Asynchronous Design Seminar at University of Verona – Lecture Notes 5

Asynchronous Control Circuits – One-Hot FSMs and PTnet Theory

Contents

- One-Hot FSM Templates
  - Linear States
  - Choice States
- Hollaar's and David's Approaches
  - SI vs. non SI operation during State Transitions
- Extensions to PTnets
  - Fork/Join Structures
- Hazard-freeness of Combinational Logic
  - How to check?

One-Hot/One-Cold Encoding

- What is unique to One-Hot/One-Cold State Codes?

State Number	Sequential Encoding	Gray Encoding	Johnson Encoding	One-hot Encoding
0	0000	0000	00000000	0000000000000001
1	0001	0001	00000001	0000000000000010
2	0010	0011	00000011	0000000000000100
3	0011	0010	00000111	0000000000001000
4	0100	0110	00001111	0000000000010000
5	0101	0111	00011111	0000000000100000
6	0110	0101	00111111	0000000001000000
7	0111	0100	01111111	0000000010000000
8	1000	1100	11111111	0000000100000000
9	1001	1101	11111110	0000001000000000
10	1010	1111	11111100	0000010000000000
11	1011	1110	11111000	0000100000000000
12	1100	1010	11110000	0001000000000000
13	1101	1011	11100000	0010000000000000
14	1110	1001	11000000	0100000000000000
15	1111	1000	10000000	1000000000000000

One-Hot/One-Cold Encoding

- One-Hot encoding is not actually an encoding per se
  - Simple and unique assignment of binary codes to symbolic states
- Very commonly used in industry, why?
  - Fast in terms of CAD tools time → no computation required with respect to binary encoding states
  - Fast in terms of circuit operation → no encoding/decoding logic required in the fan-in/fan-out of state variables
  - One-Hot FSM is compositional → easy to break to pieces
- Disadvantage?
  - Number of binary signals linearly dependent on number of states
  - In many cases this is not a problem as FSMs are usually small

One-Hot/One-Cold Encoding in Asynchronous Design

- In asynchronous design, encoded states may present **races** in state transitions
  - Presented earlier in Hazards/Race analysis Methods
- One-Hot Encoding transitions are **regular**
  - Two signal transitions per state transition
  - 1→ 0 and 0 → 1 depending on current/next state
- Is One-Hot Encoding race-free?
  - NO! But the race is regular and can thus be rectified by using a regular circuit structure
- One-hot Critical Race (for any n states add 0s):
  - 10 → 11 → 01 (2 states – unique)
  - 100→110→010→011→001 (3 states – not unique)

One-Hot Critical Race

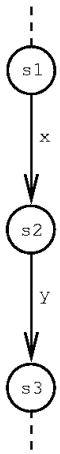
- For any two state transition (for any n states add 0s):
  - **Correct** transition: 10 → 11 → 01
  - *Erroneous* transition: 10 → 00 → XX
- Avoiding the Race imposes a Race Constraint
  - ***Enter the next state before leaving the current state***
  - Only the first transition is acceptable: 10 → 11 → 01
  - No interference between next/current state (11) due to one-hot encoding!
- How can the Race Constraint be satisfied?
  - As a timing constraint for the state set and reset paths
  - By circuit modification

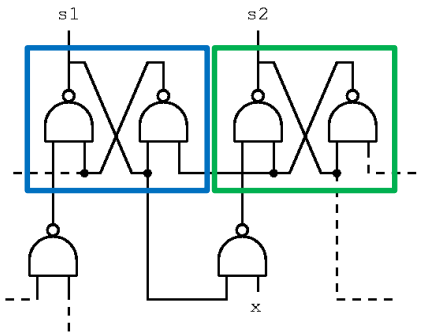
One-Hot FSMs

- Regular Template Circuit Structure
  - State is SR Latch
    - Implemented using standard-cells or as a single latch cell
  - Set and Reset conditions implemented typically in 2-level Logic
  - **Set** condition =
    `Previous State AND Enter Conditions for Current State`
  - **Reset** condition = `Next State (Entered)`
- May be implemented as SI or Fundamental Mode
  - Depends on Environment constraints
  - Depends on Local Timing constraints required for Correct operation

One-Hot FSMs – Linear FSM Example

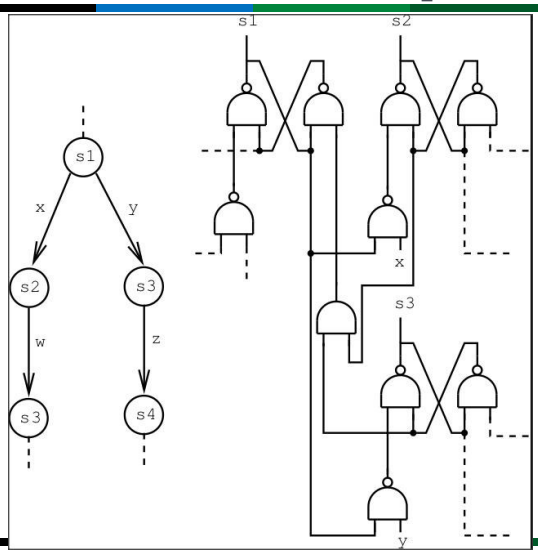
- Each One-hot state may be implemented using an SR latch
- **Set** input
  - State entering conditions
- **Reset** input
  - State exit conditions
- Both Active-low due to NAND SR latch
- Regular Circuit Structure!





