2
CHAPTER

Kinematic Geometry of Human Motion: Body Posture

This chapter is about methods used to define a body posture (joint configuration). The position of a body segment can be defined relative to

- a global system of coordinates (e.g., segment coordinates, absolute coordinates, World coordinates),
- a somatic system attached to the whole body (somatic coordinates), or
- an adjacent segment (joint coordinates).

Mixed systems are also used. In such a system the orientation of one body segment is determined through the global reference system, and the orientation of the second segment is realized through the joint coordinates. For example, the thigh position measured relative to the vertical can be plotted versus the shank position taken relative to the thigh. A system of coordinates is often referred to as a *space*. Hence, the expression "joint space" is a synonym for "joint coordinates."

Section **2.1** addresses the techniques used for describing relative orientation of adjacent body segments. It starts by explaining the difference between the technical and somatic reference systems; then the various techniques used for describing joint orientation are discussed, particularly the clinical reference system, globographic presentation, segment coordinate system, and—the most popular approach—joint rotation convention. Kinematic chains are discussed in Section **2.2**. In this section, the notion of degrees of freedom and the mobility of kinematic chains are discussed and various modifications of the Gruebler's formula are used to determine the total number of degrees of freedom of kinematic chains. This section also concentrates on open kinematic chains and end-effector mobility. Kinematic models and mobility of the human body are also described, as are constraints on human movement and the position of kinematic chains. In the discussion of kinematic chain position, two simple chains are considered: a two-link planar and three-link planar. Multilink chains are also addressed and transformation analysis of kinematic chains is highlighted. In this Section, I will also consider the Denavit-Hartenberg convention broadly used for describing kinematic chains. Section **2.3** is devoted to several applications of kinematic geometry in biomechanics. The discussion focuses on the internal representation of the immediate extrapersonal space and body posture.

2.1 JOINT CONFIGURATION

Representative papers: Kinzel, Hall, & Hilberry, 1972; Woltring, 1991.

Generally, the relative motion of one human body segment with regard to an adjacent one is a combination of translation and rotation. However, if the object of the study is gross human motion, such as walking or a gymnastic exercise, the translation can be disregarded due to its small magnitude and joint motion is analyzed as pure rotation. In Chapters **2** and **3**, the joint motions are considered pure rotations around fixed axes. The following simplifying assumptions are repeatedly made:

- If the joint exhibits rotation around more than one axis, the axes of rotation intersect at one point.
- The axes of rotation coincide with the joint reference system of coordinates.
- The axes of rotation coincide with the anatomic axes (for instance, the flexion-extension axis coincides with the frontal axis passing through the joint center).

More complex forms of joint motion, as well as specific systems used for particular joints, will be described in Chapters **4** and **5**.

2.1.1 Technical and Somatic Systems

Representative papers: Cappozzo & Gazzani, 1990; Söderkvist & Wedin, 1993.

Methods used to describe joint configuration or body posture differ primarily in the manner in which the local reference frame is fixed within a body segment. All local systems can be classified as either technical or somatic.

Technical reference systems are fixed with technical devices, such as goniometers or skin markers. For instance, if goniometers or other transducers are used, they are located externally on the body surface rather than fixed directly at the origin of the segment's local reference frame. Technical reference systems are related to somatic systems by translation and rotation. Experimental data registered in a technical system of coordinates are subsequently expressed in the corresponding somatic frame. The 4 × 4 transformation matrix method, described in Section **1.2.5.1**, can be used for this purpose.

The main problem associated with technical reference systems is accuracy. The errors made when recording human body movement are divided into *errors of measurement*, for example skin marker position in space, and *errors of transformation*, which occur when data are transformed from a technical reference frame (skin markers) to a somatic reference frame (body segment position). To minimize the errors of measurement, various smoothing procedures, which are not described in this textbook, are used. The errors of measurement depend on, among other factors, the configuration of the markers. For instance, if all of the skin markers are located along a straight line, accuracy decreases. It has been shown that accuracy depends on the *condition number* of the configuration. To define the condition number, assume that we have n skin markers on a rigid body. Determine the projected distance of each landmark from the geometric center of the configuration along axes x, y, and z. The sum of the squared distances taken along axis x is σ_x^2. The axes are selected in such a way that $\sigma_z^2 \geq \sigma_x^2 \geq \sigma_y^2$. For an absolute error, i.e., for the error in determining body position in an external frame, the condition number is determined by

