Oct. 25, 2012

I. Exercises Representation Theory

1. Exercise: Let $Q = (Q_0, Q_1)$ be a (non-empty) finite quiver without oriented cycles.

a) There always exist sinks and sources in Q.

b) The vertices of Q can be labelled by $1, \dots, n$ such that: if $i \to j$ is an arrow in Q_1, then $i < j$. 10 P.

(Hint: Induction by $n = |Q_0|$; consider the set of all sinks of Q.)

2. Exercise: Let $Q = (Q_0, Q_1)$ be a quiver without oriented cycles. For each $i \in Q_0$ let S_i be the K-linear representation given by

$$S_i(j) = \begin{cases} K & j = i \\ 0 & j \neq i, \end{cases}$$

and with $S_i(a) = 0$ for all $a \in Q_1$.

a) S_i is a simple representation.

b) For all $i, j \in Q_0$ with $i \neq j$ we have $S_i \not\simeq S_j$.

c) Let $V \neq 0$ be a representation of Q. Show that there is an $i \in Q_0$ and a monomorphism $S_i \to V$; there is a $j \in Q_0$ and an epimorphism $V \to S_j$. (Hint: Use part b) of Exerc. 1.) What can we conclude, if in particular V is a simple representation? 10 P.

3. Exercise: Let Q be the quiver consisting of precisely one vertex 1 and one loop $a\colon 1 \to 1$. For each $\alpha \in K$ let S_α be the K-linear representation with $S_\alpha(1) = K$ and linear map $S_\alpha(a) : K \longrightarrow K$ given by $S_\alpha(a)(x) = \alpha x$. Show that all S_α are simple representations such that $S_\alpha \not\simeq S_\beta$ for all $\alpha \neq \beta$. 10 P.