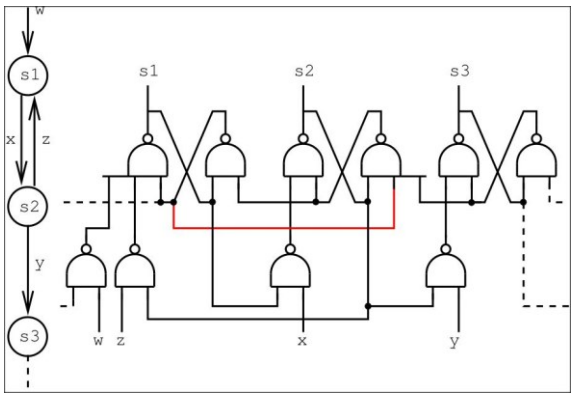
One-Hot FSMs – Choice FSM Example

- Choice can easily be implemented
  - Reset condition for state `s1` is the Set condition of `s2` OR `s3`
  - Transforms to AND due to active-low Reset
- Similarly for return from Choice
  - Reset for predecessors is Set of successor



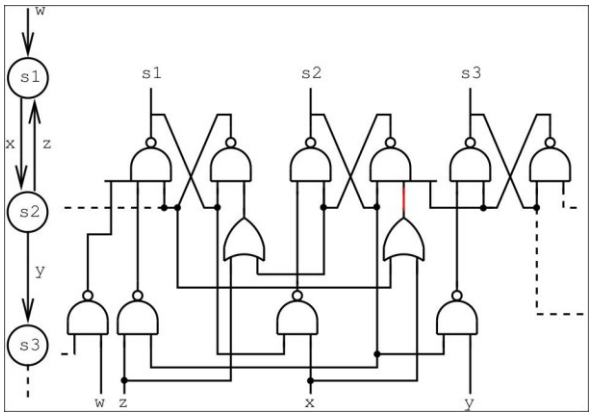
Scale-of-Two Loops

- For correct operation *Set* and *Reset* conditions must be **Mutually-Exclusive**
- If the predecessor and successor states are the same this correctness condition is violated if Set/Reset conditions overlap in time
- What if both `x` and `z` are high in RHS?



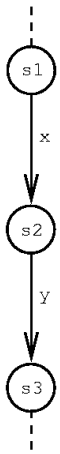
Scale-of-Two Loops - Solutions

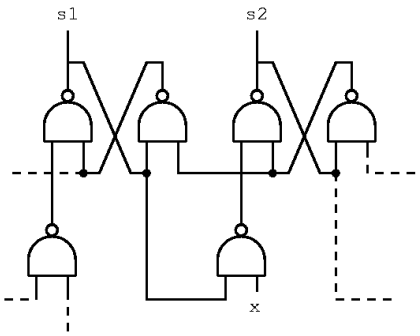
- To ensure Mutually-Exclusive Set and Reset signals we can either:
  - **Add intermediate *dummy* state** (SI operation)
  - **Add transition signals to Reset conditions** (normally ignored)



One-Hot Race

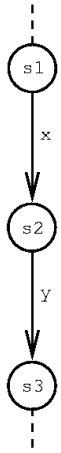
- Is this approach Race-Free?
  - NO!!!
  - Why not?
- It ***does not enforce*** One-Hot Race-Free behaviour:
  - 10 → 11 → 01
- Can it be made Race-Free?

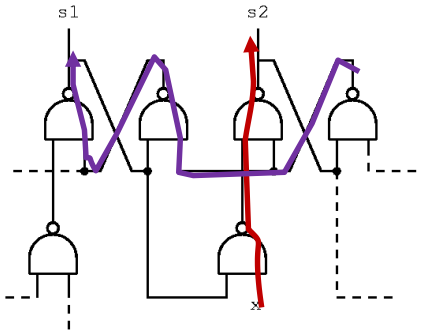




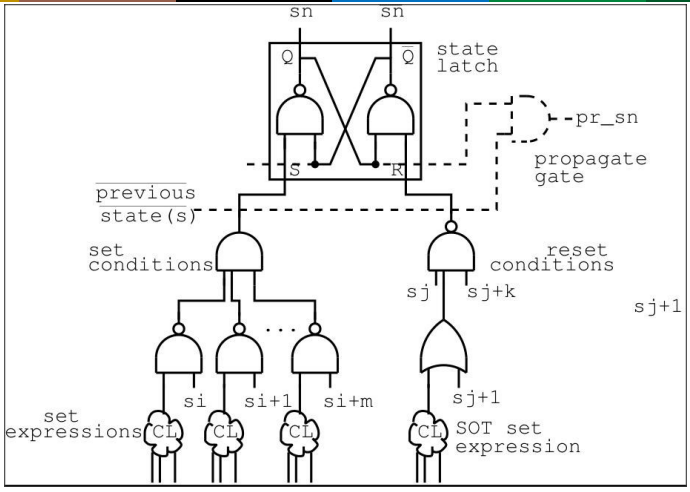
One-Hot Race

- Can it be made Race-Free?
  - Yes
- Two approaches:
  - Ensure Set path is faster than Reset path (for two or *more states*!!!)
- SI Solution:
  - Add a propagate gate (current state and NOT previous state)
  - `PR = CS.PS'`
  - Use `PR_sn` instead of sn





General One-Hot FSM State Structure



- Propagate gate (`pr_sn`) ensures SI operation

What about Hazards? Is this a Hazard-free Approach?

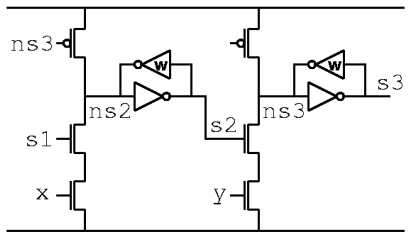
- It is possible for the Set and Reset Logic to contain Logic Hazards
- How can this be resolved?
- Hazards may be analysed using Fundamental or SI Analysis Techniques
- Logic Hazards can be removed by restructuring Set/Reset Logic

Other Observations about One-Hot FSMs

- It is possible to extend One-Hot FSMs to model FSMs with Forks and Joins!
  - Allow multiple One-hot states to be high between fork and join states
- How?
  - Modify reset conditions

CMOS One-Hot FSMs

- *Possible to implement One-Hot FSMs at Transistor-Level*
- Set and Reset Conditions become **Pull-up** and **Pull-down** networks of State-storing gates



Example where CSC has no Solution!

