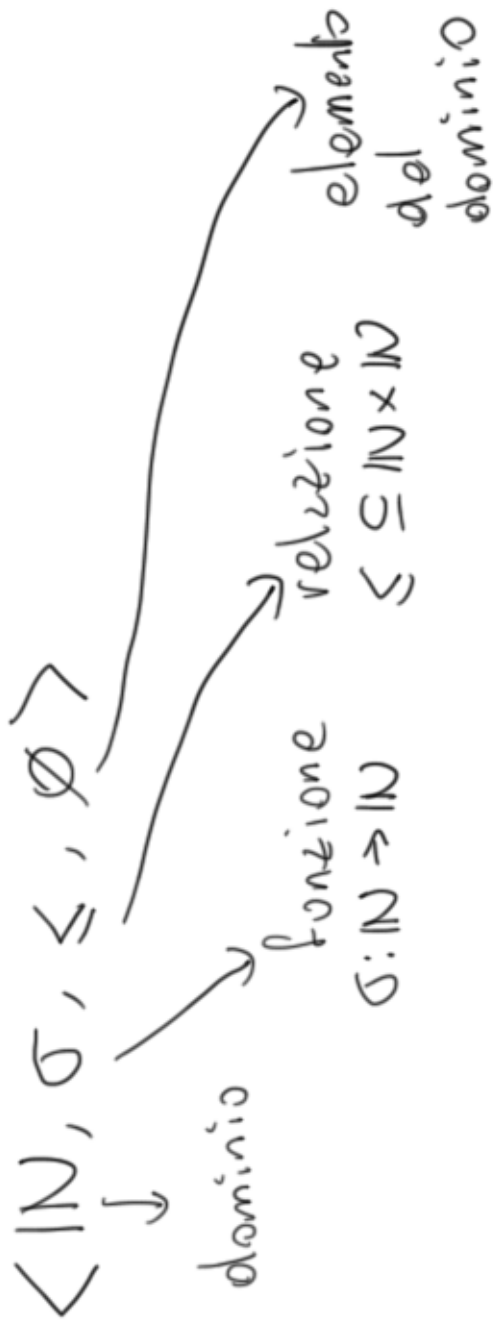




$$\begin{array}{c}
 \langle \mathbb{R}, \exp, \leq, \pi \rangle \xrightarrow{\quad} \pi \in \mathbb{R} \\
 \downarrow \text{exp} \quad \downarrow \text{exp} \quad \downarrow \text{exp} \\
 \mathbb{R} \rightarrow \mathbb{R} \quad \mathbb{R} \rightarrow \mathbb{R} \quad \mathbb{R} \rightarrow \mathbb{R} \\
 x \mapsto e^x \quad x \mapsto e^x \quad x \mapsto e^x \\
 \subseteq \mathbb{R} \times \mathbb{R}
 \end{array}$$

$\langle \mathbb{R}, +, -, \leq, \{\pi, e\} \rangle$

$\langle 2, 2; 2; 2 \rangle$

exponent.

$\langle \mathbb{R}, -, \cdot, +, \exp \rangle$

invers. segno

sottr.

somm.

$\langle 1, 2, 2, 1; -; - \rangle$

$-: \mathbb{R} \rightarrow \mathbb{R}$

$\cdot: \mathbb{R}^2 \rightarrow \mathbb{R}$

$+: \mathbb{R}^2 \rightarrow \mathbb{R}$

$\exp: \mathbb{R} \rightarrow \mathbb{R}$

$$\left[\langle A; f_1, f_2; R_1, R_2, R_3; c_1, c_2 \rangle \right. \\ \left. \langle 2, 4; 1, 3, 6; 2 \rangle \right]$$

$$\langle A; f_1, f_2; R_1, R_2, R_3; c_1, c_2 \rangle$$

$$f_1: A^2 \rightarrow A \quad R_2 \subseteq A^3$$

$$f_2: A^4 \rightarrow 4 \quad R_3 \subseteq A^6$$

$$R_1 \subseteq A$$

$$|R_1, R_2| = 2$$

$$f: A \xrightarrow{s} A \xrightarrow{\text{ar} \circ f} R$$

$$\langle A, \dots, f \rangle$$

$$f: A^n \rightarrow A$$

$$A^n \quad n > 0$$

$$A^0$$

$$\mathbb{R}^n$$

$$A \times A$$

$$(A \times A) \times A \stackrel{?}{=} A \times (A \times A)$$

$$((a, b), c) \quad (a, (b, c))$$

$$A^n \quad n > 0$$

$$A^n = \{ \underbrace{(a_1, \dots, a_n)}_{\substack{\text{---} \\ \text{---}}} \mid a_1, \dots, a_n \in A \}$$

$$A^2 = A \times A$$

$$\mathbb{R}^m$$

~~$$R \subseteq A^0$$

$$f: A^0 \rightarrow A$$~~

Example of a language of type $\langle 2; 2, 1; 1 \rangle$.