$$k_A = \left(\sigma_x^2 + \sigma_y^2\right)^{-1/2} \tag{2.1}$$

The condition number k_A depends on the size of the body segment because it is impossible to place the markers far apart on a small body segment. Therefore, it seems reasonable to normalize the condition number with regard to the body segment size. To do that, let us first define a 3 × n matrix, [A], with the distances of the skin markers to the center of configuration as the elements. Then, the relative condition number is defined as

$$k_R = \frac{\|A\|}{\left(\sigma_x^2 + \sigma_y^2\right)^{1/2}} \tag{2.2}$$

where $\|A\|$ is the *Frobenius norm* of matrix [A]. Matrix [A] is found by

$$\|A\| = \left(\sum_{i,k} a_{i,k}^2\right)^{1/2} \tag{2.3}$$

To calculate the Frobenius norm, the sum is computed over all the elements of the matrix [A]. A practical recommendation is to place the markers as far apart as possible.

The errors of transformation depend on two factors: first, accuracy of the relative location of the skin markers and bony landmarks, and second, displacement of the skin markers during human motion. Location of the technical reference frame with regard to the body, especially the skeletal bones, changes during human movement. This is due to movement of the skin and underlying tissues relative to the bony landmarks. There are two sources for such displacement: angular joint motion and inertial forces caused by high accelerations and impacts, like those caused by heelstrikes in walking. In experimental settings, the displacement of skin markers and transducers should be reduced as much as possible or the errors caused by this displacement should be corrected mathematically.

Somatic systems are classified into three types based on (a) anatomic landmarks; (b) mechanical points and axes; and (c) a combination of (a) and (b) used to define the reference frame. Results may differ substantially when alternative systems are employed (Figure 2.1). This is because the reference frames are slightly rotated and translated relative to each other. Also, a movement pattern that is considered as pure rotation around a reference axis in one frame may be expressed as a complex general motion in another local system.

2.1.2 The Clinical Reference System

Representative publication: *American Academy of Orthopaedic Surgeons,* 1965.

This system is most often used by practitioners and is commonly taught in anatomy and kinesiology classes. The clinical system is defined as follows:

1. Sagittal, frontal, and transverse planes are defined when the body is in the anatomic position (see "Cardinal Planes and Axes of the Human Body" on page 13 and Figure 1.9).
2. The components of segmental motion with regard to the anatomic position are defined as
   a. motion of the segment in a sagittal plan (flexion-extension),
   b. motion of the segment away from or toward the midsagittal plane (abduction-adduction), and
   c. rotation about the long axis of the segment (internal-external rotation or supination-pronation; the terminology differs depending on the joint).