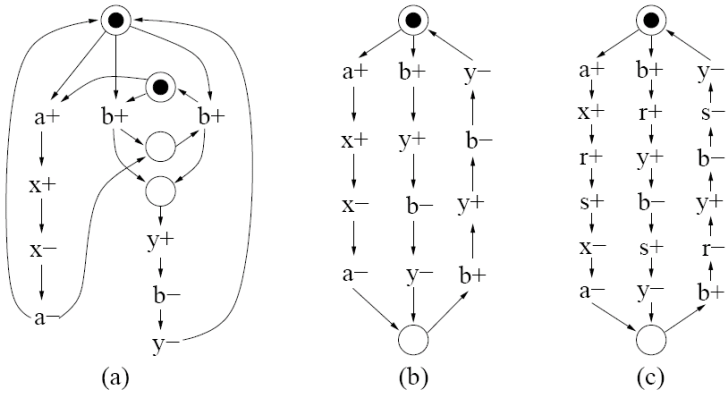


Figure 5: Example with unsolvable CSC conflict.

DAC'06: Carmona and Cortadella: "State Encoding of Large Asynchronous Controllers"

Contents

- Fundamental Definitions
  - Net, Marking, etc.
- PTnet Classes
- Siphons/Deadlocks and Traps
  - Handles
- S-Covers and T-Covers
- An Algorithm for S-Covering FCPTnets

PTnet Definitions

- Definition [**PTnet**]:
  - A Place Transition net (or PTnet) is a triple $N = (S, T, F)$, where
    - S is the set of *places*,
    - T is the set of *transitions* ($S \cap T = \varnothing$)
    - $F \subseteq (S \times T) \cup (T \times S)$ is the *flow relation*
- Elements of $S \cup T$ are called nodes
- *pre-set* $\bullet x$ of x in $(S \cup T)$ is given by
  - $\bullet x = \{y \text{ in } S \cup T \mid (y, x) \text{ in } F\}$
- *post-set* $x\bullet$ of x in $(S \cup T)$ is given by
  - $x\bullet = \{y \text{ in } S \cup T \mid (x, y) \text{ in } F\}$
- A *Marking M* is a function $M: S \rightarrow \mathbb{N}$
- A *Marked PTnet* $<N, M0>$ is a PTnet with an initial marking

PTnet Definitions

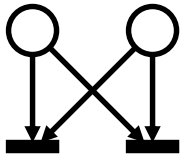
- A transition t in T is *enabled* at marking M iff:
  - For all p in •t: M(p) > 0
- When t *fires* at M a new marking M' is produced:
  - M'(p) = M(p) – $\mathbb{F}$(p, t) + $\mathbb{F}$(t, p) ($\mathbb{F}$ is characteristic function of F)
  - M[t>M' denotes that M' is reachable from M by the occurrence of transition t
- The set of markings reachable from the initial marking M0 by the occurrence of a sequence of transitions $\sigma = t_1 t_2 \ldots t_n$ is denoted by *R(N, M0)*

PTnet Definitions – Net Subclasses

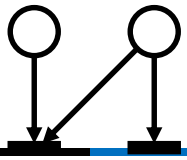
- Definition [**S-graph – State Machine**]:
  - A net N = (S, T, F) is called an *S-graph or State Machine,* iff each transition has exactly one input place and one output place
  - for all t in T: |•t| = 1 = |t•|
- Definition [**T-graph – Signal Transition Graph**]:
  - A net N = (S, T, F) is called an *T-graph or STG,* iff each place has exactly one input transition and one output transition
  - for all s in S: |•s| = 1 = |s•|

PTnet Definitions – Net Subclasses

- Definition [**Free-Choice**]:
  - N is called *free-choice* iff all p in S such that |p•|>1, •(p•) = p

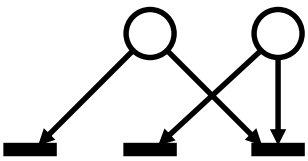


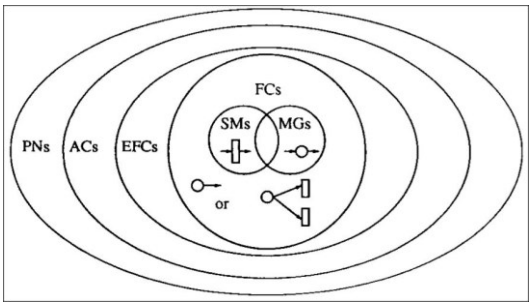
- Definition [**Asymmetric-Choice**]:
  - N is called *asymmetric-choice* (or simple net) iff for every two places p1, p2, p1•∩ p2• ≠ ∅ ➔ p1• ≤ p2• or p1• ≥ p2•



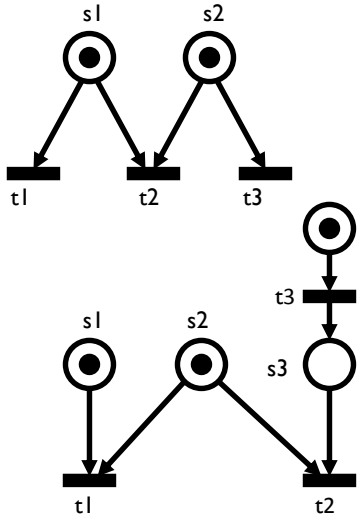
PTnet Definitions – Net Subclasses

- Definition [**General PTnet**]:
  - N is called *general PTnet* iff for every two places p1, p2, p1•∩ p2• ≠ ∅ ➔ p1• ≦ p2• or p1• ≧ p2•



