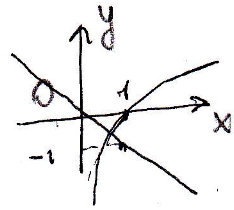
2) Studiare il grafico di $f(x) = -x^{-1}\ln x - 1 = -\frac{\ln x}{x} - 1 = -\frac{\ln x + x}{x}$

Ris

- dominio $A = \{x \in \mathbb{R} ; x > 0\}$
- segno $-\frac{\ln x + x}{x} > 0$ sse $\ln x + x < 0$ sse $\ln x < -x$



Sia $\bar{x} \in (0,1)$ t.c. $\ln \bar{x} = -x$

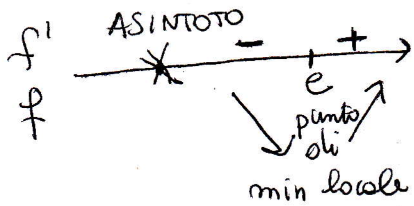
$\Rightarrow$ $f(x) > 0$ se $0 < x < \bar{x}$

$f(x) = 0$ se $x = \bar{x}$

$f(x) < 0$ se $x > \bar{x}$

- intersezione con gli assi: $(\bar{x}, 0)$ nessuna intersezione con l'asse y
- $f \in C^\infty(A)$ $\quad f'(x) = \left(-\frac{\ln x}{x} - 1\right)' = \frac{\ln x - 1}{x^2}$

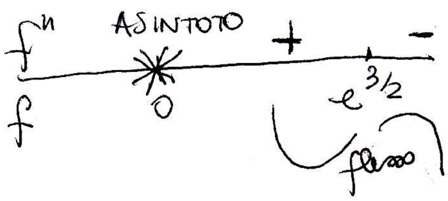
$f'(x) > 0$ sse $\frac{\ln x - 1}{x^2} > 0$ sse $\ln x > 1$ sse $x > e$



$f(e) = -\frac{\ln e}{e} - 1 = -\frac{1}{e} - 1$ minimo locale

- $f''(x) = \frac{\frac{1}{x}x^2 - (\ln x - 1)2x}{x^4} = \frac{x + 2x - 2x\ln x}{x^4} = \frac{x(3 - 2\ln x)}{x^4} = \frac{3 - 2\ln x}{x^3}$

$f''(x) > 0$ sse $\frac{3 - 2\ln x}{x^3} > 0$ sse $3 - 2\ln x > 0$ sse $\ln x < \frac{3}{2}$ sse $0 < x < e^{3/2}$



$f(e^{3/2}) = -\frac{\ln e^{3/2}}{e^{3/2}} - 1 = -\frac{3}{2}e^{-\frac{3}{2}} - 1$

$\lim_{x \to +\infty} -\frac{\ln x}{x}$ in $\left[\frac{\infty}{\infty}\right]$

$y = -1$ asintoto orizzontale

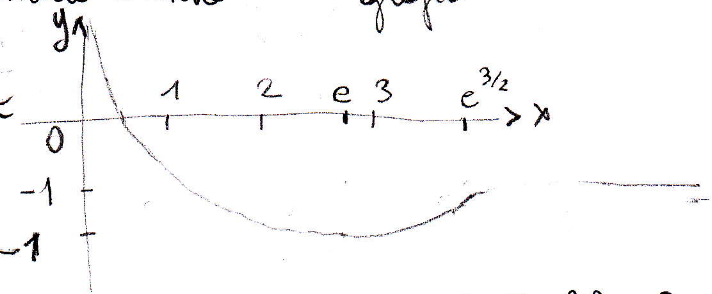
$\Uparrow$

$\stackrel{H}{=} \lim_{x \to +\infty} \frac{-\frac{1}{x}}{1} = \lim_{x \to +\infty} -\frac{1}{x} = 0 \Rightarrow \lim_{x \to +\infty} -\frac{\ln x}{x} - 1 = -1$

$\lim_{x \to 0^+} -\frac{\overset{\to -\infty}{\ln x}}{\underset{\to 0^+}{x}} - 1 = +\infty \Rightarrow f$ non è superiormente limitata

$\Downarrow$ $x = 0$ asintoto verticale

grafico



$\Rightarrow$ $x_0 = e$ punto di minimo assoluto

EX 2 file A