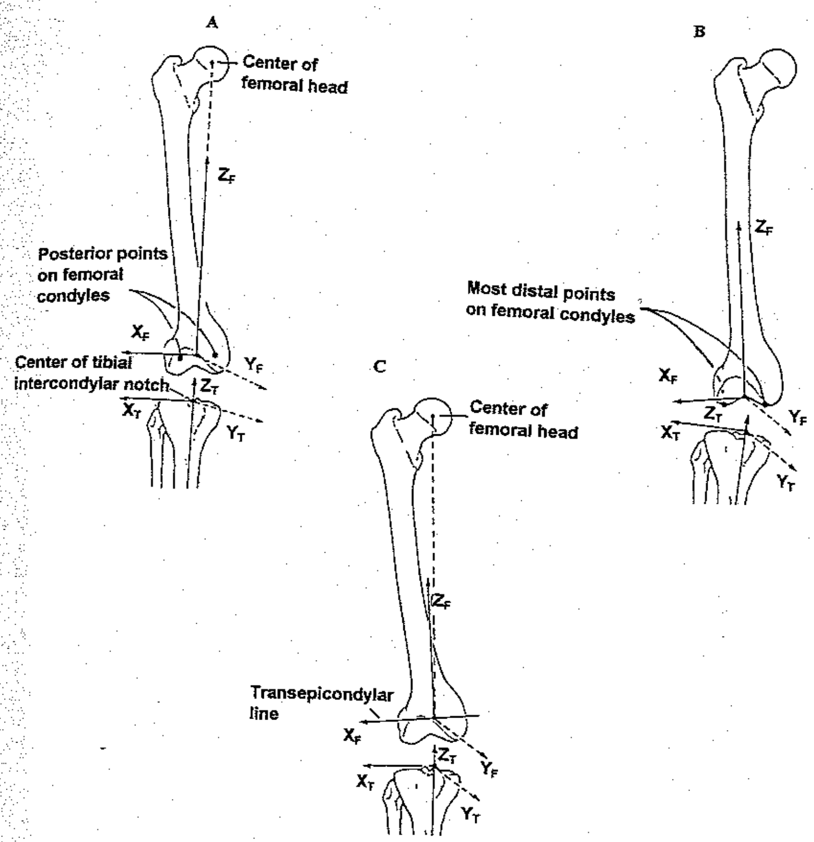

Figure 2.1 Somatic reference systems for a knee joint. A. A mechanically based system. B. This system is based on the anatomic axes of the femur and the tibia. C. This system is based on the transepicondylar line (anatomic) and the tibial mechanical axis. Experimentally measured differences in abduction-adduction at maximal swing phase flexion during walking equal 4° between A and C and 7° between B and C.

A. is revised from Grood, E.S. & Suntay, W.J. (1983). A joint coordinate system for the clinical description of three-dimensional motion: application to the knee. *Journal of Biomechanical Engineering, 105,* 136-144. Adapted by permission from ASME International.

B. is adapted by permission from Lafortune, M.A. (1984). *The use of intra-cortical pins to measure the motion of the knee joint during walking.* Unpublished doctoral dissertation, Pennsylvania State University. Adapted by permission from the author.

C. is adapted from *Journal of Biomechanics, 23,* Pennock, G.R. & Clark, K.J., An anatomy-based system for the description of the kinematic displacements in the human knee, 1209-1218, 1990, with kind permission from Elsevier Science Ltd, The Boulevard, Langford Lane, Kidlington 0X5 1GB, UK.

FROM THE SCIENTIFIC LITERATURE

Converting Data From a Technical to a Somatic System

Source: Chao, E.Y.S. (1980). Justification of triaxial goniometer for the measurement of joint rotation. *Journal of Biomechanics, 13,* 989-1006.

Converting data from a technical to a somatic system may be a laborious problem. Consider, as an example, triaxial goniometers. Ideally, the axes of the goniometer should coincide with the joint rotation axes (Figure 2.2). If the goniometer assembly could be placed at the center of the joint, it would measure the joint angles without distortion. Placing the goniometer in the center of the joint is technically impossible, however. The goniometer assembly is fixed to the side of the joint. The axes of the goniometer, especially the axis of internal-external rotation, do not coincide

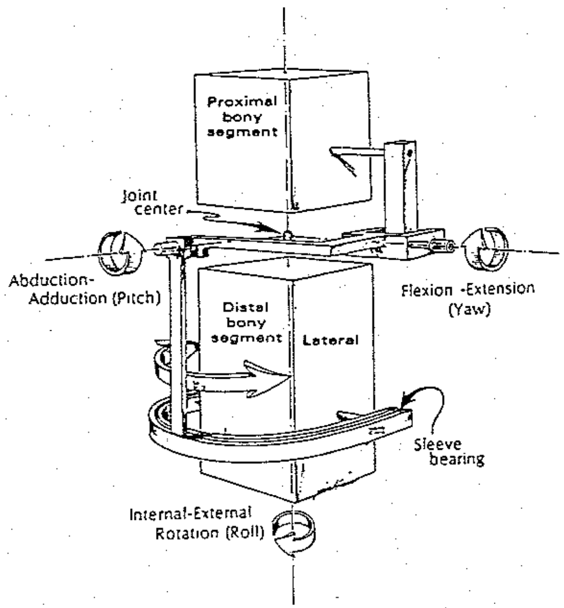

Figure 2.2 A schematic diagram of a skeletal joint and its three-dimensional rotation as measured by an ideal triaxial goniometer.
Reprinted from *Journal of Biomechanics, 13,* Chao, E.Y.S., Justification of triaxial goniometer for the measurement of joint rotation, 989-1006, 1980, with kind permission from Elsevier Science Ltd, The Boulevard, Langford Lane, Kidlington 0X5 1GB, UK.

with the joint axes of rotation. This arrangement creates error and makes the results of the individual angular measurements interdependent.