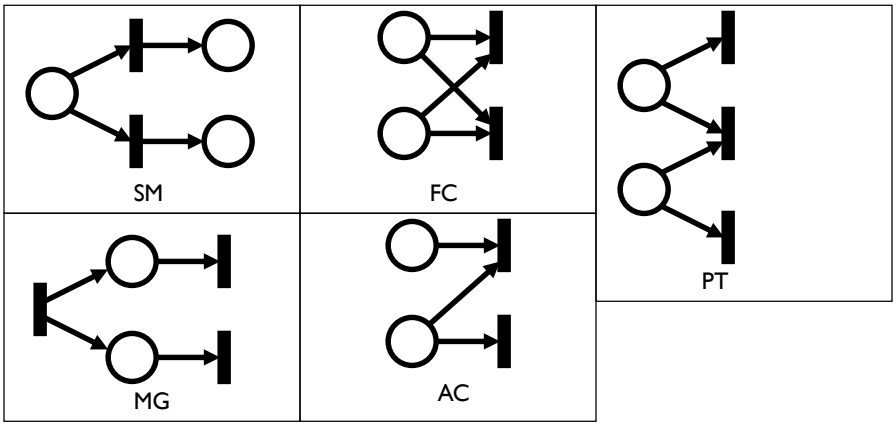


Asymmetric or Symmetric Confusion



- Cases where Choice and Concurrency are combined are referred to as **confusion**
  - Symmetric Confusion
    - **t1, t3 are concurrent but in conflict with t2**
  - Asymmetric Confusion
    - **t1, t2 are in conflict, if t3 fires before t1!**
    - **t1, t3 are concurrent**

PTnet Classification Revisited



PTnet Definitions - Subnets

- Definition [**S-Components and T-Components**]:
  - N' = (S', T', F') is a *subnet* of N = (S, T, F) iff S' ≤ S, T' ≤ T and F' = F ∩ ((S' x T') U (T' x S'))
  - N' is an ***S-component*** (***T-component***) of N iff N' is a strongly connected S-graph (T-graph) and| T' = •S'∩S'• (S' = •T'∩T'•).
  - N' is generated by a set X' ≤ S ∩ T iff:
    S' = (X' ∩ S) U •(X' ∩ T) U (X' ∩ T)•
    T' = (X' ∩ T) U •(X' ∩ S) U (X' ∩ S)•

PTnet Definitions – Behavioural Properties

- Definition [**Bounded Net**]:
  - A *marked net (N, M0)* is bounded iff:
  - for all p in S, there exists k in $\mathbb{N}$, s.t. for all markings M reachable from M0: M(p) ≤ k
- Definition [**Structurally Bounded Net**]:
  - A net N is *structurally bounded* iff it is bounded for any initial marking M0.
- Definition [**Liveness**]:
  - A transition t in T is *live* in (N, M0) iff:
    - For all M in R(N, M0) there exists M' in R(N, M): M' enables t.
  - The marked net (N, M0) is *live* iff all t in T are live.
  - N is *structurally live* iff there exists initial marking M0 such that (N, M0) is live

PTnet Definitions – Graph Properties

- Definition **[Path]**:
  - A *path* of a net N=(S, T, F) is an alternating sequence $\pi = (x_0 f_0 x_1 \ldots f_{r-1} x_r)$ of elements X = S U T such that: for all I, $0 \le i \le r - 1 : f_i = (x_i, x_{i+1})$ in F
  - A path is *elementary* iff all x_i are distinct except x_0 and x_r
  - A *circuit* is a path such that $x_0 = x_r$
  - A circuit is *elementary* iff it is elementary as a path
- Definition**[Alternating Circuit]**
  - Let N = (S, T, F) be a net. A circuit Γ of N (not necessarily elementary) is an *alternating circuit* iff for all arcs in Γ of the form (p, t) the equality •t = {p} holds

PTnet Definitions - Subnets

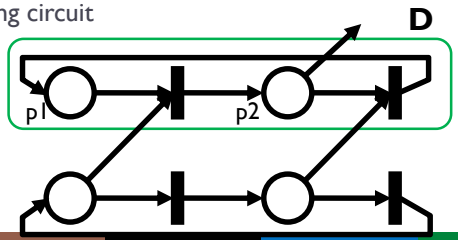
- Definition **[Well-formed Net]**:
  - A Net N is *well-formed* if **there exists a marking** M0 of N such that (N, M0) is a live and bounded system
- Thus, the net is not necessarily live at the current marking…
- Theorem **[S-Components and Well-Formed Nets]**:
  - Well formed Nets are covered by S-Components

PTnet Definitions – Siphons and Traps

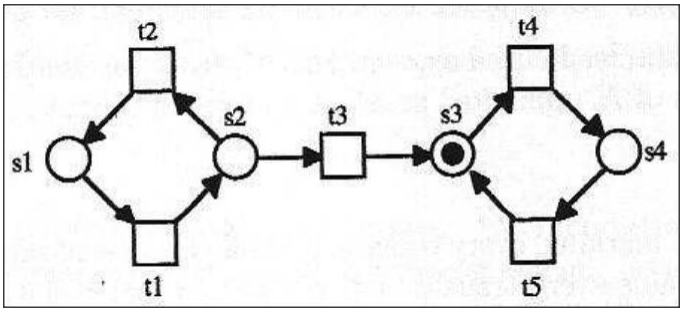
- Definition [**Siphons (Deadlocks) and Traps**]:
  - Let N = (S, T, F) be a net.
  - D ≤ S is a *siphon* (*deadlock*) iff D ≠ ∅ and •D ≤ D•
  - Θ ≤ S is a *trap* iff Θ ≠ ∅ and Θ• ≤ •Θ
  - A siphon (deadlock) or trap is *minimal* iff there exists no deadlock or trap D' such that D' ≤ D
  - A siphon (deadlock) or trap is *strongly-connected* iff the subnet generated by D U •D is strongly connected
- **Commoner's Theorem**
  - An FC System is live iff each siphon contains a marked trap
    - Each cycle is marked

