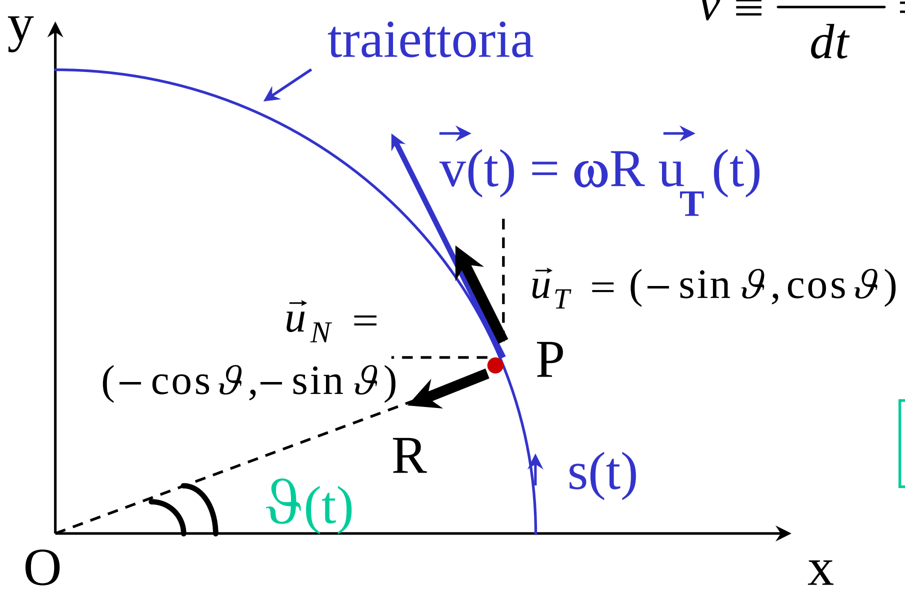
Moto circolare uniforme:

velocità con modulo costante:

coordinata curvilinea $s(t)=R\vartheta(t)$

$$v \equiv \frac{ds(t)}{dt} = R\frac{d\vartheta(t)}{dt} = \omega R$$



"velocità angolare"

$$\omega \equiv \frac{d\vartheta(t)}{dt}$$

$$\vartheta(t) = \vartheta_0 + \omega t$$

$$x(t) = R\cos\vartheta(t) \qquad y(t) = R\sin\vartheta(t)$$

$$\Rightarrow \quad v_x(t) = \frac{dx(t)}{dt} = -R\sin\vartheta(t)\frac{d\vartheta}{dt} \equiv -R\omega\sin\vartheta(t)$$

$$v_y(t) = \frac{dy(t)}{dt} = R\cos\vartheta(t)\frac{d\vartheta}{dt} \equiv R\omega\cos\vartheta(t)$$

$$\Rightarrow \quad \vec{v}(t) = (v_x(t), v_y(t)) = R\omega(-\sin\vartheta(t), \cos\vartheta(t))$$

$\vec{u}_T$

$$\Rightarrow \quad \vec{v}(t) = R\omega\vec{u}_T(t) = v\vec{u}_T(t)$$

$$a_x(t) = \frac{dv_x(t)}{dt} = -R\omega\cos\vartheta(t)\frac{d\vartheta}{dt} = -R\omega^2\cos\vartheta(t)$$

$$a_y(t) = \frac{dv_y(t)}{dt} = -R\omega\sin\vartheta(t)\frac{d\vartheta}{dt} = -R\omega^2\sin\vartheta(t)$$

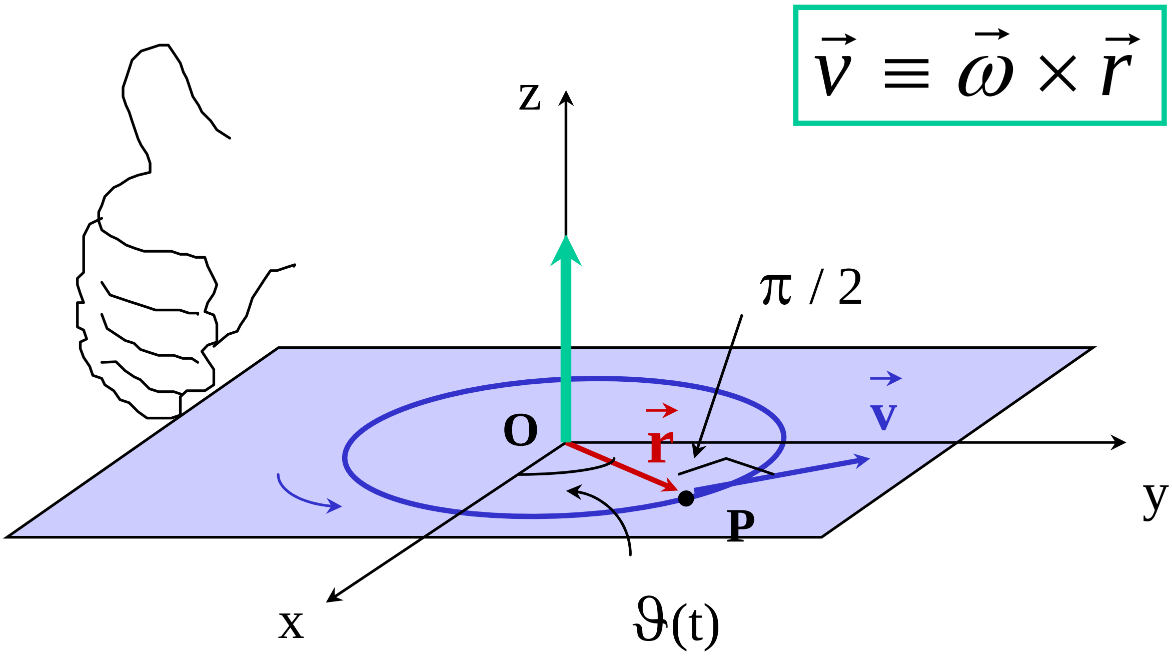
$$\Rightarrow \quad \vec{a}(t) = (a_x(t), a_y(t)) = R\omega^2(-\cos\vartheta(t), -\sin\vartheta(t))$$

$\vec{u}_N$

$$\Rightarrow \quad \vec{a}(t) = R\omega^2\vec{u}_N(t) = \frac{v^2}{R}\vec{u}_N(t)$$

Moto circolare in notazione vettoriale:

Vettore velocità angolare $\vec{\omega}$



$$\omega \equiv \frac{d\vartheta(t)}{dt}$$

$\Rightarrow$

$\vec{\omega}$ è $\perp$ al piano del moto, con verso definito dalla "regola della mano destra"

Infatti:

$$\left|\vec{\omega} \times \vec{r}\right| \equiv \omega r \sin\frac{\pi}{2} = \omega r \equiv v = \frac{ds(t)}{dt} = r\,\frac{d\vartheta(t)}{dt} \quad \Rightarrow \quad \omega \equiv \frac{d\vartheta(t)}{dt}$$

Vettore accelerazione angolare $\vec{\alpha}$

$$\vec{\alpha} \equiv \frac{d\vec{\omega}(t)}{dt}$$

Accelerazione:

$$\vec{a} \equiv \frac{d\vec{v}(t)}{dt} = \frac{d(\vec{\omega} \times \vec{r})}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$\frac{d\vec{\omega}}{dt} \equiv \vec{\alpha} \qquad \frac{d\vec{r}}{dt} = \vec{v} \equiv \vec{\omega} \times \vec{r}$$

$$\Rightarrow \quad \vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$\vec{\alpha} \times \vec{r} = \vec{a}_T \qquad \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \vec{a}_N$