For example, if the body segment is undergoing internal rotation, the goniometer measurements of the flexion angle and the abduction angle are not equal to the actual joint angles. This difference is called *cross-talk,* and it can be large (Figure 2.3).

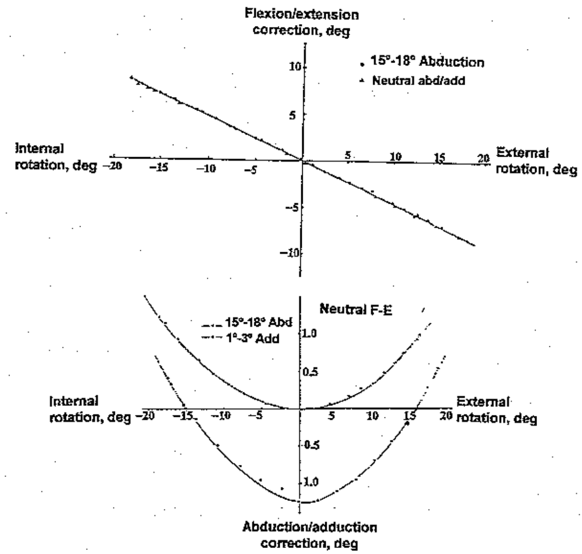

Figure 2.3 The influence ("cross talk") of the internal-external rotation on the recorded values of the flexion-extension angle (upper panel) and the abduction-adduction angle (bottom panel). The actual flexion angle α (not marked on the figure) can be obtained from the goniometer readings as $\alpha = \alpha_{gon} \pm 0.48\psi_{gon}$, where α_{gon} is the flexion angle and ψ_{gon} is the axial rotation measured by the goniometer. The correction pattern for the abduction angle is nonlinear and depends on the initial angle of abduction. The actual abduction angle β can be calculated as $\beta = \beta_{gon} + c\left(1 - e^{k|\psi_{gon}|}\right)$, where β_{gon} is abduction measured at the goniometer, $|\psi_{gon}|$ is the absolute value of axial rotation measured at the goniometer, and c and k are constants depending on the initial abduction-adduction angle at the joint.
Reprinted from *Journal of Biomechanics, 13,* Chao, E.Y.S., Justification of triaxial goniometer for the measurement of joint rotation, 989-1006, 1980, with kind permission from Elsevier Science Ltd, The Boulevard, Langford Lane, Kidlington 0X5 1GB, UK.

See **DATA**, *p. 86*

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To arrive at these results, the author performed a commendable job. The goniometer linkage and the skeletal joint system were modeled as a seven-bar, nine degree of freedom (DOF), spatial mechanism (Figure 2.4). Denavit-Hartenberg transformation matrices were employed to solve for the unknown joint motion based on the recorded goniometer readings. Both the notion of DOF and the Denavit-Hartenberg convention will be explained later in this chapter (see Section **2.2.1** and Section **2.2.5.3**).