PTnet Definitions – Siphons and Traps

- Theorem[**Minimal Siphon for General PTnets**]
  - Let N = (S, T, F) be a net, D ≤ S a siphon of N and N_D the subnet of N generated by D U •D.
  - D is minimal iff there exists a *circuit* Γ in N_D (not necessarily elementary) that passes through all places of D such that for every transition t in Γ either:
  - |•t ∩ D| = 1 (*if net is FC*) or
  - |•t ∩ D| ≥ 2 and the places of (•t ∩ D) belong to an alternating circuit

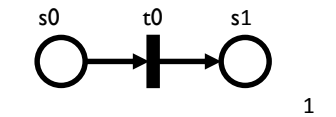


PTnet Definitions – Siphons and Traps



- The set {s1, s2} is a siphon; the set {s3, s4} is a trap

Siphon/Trap Examples



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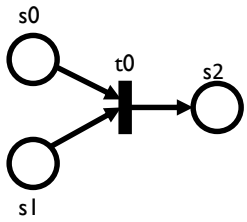
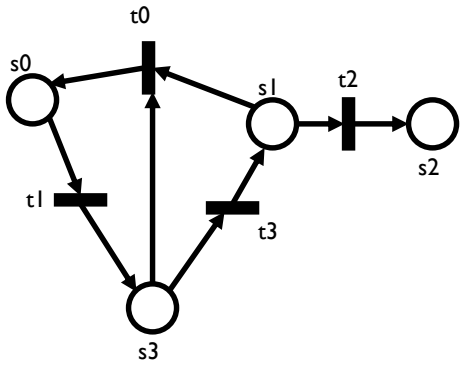


2

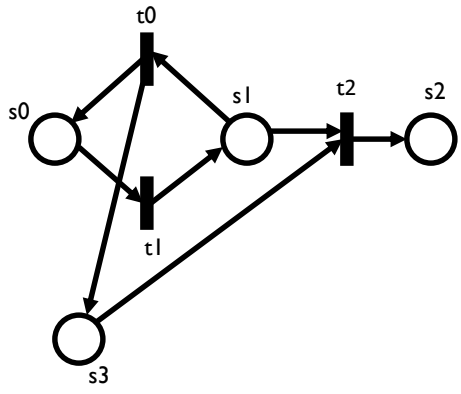
- 1:
  - s0 is a siphon (minimal)
  - s1 is a trap
- 2:
  - {s0, s1} is a siphon
    - NOT minimal, as structurally {s0}, {s1} are also siphons

Siphon/Trap Examples



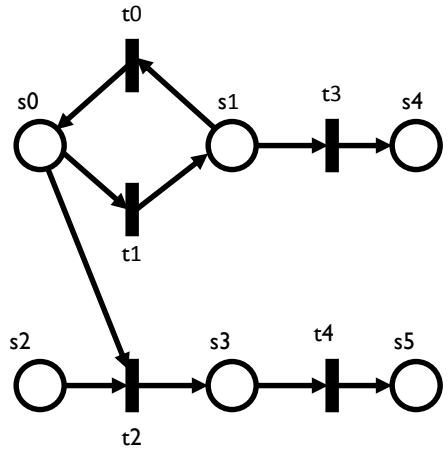
- Siphon
  - D = {s0, s1, s3}
  - •t0 = {s1, s3}
  - |•t0 ∩ {s1, s3}| = 2
  - D is not minimal
- Minimal Siphon
  - {s0, s3}

Siphon/Trap Examples



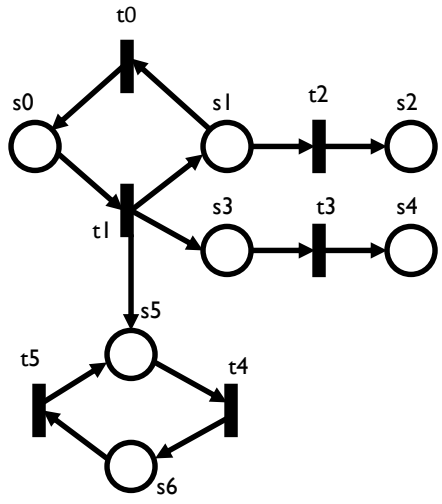
- Minimal Siphon
  - {s0, s1}
- Minimal Trap
  - {s2}

Siphon/Trap Examples



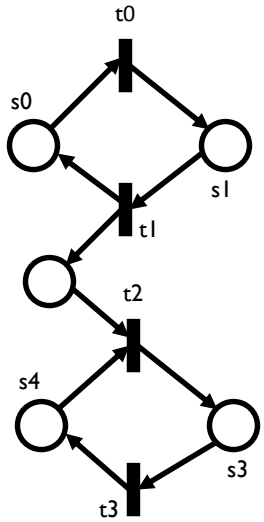
- Minimal Siphon
  - {s2}
- Minimal Traps
  - {s4}, {s5}

Siphon/Trap Examples



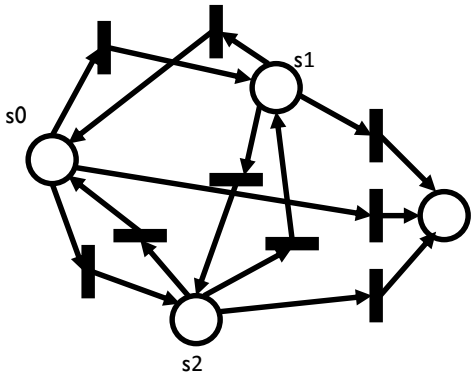
- Minimal Siphon
  - {s0, s1}

Siphon/Trap Examples



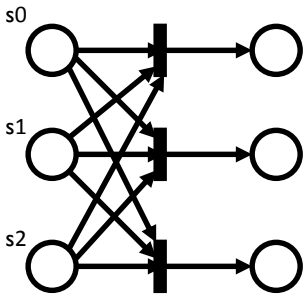
- Minimal Siphons/Traps
  - {s0, s1}
  - {s3, s4}