predicate symbols: $L, =$

function symbols: p, i

constant symbol: e

$x_0, x_1, \dots$
 $x, y, z, \dots$

$p(x, y)$

$p(i(x), p(x, y))$

$i(p(x, i(i(x))))$

$\text{int } p(\text{int}, \text{int});$

$\text{int } i(\text{int});$

$\text{int } x;$

$i(p(x, i(x)));$

Example of a language of type $\langle 2; 2, 1; 1 \rangle$.

predicate symbols: $L, =$

function symbols: p, i

constant symbol: e

$x_0, x_1, \dots$
 $x, y, z, \dots$

$$L(x, y) \quad L(x, y)$$

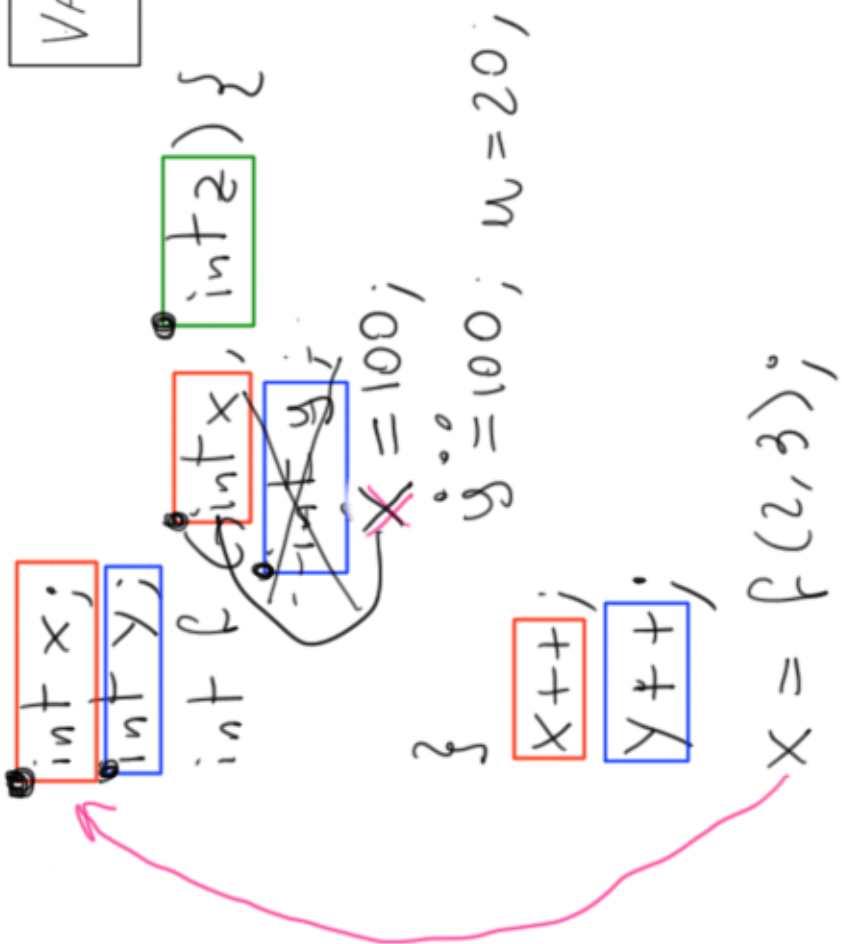
$$((\forall x) A)$$

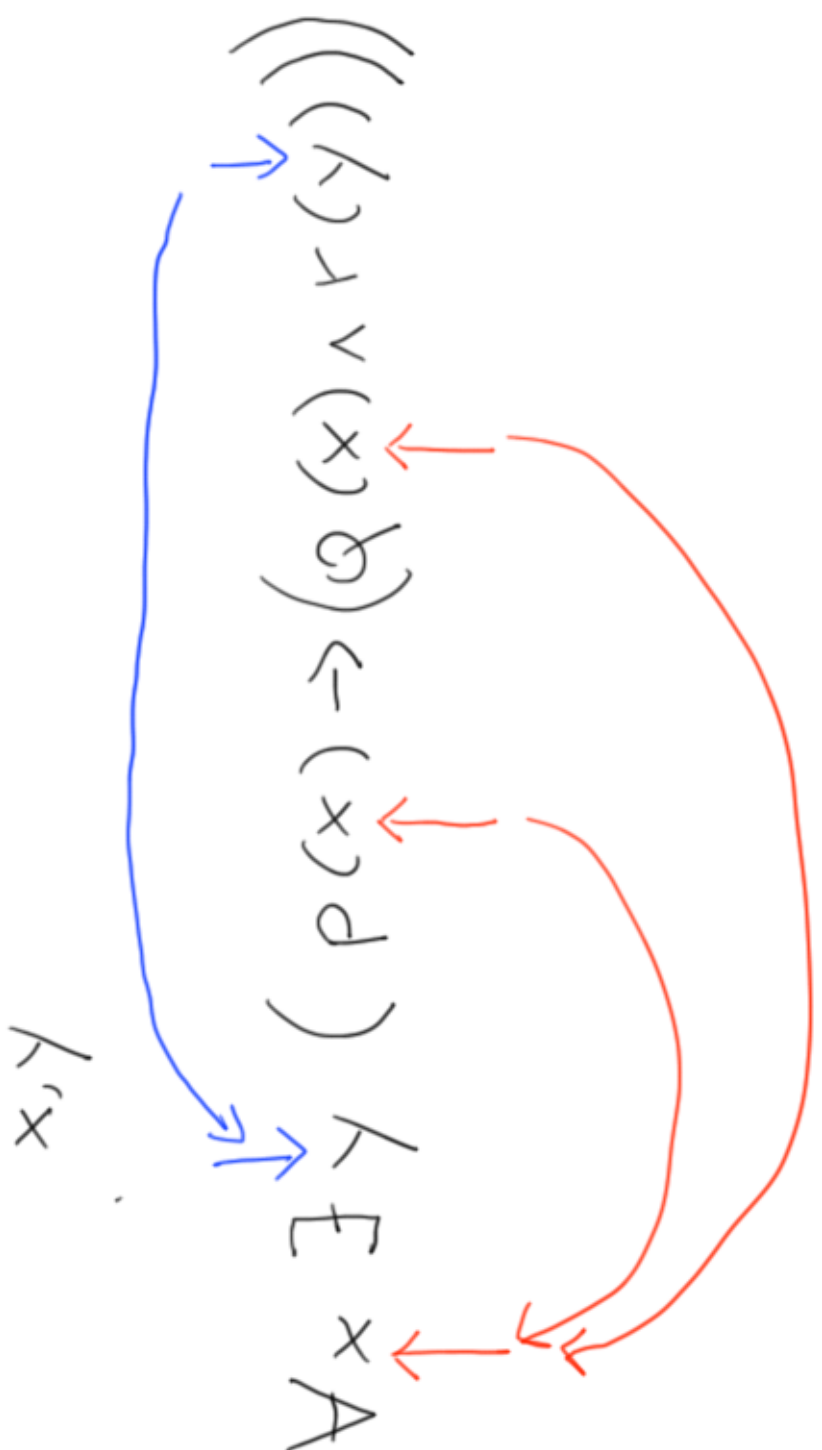
$$x = y \rightarrow L(x, y)$$

$$\forall x \exists y L(x, y)$$

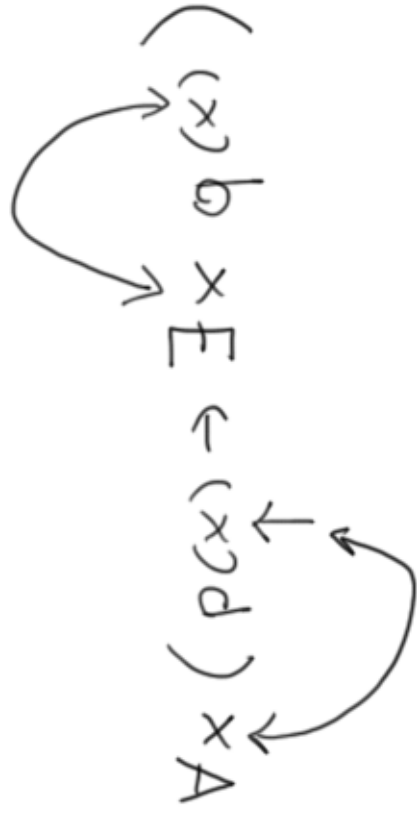
$$(\exists y \exists z (L(p(x, y), i(z)) \rightarrow (\forall x (\forall x A) \rightarrow x = y)))$$

VAR = {x, y, z, u}





$\{x\}$