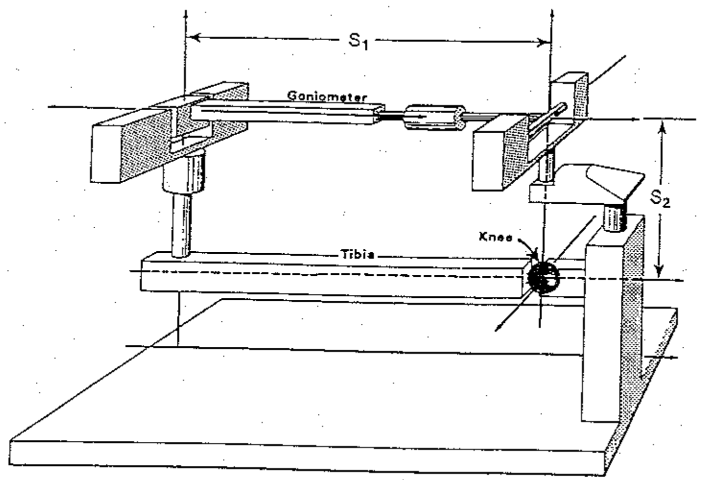

Figure 2.4 The nine-DOF spatial model used to simulate the triaxial goniometer as applied for the measurement of human knee joint motion. The model has eight DOF in rotation and one in translation.
Reprinted from *Journal of Biomechanics, 13,* Chao, E.Y.S., Justification of triaxial goniometer for the measurement of joint rotation, 989-1006, 1980, with kind permission from Elsevier Science Ltd, The Boulevard, Langford Lane, Kidlington 0X5 1GB, UK.

These motions are defined with regard to the cardinal anatomic planes. A combination of flexion-extension and abduction-adduction resulting in a circular movement is called *circumduction.*

Included and anatomic joint angles are also defined (Figure 2.5). The *included joint angle* is located between the longitudinal axis of the two segments defining a joint. The *anatomic joint angle* is that angle through which the joint would have to be moved to take it from the anatomic position to the position of

interest. From a purely geometric standpoint, the included angles can be viewed as *internal* and the anatomic angles as *external* joint angles.

The clinical reference system provides anatomically meaningful definitions of main segmental movements. The system is convenient when joint motion is performed from a standard anatomic, or neutral, position. However, when a joint rotation commences from a nonneutral configuration, the clinical system is not suitable. Also, it is not an appropriate format to describe complex movement patterns. This limitation can be illustrated with the following example. Imagine a subject who performs three shoulder joint motions in succession (a reader may perform this simple experiment). The joint movements begin with the shoulder in a dependent posture and are (1) flexion of 90°; (2) horizontal extension (abduction) of 90°; and (3) adduction. After such movements, the subject finds his or her hand in a pronated position (anatomic posture), although pronation was not performed. This phenomenon is called Codman's paradox (after E.A. Codman, who described it in 1934). This phenomenon occurs because rotations in the clinical system are not defined in accordance with the requirements of kinematics.

To define a joint rotation in three-dimensional space, the axis of rotation must be explicitly defined. In the clinical system, the rotations, especially abduction-adduction, are defined relative to the plane rather than the axis. When a rotation is defined relative to the plane, the axis of rotation may vary. For example, when an arm is flexed at the shoulder joint, the movement away from the midsagittal plane is definitely performed relative to a different axis than when the arm is oriented vertically. Additionally, even when the axes of rotation are indicated, they are described ambiguously, that is, without regard

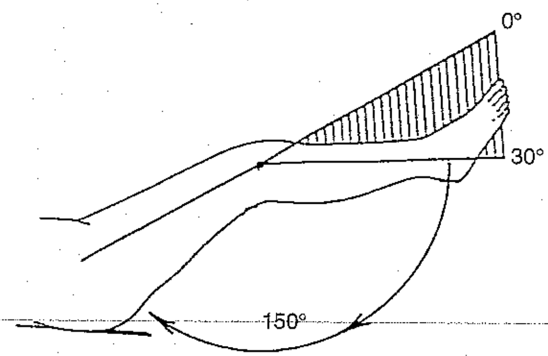

Figure 2.5 Included and anatomic angles used to designate joint configuration. The shaded area (= 30°) represents the anatomic angle. The included angle equals 150° in this example.

to the requirements of a valid kinematic description. For example, in the framework of the clinical reference system flexion is defined as the rotation about a joint frontal axis. There is no problem with such a definition when the motion begins with the arm at the side of the body. Yet, when the arm is adducted 90°, rotation around the frontal axis can barely be called flexion anymore; rather, it is internal or external rotation.

From the foregoing examples, we can see that three-dimensional joint motion cannot, in principle, be described unequivocally by such a triad as "flexion-extension, abduction-adduction, and internal-external rotation." To describe movement unambiguously, a reference system must be defined according to the requirements of kinematics.

