$\Box \iff \times R_Y$

$\bigcirc \iff \exists z \mathcal{N}\, u$

$\mathcal{N}^* \subseteq R$

$R = \mathcal{N}^*$

$\Gamma \vdash \varphi$ if $\varphi \in \Gamma$

$\Gamma', \varphi \vdash \varphi$

$\Gamma \cup \{\alpha\}$

Γ, α

• φ φ

$\Gamma \vdash \varphi,\ \Gamma' \vdash \psi \Rightarrow \Gamma \cup \Gamma' \vdash \varphi \wedge \psi$

Γ, Γ'

$hp(\mathcal{D})$ = insieme delle ipotesi non cancellate di $\mathcal{D}$

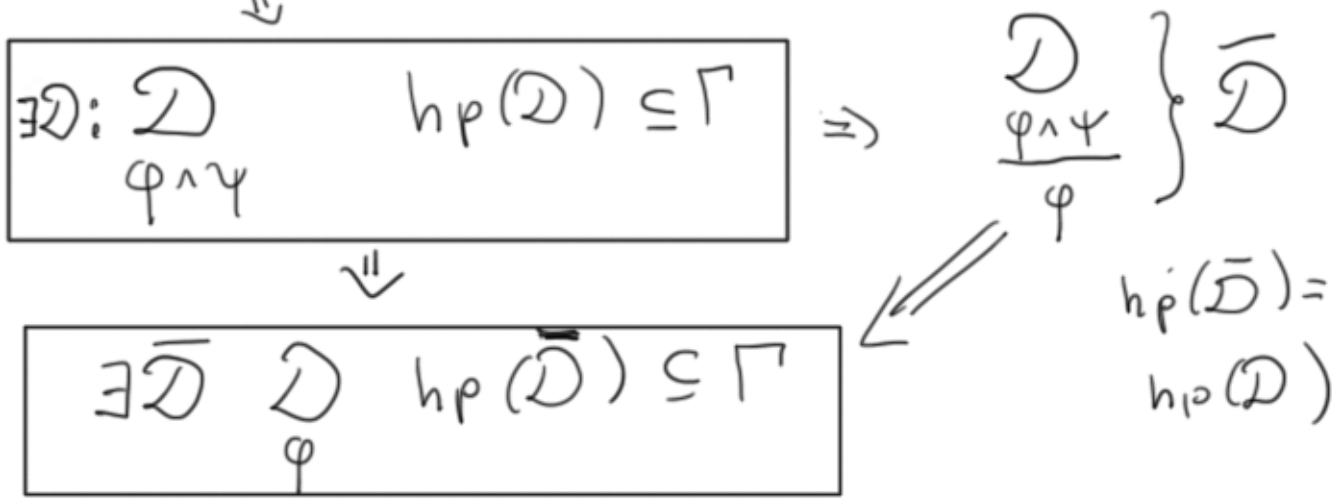
$$\left[\begin{array}{c}\mathcal{D}\\ \varphi\end{array}\ hp(\mathcal{D}) \subseteq \Gamma\right] \quad \left[\begin{array}{c}\mathcal{D}'\\ \psi\end{array}\ hp(\mathcal{D}') \subseteq \Gamma'\right]$$

$$\mathcal{D}^* \left\{ \begin{array}{c} \begin{array}{cc}\mathcal{D} & \mathcal{D}'\\ \varphi & \psi\end{array} \\ \hline \varphi \wedge \psi \end{array} \right.$$

$hp(\mathcal{D}^*) = hp(\mathcal{D}) \cup hp(\mathcal{D}') \subseteq \Gamma \cup \Gamma'$

$$\Gamma \vdash \varphi \wedge \psi \Rightarrow \left(\Gamma \vdash \varphi \text{ and } \Gamma \vdash \psi \right)$$

1.) $\Gamma \vdash \varphi \wedge \psi \Rightarrow \Gamma \vdash \varphi$



$$\Gamma, \varphi \vdash \psi \Rightarrow \Gamma \vdash \varphi \to \psi$$

$$\exists \mathcal{D} \quad \begin{matrix} \varphi \\ \mathcal{D} \\ \psi \end{matrix} \quad \& \quad hp(\mathcal{D}) \subseteq \Gamma \cup \{\varphi\}$$

$$\overline{\mathcal{D}} = \left\{ \dfrac{\begin{matrix} [\varphi] \\ \mathcal{D} \\ \psi \end{matrix}}{\varphi \to \psi} \right.$$

$$hp(\overline{\mathcal{D}}) = hp(\mathcal{D}) - \{\varphi\} \subseteq \Gamma$$

$$\Downarrow$$

$$\Gamma \vdash \varphi \to \psi$$

$\vdash \Phi \to (\psi \to \Phi)$ $\models \varphi \to (\psi \to \varphi)$

$\overset{\mathcal{D}}{\varphi \to (\psi \to \varphi)}$ $hp(\mathcal{D}) = \emptyset$

$$\dfrac{\dfrac{[\varphi]^1}{\psi \to \varphi}}{\varphi \to (\psi \to \varphi)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\alpha]^2\ [\beta]^1}{\alpha\wedge\beta}\ [(\alpha\wedge\beta)\to\gamma]^3}{\gamma}}{\beta\to\gamma}\,1}{\alpha\to(\beta\to\gamma)}\,2}{((\alpha\wedge\beta)\to\gamma)\to(\alpha\to(\beta\to\gamma))}$$

