$\langle \mathbb{N}, \sigma, \leq, \emptyset \rangle$

dominio

funzione
$\sigma : \mathbb{N} \to \mathbb{N}$

relazione
$\leq \subseteq \mathbb{N} \times \mathbb{N}$

elemento
del
dominio

$\langle S, \sqsubseteq \rangle$ ordine parziale

dominio

$\sqsubseteq \subseteq S \times S$

$\langle \mathbb{R}, \exp, \leq, \pi \rangle$ $\pi \in \mathbb{R}$

$\exp : \mathbb{R} \to \mathbb{R} \quad x \mapsto e^x$

$\leq \; \subseteq \mathbb{R} \times \mathbb{R}$

$$\langle \mathbb{R}, +, -, \leq, \{\pi, e\} \rangle$$

$$\langle 2, 2; 2; 2 \rangle$$

inver. segno exponent.

$$\langle \mathbb{R}, -, \dot{-}, +, \exp \rangle$$

sottr. somma

$$\langle 1, 2, 2, 1; -; - \rangle$$

$-: \mathbb{R} \to \mathbb{R}$ $\quad$ $+: \mathbb{R}^2 \to \mathbb{R}$ $\quad$ $\exp: \mathbb{R} \to \mathbb{R}$

$\dot{-}: \mathbb{R}^2 \to \mathbb{R}$

$$\left[\begin{array}{l} \langle A; f_1, f_2; R_1, R_2, R_3; c_1, c_2 \rangle \\ \quad \langle 2, 4; 1, 3, 6; 2 \rangle \end{array}\right.$$

$$\langle A; f_1, f_2; R_1, R_2, R_3; c_1, c_2 \rangle$$

$f_1 : A^2 \to A$ $\qquad$ $R_2 \subseteq A^3$

$f_2 : A^4 \to 4$ $\qquad$ $R_3 \subseteq A^6$

$R_1 \subseteq A$ $\qquad$ $|\{c_1, c_2\}| = 2$

arietà di f

$f : A^{n} \to A$

$R \subseteq A^{n}$ → arietà di R

$\langle A, \ldots f \ldots \rangle$

$f : A^{k} \to A$

A^n $\quad n > 0$

A^0

$A \times A$ $\quad \mathbb{R}^n$

$(A \times A) \times A \overset{?}{=} A \times (A \times A)$

$((a, b), c) \quad (a, (b, c))$

$A^n \qquad n > 0$

$$A^n = \{ \overbrace{(a_1, \ldots, a_n)} \mid a_1, \ldots, a_n \in A \}$$

$$A^2 = A \times A$$

$\mathbb{R}^m$

$$\cancel{R \subseteq A^0}$$

$$\cancel{f : A^0 \to A}$$

Example of a language of type ⟨2;2,1;1⟩.

predicate symbols: L, =
function symbols: p, i
constant symbol: e

$x_0, x_1, \ldots$

$x, y, z, \ldots$

$p(x, y)$

$p(i(x), p(x, y))$

$i\left(\boxed{p(\boxed{x}, \boxed{i(\boxed{x})})}\right)$

```
int p(int, int);
int i(int);
int x;
i(p(x, i(x)));
```

Example of a language of type ⟨2;2,1;1⟩.

predicate symbols: L, =
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$x_0, x_1, \ldots$
$x, y, z, \ldots$

$L(x,y)$ $L(x,x)$ $((\forall x) A)$

$x=y \to L(x,y)$

$\forall x \exists y\ L(x,y)$

$(\exists y \exists z (L(p(x,y), i(z)))) \to (\forall x \forall y\ x=y)$

$VAR = \{x, y, z, u\}$

```
int x;
int y;
int f(int x, int z) {
    ~~int y;~~
    x = 100;
    y = 100; u = 20;
}
x++;
y++;
x = f(2, 3);
```

$$\forall x \exists y (p(x) \rightarrow (q(x) \lor r(y)))$$

x, y

$\{x\}$

$\forall x\, (p(x) \to \exists x\, q(x))$

$$\alpha = \forall x \left(p(x, y) \wedge \exists y \, q(y) \right)$$

$*$

$$BV(\alpha) = \{x, y\}$$

$$FV(\alpha) = \{y\}$$

$$Q \in \{\forall, \exists\}$$

$$\cdots \underbrace{(Qx \,.\, \beta)} \cdots$$

$$\alpha = (p(x) \to (\exists x\, (q(x,y) \wedge r(z)))) \vee q(x,y)$$

$$FV(\alpha) = \{x, y, z\}$$

$$BV(\alpha) = \{x\}$$

$$\alpha \equiv (\forall x\, p(x)) \wedge q(y))$$

$$FV(\alpha) = \{y\}$$

$$BV(\alpha) = \{x\}$$

$$\dfrac{\begin{matrix}\mathcal{D}\\ \varphi(x)\end{matrix}}{\forall x\, \varphi(x)} \quad x \notin FV(hp\,\mathcal{D})$$

$$\Downarrow$$

$$\dfrac{\begin{matrix}\mathcal{D}\\ \varphi(x)\end{matrix}}{\forall x\, \varphi(x)}$$

$\Leftarrow$ x è un valore generico che non dipende dalle ipotesi

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\forall x \forall y\, \varphi(x,y)]^1}{\forall y\, \varphi(x,y)}}{\varphi(x,y)}}{\forall x\, \varphi(x,y)}}{\forall y \forall x\, \varphi(x,y)}}{(\forall x \forall y\, \varphi(x,y)) \to (\forall y \forall x\, \varphi(x,y))}\,1
$$