2.1.3 Globographic Representation

With a globographic representation, an attitude of a body segment is given in spherical coordinates. This representation is used for descriptions of a spherical joint position; for example, the position of the upper arm with regard to the trunk. When a proximal coordinate system originating in the shoulder joint is fixed with the trunk, the longitudinal axis of the upper arm passes through the origin. The angular position of this line is shown on a globe with meridians and parallels and can be described by two angles (Figures 2.6 and 2.7). Such a representation is free from the artificiality of the cardinal anatomic planes. The angles are calculated along meridians (angle α, for motion in the vertical plane) and parallels (angle β, for horizontal circular movement of the representative point). A unit vector **V**, given by its spherical coordinates, α and β, can be written in a Cartesian system as

$$\mathbf{V} = \begin{bmatrix} \sin\alpha \sin\beta \\ \cos\alpha \\ \sin\alpha\cos\beta \end{bmatrix} \tag{2.4}$$

The globographic angles are different from Euler's angles because the latter are defined as consecutive rotations. Thus, the second and third rotations are performed about the new axes in the movable coordinate systems. Globographic angles are calculated about a fixed system of coordinates; α and β together define a single rotation about an oblique (usually) axis through the joint center. When a twist (third) angle is calculated, the reference (or zero) position is different when Euler's or globographic angles are used. When the globographic method is used to describe consecutive displacements, an *induced twist* may be seen; final values of the twist are different when the body segment moves directly from position A to position C compared with when it moves from A to B and then to C. In a globographic system, $A \to C \neq A \to B \to C$.

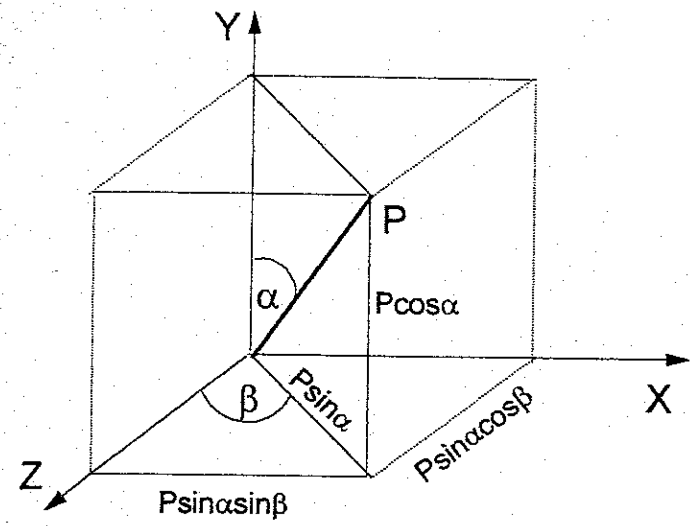

Figure 2.6 Angle convention used in a spherical (globographic) presentation.

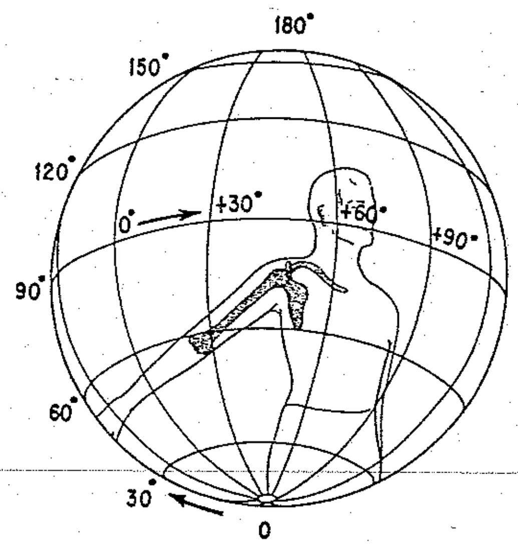

Figure 2.7 Globographic presentation of shoulder movement.