Siphon Example and Complexity



- Places s0, s1, s3 are choice places to all others
- Minimal Siphon
  - {s0, s1, s2}
- Are NOT Siphons
  - {s0, s1}, {s1, s2}, {s0, s2}
  - {s0}, {s1}, {s2}
- Complexity ~O(S,T)

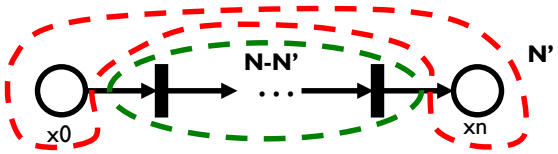
Siphon Example and Complexity



- Free-Choice structure between multiple place siphons
  - Minimal siphons = {s0}, {s1}, {s3}
- Siphons
  - {s0}, {s1}, {s3}
  - {s0, s1}, {s0, s2}, {s1, s2}
  - {s0, s1, s2}
- The number of Siphons is **WC Exponential!**
- *Should not aim to explore ALL siphons*

PTnet Definitions – Graph Properties

- Definition [**Handle**]:
  - Let N' be a partial subnet of N.
  - An elementary path $\pi = (x_0 f_0 x_1 \ldots f_{n-1} x_n)$ is a *handle* of N' iff $\pi \cap N' = \{x_0, x_n\}$



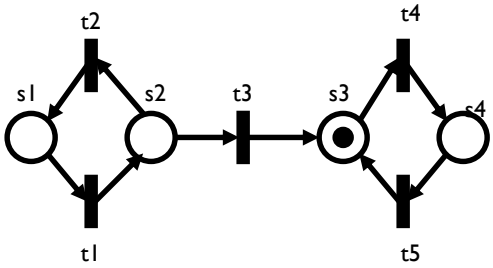
PTnet Examples

PTnet Examples - 1

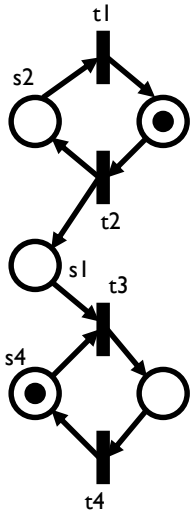
1.

B, L

2.



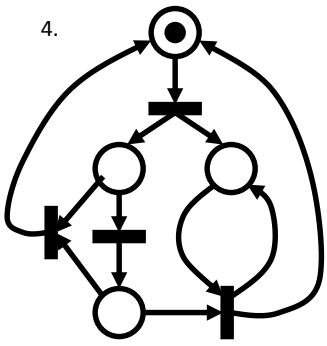
B, L'



B', L

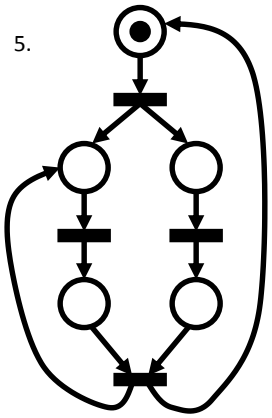
PTnet Examples - 2

4.



B', L'

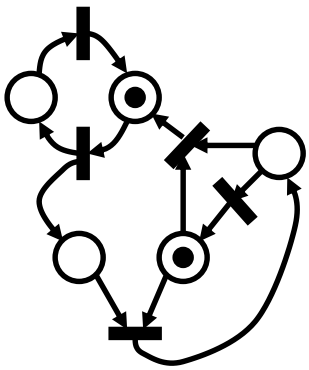
5.



B', L

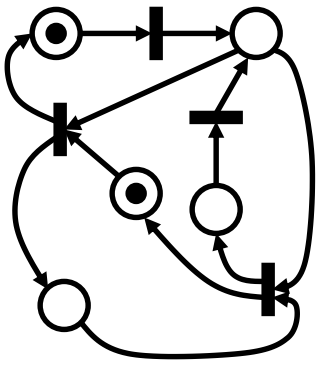
PTnet Examples - 3

6.



B', L'

7.



B, L

Minimal Siphons and S-Components – Esparsa/Kemper Algorithm

Get Handle Algorithm

Algorithm Get_Handle(S, S', F, t, p)

```
// S ∩ S' = ø, p in S, t in S' and t is in •p //
i = 1; Stack = empty;
forall x in S' num(x) = 0;
forall x in S num(x) = -1;

push(Stack, p);
if (handle-DFS(t) == 0)
{
  return NULL;
  printf("No handle exists\n");
}
else
{
  return Stack; // Stack contains Handle //
}
```

Algorithm handle-DFS(v)

```
num(v) = i; i = i + 1;

push(Stack, v);

forall (w in •v)
  if (num(w) == -1) // start node of handle //
  {
     push(Stack, w);
     return 1;
  }
forall (w in •v)
  if (num(w) == 0) // new non-start node //
     if (handle-DFS(w) == 1) return 1;

pop(Stack, v);
return 0;
```

- N is strongly-connected FC-Net with S, S' ≤ P U T
- S ∩ S' = Ø, p in S, t in S', t is in •p, Outputs handle H = (x_0, x_1, …, x_{n-2}, t, p) or 0
- Num(v) values:
  - num(v) == -1 ➔ v in S, start point of handle, num(v) ==0 ➔ v in S', not visited before
  - num(v) > 0 ➔ v has been visited before and either (a) no handle was found or (b) has been reached again. In both cases v cannot belong to the handle