$$\dfrac{\begin{array}{|c|}\hline [(\alpha\wedge\beta)\to\gamma]^1 \\ \mathcal{D} \\ \alpha\to(\beta\to\gamma) \\ \hline \end{array}}{((\alpha\wedge\beta)\to\gamma)\to(\alpha\to(\beta\to\gamma))}\to_1$$

$$\vdash \alpha \to \neg\neg\alpha$$

$$\vdash \neg\neg\alpha \to \alpha$$

$$\dfrac{\dfrac{\dfrac{[\neg\alpha]^1\ [\neg\alpha]^2}{\bot}}{\alpha}\,1}{(\neg\alpha)\to\alpha}$$

$$\dfrac{\dfrac{[\neg\alpha]}{\mathcal{D}}}{\dfrac{\bot}{\alpha}}$$

$$\vdash (\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma))$$

$$\dfrac{\dfrac{\dfrac{[\alpha \to (\beta \to \gamma)]^{2} \quad \mathcal{D} \quad \alpha \to \gamma}{(\alpha \to \beta) \to (\alpha \to \gamma)}}{(\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma))}\,1}$$

OBIETTIVO : TROVARE $\mathcal{D}$

$$\mathcal{D} \begin{cases} \alpha \to (\beta \to \gamma) & \alpha \to \beta & [\alpha]^{3} \\ & \overline{\mathcal{D}} & \\ & \gamma & \\ & \alpha \to \gamma & 3 \end{cases}$$

$$\left.\begin{array}{c} \dfrac{\alpha \quad \alpha \to \beta}{\beta} \qquad \dfrac{\alpha \quad \alpha \to (\beta \to \gamma)}{\beta \to \gamma} \\ \hline \gamma \end{array}\right\} = \overline{\mathcal{D}}$$

$$\vdash ((\neg\alpha \to \beta) \wedge (\neg\alpha \to \neg\beta)) \to \alpha$$

$$\dfrac{\dfrac{\dfrac{[\neg\alpha]^1 \quad \dfrac{[(\neg\alpha\to\beta)\wedge(\neg\alpha\to\neg\beta)]}{\neg\alpha\to\beta}}{\beta} \quad \dfrac{[\neg\alpha]^1 \quad \dfrac{[(\neg\alpha\to\beta)\wedge(\neg\alpha\to\neg\beta)]^2}{\neg\alpha\to\neg\beta}}{\neg\beta}}{\bot}}{\alpha}$$

$$\dfrac{}{((\neg\alpha\to\beta)\wedge(\neg\alpha\to\neg\beta)) \to \alpha}$$

$\vdash \alpha \to \alpha$

$$\frac{[\alpha]}{\alpha \to \alpha}$$

$$\frac{\begin{array}{c}[\alpha)\\ \vdots \\ \beta\end{array}}{\alpha \to \beta}$$

$\Gamma \models \varphi$ if $\varphi \in \Gamma$

$\Gamma \models \varphi, \Gamma' \vdash \psi \Rightarrow \Gamma \cup \Gamma' \models \varphi \wedge \psi$

$\Gamma \models \varphi \wedge \psi \Rightarrow \Gamma \models \varphi$ and $\Gamma \models \psi$

$\Gamma \cup \varphi \models \psi \Rightarrow \Gamma \models \varphi \rightarrow \psi$

$\Gamma \models \varphi, \Gamma' \models \varphi \rightarrow \psi \Rightarrow \Gamma \cup \Gamma' \models \psi$

$\Gamma \models \bot \Rightarrow \Gamma \models \varphi$

$\Gamma \cup \{\neg\varphi\} \models \bot \Rightarrow \Gamma \models \varphi$

$\Gamma \models \alpha \Leftrightarrow \forall v$
$[\Gamma]_v = 1 \Rightarrow$
$[\alpha]_v = 1$

$$\vdash (\varphi \wedge \psi) \vee \sigma \rightarrow (\varphi \vee \sigma) \wedge (\psi \vee \sigma).$$

γ $\qquad$ ω

$$\dfrac{\gamma \vee \sigma \qquad \begin{matrix}[\gamma] \\ \vdots \\ \omega\end{matrix} \qquad \begin{matrix}(\sigma) \\ \vdots \\ \omega\end{matrix}}{\omega}$$

$\vdash \gamma \vee \sigma \rightarrow \omega$ $\qquad$ γ $\qquad$ $\equiv$ $\qquad$ $\varphi \wedge \psi$

$\vdots$ $\qquad$ $\vdots$

ω $\qquad$ $(\varphi \vee \sigma) \wedge (\psi \vee \sigma)$

$$\dfrac{\dfrac{\dfrac{\varphi \wedge \psi}{\varphi}}{\varphi \vee \sigma} \qquad \dfrac{\dfrac{\varphi \wedge \psi}{\psi}}{\psi \vee \sigma}}{(\varphi \vee \sigma) \wedge (\psi \vee \sigma)}$$

$$\dfrac{\dfrac{\sigma}{\varphi \vee \sigma} \qquad \dfrac{\sigma}{\psi \vee \sigma}}{(\varphi \vee \sigma) \wedge (\psi \vee \sigma)}$$