$$\alpha = \forall x (p(x, y) \wedge \exists y q(y))$$

$$BV(\alpha) = \{x, y\}$$

$$FV(\alpha) = \{y\}$$

$$\begin{aligned} F \vee (\alpha) &= \{x, x\} \\ B \vee (\alpha) &= (x) \vee B \end{aligned}$$

$$(K, x) b \wedge \left(\left(\left((z) \vdash \vee (K, x) b \right) \times E \right) \leftarrow (x) d \right) = \alpha$$

$$\dots \underbrace{(d \cdot x \mathcal{D})} \dots$$

$$\{E, A\} \ni \mathcal{D}$$

$$(\exists x (\forall y (p(x,y) \rightarrow q(y)))) \equiv \exists x$$

$$FV(x) = \{x\}$$

$$BV(x) = \{x\}$$

$$\mathcal{D} \frac{\varphi(x)}{\forall x \varphi(x)}$$

$$x \notin FV(hp\mathcal{D})$$



$\mathcal{D}$

$$\frac{\varphi(x) \Leftarrow x \bar{e} \text{ un valore generico}}{\forall x \varphi(x)} \text{ che non dipende dalle ipotesi}$$

$$\frac{((k'x)\phi \times_A kA) \in ((k'x)\phi \vdash_{A^{\times A}})}{((k'x)\phi \vdash_{\times_A} kA)}$$