Get Minimal Siphon (Deadlock) Algorithm

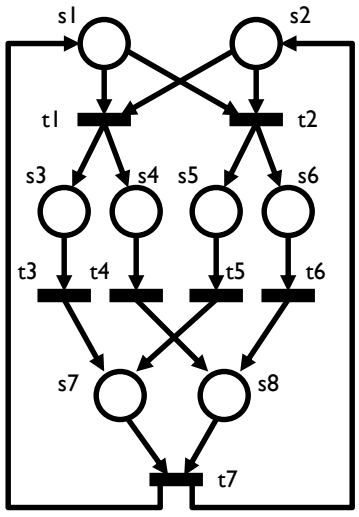
Algorithm Get_Minimal_Deadlock(P,T, F, p, D,Td)

```
Pc = {p}; Tc = 0; // current sets of Places and Transitions

while (there exists p' in Pc, there exists t in •p' and t not in Tc)
{
    H = Get_Handle((Pc U Tc), (P U T)-(Pc U Tc), F, p', t);
    Pc = Pc U (H ∩ P); // discard multiple instances of places
    Tc = Tc U (H ∩ T); // and transitions
}
D = Pc; Td = Tc;

return D, Tc; // return Deadlock places and transitions
```

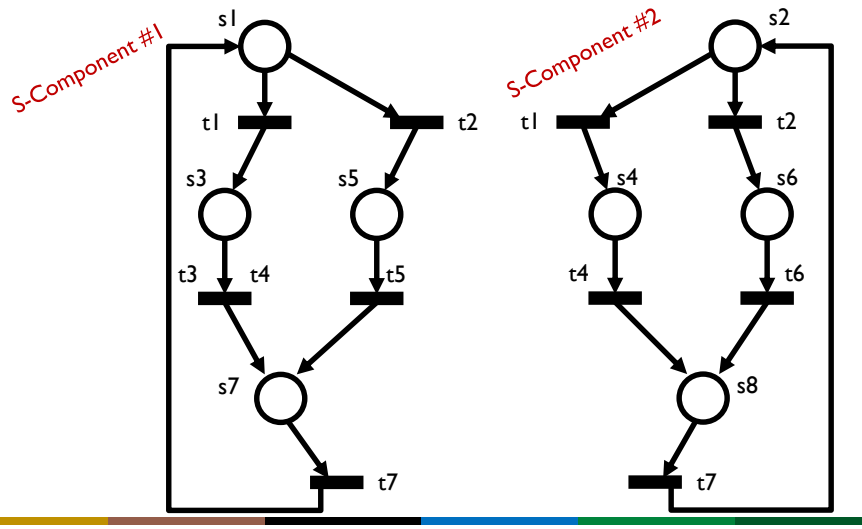
- N = (P,T, F) strongly-connected FC-Net with p in P
- D is minimal deadlock D ≤ P, containing P
- To be an S-Component, minimal deadlock D must satisfy:
  - It is strongly connected
  - For all t in D: |•t∩D| = |t•∩D| = 1

S-Covering - Example

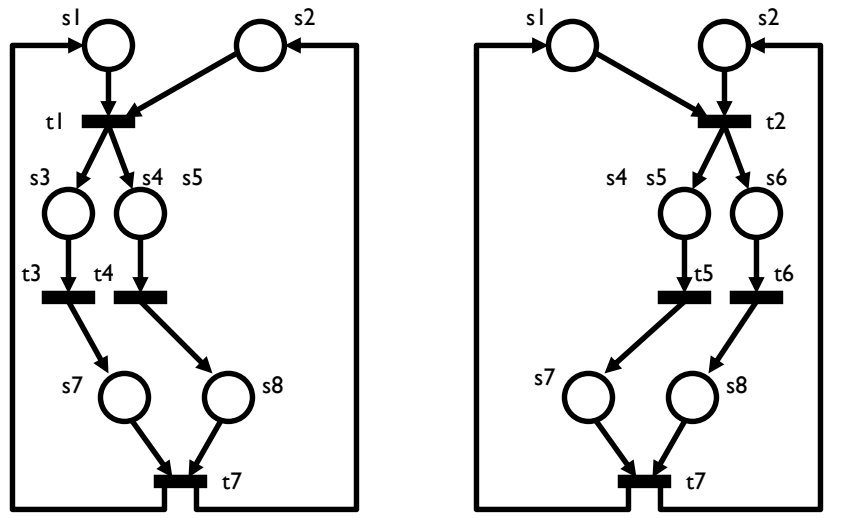
- Original Net:



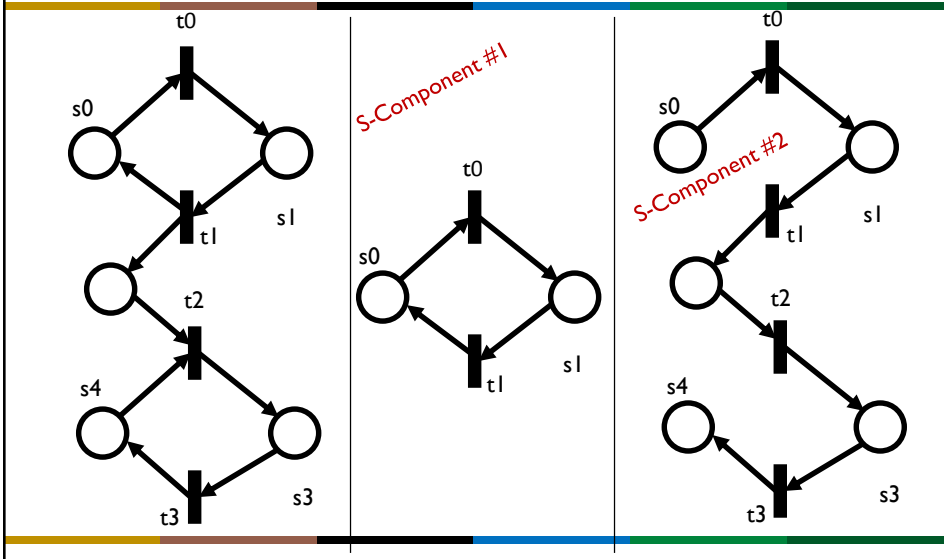
S-Covering – Example – S-Cover



T-Covering – Example T-Cover



S-Cover for non Well-formed Net



Minimal Siphons – Cordone et. al (PIPE2) Algorithms

Rules for Siphon Extraction/Minimisation

- Definition **[PTnet Reduction Function RED]**
  - Let G = (P,T, F) be a PTnet and ~P ≤ P.
  - The reduction function RED is defined as follows:
  - ~G = *red*(G, ~P), where the reduced net ~G = (~P, ~T, ~F) is defined by:
    - ~T = {t in T | (•t U t•) ∩ ~P ≠∅}
    - ~F(p, t) = F(p, t), ~F(t, p) = F(t, p), forall p in ~P, t in ~T
- Rule 1 [*place subset reduction*]
  - Let G = (P,T, F) be PTnet and ~P ≤ P
  - Set of siphons of G contained in ~P is the same as siphons of reduced net ~G = *red*(G, ~P)

Rules for Siphon Extraction/Minimisation

- Places with ***empty subset*** and places with ***all net transitions*** as post-set are special cases!
- Rule 2 [*empty subset places*]
  - Let G = (P,T, F) be PTnet and _P ≤ P such that •_P = ∅
  - Then, minimal siphons of G are minimal siphons of ~G = *red*(G, P - _P), **plus the individual places in _P**
- Rule 3 [*all net transitions in post-set places*]
  - Let G = (P,T, F) be a PTnet such that P• = T
  - Then P is a siphon

Rules for Siphon Extraction/Minimisation

- Places in ***post-set of transitions with empty pre-set cannot belong to a siphon*** and can be eliminated
- If all transitions are in post-set of some place, T = P• then P is a siphon (see Rule 3)
- Rule 4 [*trap places elimination*]
  - Let G = (P,T, F) be a PTnet and let _T ≤ T be such that
  - •_T = Ø. Then, if _P = _T•, G has same siphons as
  - ~G = *red*(G, P - _P)
- Rule 5 [*redundant places*]
  - Let G = (P,T, F) be a PTnet and S ≤ P a siphon of G.
  - If there exists _p in S: for all t in _p• either
    (t• ∩ S) > {_p} or
    (•t ∩ S) = Ø, then S – {_p} is also a siphon of G

Rules for Siphon Extraction/Minimisation

- Rule 6 [*Siphon Minimality*]
  - Let G = (P,T, F) be a PTnet and S ≤ P a siphon of G.
  - S is minimal for G, iff all reduced nets:
  - ~Gp = *red*(G, S-{p}) for all p in S do not contain siphons
- Rule 7 [*Siphon Decomposition into Smaller Siphons*]
  - Let G = (P,T, F) be a PTnet and ~P = {p1, p2, …, pn} ≤ P
  - The set of Siphons of G NOT containing ~P is the Union

$$\bigcup_{i=1}^{n} S_i$$

where Si is the set of siphons of the reduced net:
~Gi = *red*(G, P – {pi}), *i.e.* containing the n places except i.

Find a Siphon - Algorithm

Algorithm Find_Siphon(G, ~P, P)

```
while (1)
{
    if ((exists p in P ∩ ~P) && (exists t in •p such that t not in P•))
    {
        S = Ø;                                  // Rule 4 //
        return;
    }
    if ((exists p in P - ~P) && (exists t in •p such that t not in P•))
        G = red(G, P - {p});                    // modifies local P and G//
    else
    {
        S = P;
        return;
    }
}
```

- ~P is a set of places (one or more) which should be contained in the siphon
- Elimination based on **Rule 4** (trap places)

Find a Minimal Siphon - Algorithn

Algorithm Find_Min_Siphon(G, ~S, ~P, P)

```
while (exists p in (P - ~P) ∩ ~S such that (•t ∩ ~S) < {p} or
                                           (t• ∩ ~S) = Ø)   // Rule 5 //
    ~S = ~S - {p};
while (1) {
    if (~S < P) G = red(G, ~S); Pnew = P - ~P;
    if (Pnew == Ø)
    {
        S = S~;
        return;                             // found minimal siphon //
    }
    forall (p in Pnew)                      // find smaller siphons of Pnew //
    {
        Gp = red(G, P - {p}); Pnew = Pnew - p;
        Sp = Find_Siphon(Gp, ~P, P);
        if (Sp != Ø)
        {
            ~S = Sp;
            break;                          // found smaller siphon - to outer loop //
        }
    }
}
```

- Computes one minimal siphon S ≤ ~S and containing ~P

Find all Minimal Siphons - Algorithm

Algorithm Find_All_Min_Siphons(G, ~P, P)

```
SS = Ø; // Minimal Siphon Set //
while ((~P == Ø) && (exists p in P such that •p = Ø)) {
    S = {p}; SS = SS U {S}; G = red(G, P - {p});          // Rule 2 //
}
~S = Find_Siphon(G, ~P, P);
if (~S == Ø) return;
S = Find_Min_Siphon(G, ~S, ~P, P);
SS = SS U S;
Pnew = S - ~P; Pold = Ø;
if (Pnew == Ø) return;
forall (p in Pnew) {
    Gp = red(G, P - {p});
    SSp = Find_All_Min_Siphons(Gp, ~P U Pold, P);          // Rule 7 //
    SS = SS U SSp;
    Pnew = Pnew - {p}; Pold = Pold U {p};
}
```

- Find_All_Min_Siphons(G, Ø) returns all minimal siphons of G
- Worst-case complexity is $O(2^n)$
  - Due to Recursive call for Rule 7