2.1.4 Segment Coordinate Systems

With segment coordinate systems, sagittal, frontal, and transverse planes are fixed within each segment. The result is a set of three reference planes for each segment that move with the segment in three-dimensional space. The intersection lines of the three planes make up the axes of the local Cartesian coordinate system. The attitude of one system, D, attached to a distal segment, relative to the second frame, P, fixed with the proximal segment, can easily be found. Although both segments may move, one segment is considered fixed and the second is considered to be moving. Because all reference axes are given, a joint angular position can be defined unambiguously with one of the described methods (rotation matrix, Euler's-Cardan's angles, or helical method).

If a set of Cardan's angles is chosen appropriately, it describes anatomic angles. For the majority of joints, the recommended sequence of Cardan's angles is (1) flexion-extension, (2) abduction-adduction, and (3) axial rotation. For the shoulder joint, the following sequence of the Euler's angles is advised: (1) rotation around the trunk-fixed vertical axis, which determines the plane of arm elevation; (2) arm elevation in the predecided plane; and (3) axial rotation. The helical method does not provide a clinical representation of three-dimensional joint motion.

Oftentimes, at least two reference frames are assigned to the body segment. One (mechanical) frame has an origin in the center of mass of the segment with the reference axes along the principal axes of inertia; this frame has no particular relation to any joint. The second frame is located at a joint and, with the joint in a neutral position, coincides with a joint reference system of the adjacent segment. With the exclusion of the so-called spin joint motion (see Chapter **4**), the axes of joint rotation do not pass through both segments forming the joint. Thus, the origin of at least one of the joint systems is physically outside the segment. The reference frame is only attached to, but not embedded into, the segment.

For two adjacent segments, four reference frames are introduced: two frames, P and D, are defined at the segments, and two, F and M, are defined at the joint (F is for "fixed" and M is for "moving"). Vectors defined in frame D can be readily presented in frame P if a homogeneous transformation matrix $[T_{PD}]$ is known:

$$P_T = [T_{PD}]P_D \tag{2.5}$$

The transformation matrix $[T_{PD}]$ can be viewed as a composition of several displacements:

$$P \rightarrow F \rightarrow M \rightarrow D \tag{2.6}$$

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Anatomic Frames for the Pelvis and Lower Limb Bones

Source: Cappozzo, A., Catani, F., Della Croce, U., & Leardini, A. (1995). Position and orientation of bones during movement: Anatomical frame definition and determination. *Clinical Biomechanics, 10,* 171-178.

Frames are proposed as a basis for standardization. The anatomic landmarks for the frames are presented in Table 2.1 and in Figures 2.8 and 2.9. The landmarks marked with (a) are used to define the anatomic frames, those indicated with (b) are used for determining (a)-type landmarks or to improve their estimation, and the (c) landmarks are recommended as supplementary.

The following reference frames are defined with respect to a subject standing in the anatomic position (Figure 2.9).

Pelvis

O_p – The origin is at the midpoint between anterior superior iliac spines (ASIS).

z_p – The z axis is oriented as the line passing through the ASISs with its positive direction from left to right.

x_p – The x axis lies in the quasitransverse plane defined by the ASISs and the midpoint between the posterior superior iliac spines (PSIS). Its positive direction is forward.

y_p – The y axis is orthogonal to the x-z plane and its positive direction is proximal.

Right and left thigh

O_t – The origin is the midpoint between the lateral and medial epicondyles (LE and ME).

y_t – The y axis joins the origin with the center of the femoral head (FH) and its positive direction is proximal.

z_t – The z axis lies in the quasifrontal plane defined by the y axis and the epicondyles and its positive direction is from left to right.

x_t – The x axis is orthogonal to the y-z plane and its positive direction is forward.

See **FRAMES,** *p. 92*

■ ■ ■ FRAMES, *continued from p. 91*

Table 2.1 Anatomic Landmarks

Hip bone		
(a)	ASIS	Anterior superior iliac spine
(a)	PSIS	Posterior superior iliac spine
(b)	AC	Center of the acetabulum
Femur		
(a)	FH	Center of the femoral head
(c)	GT	Prominence of the greater trochanter external surface
(a)	ME	Medial epicondyle
(a)	LE	Lateral epicondyle
(b) (c)	LP	Anterolateral apex of the patellar surface ridge
(b) (c)	MP	Anteromedial apex of the patellar surface ridge
(b) (c)	LC	Most distal point of the lateral condyle
(b) (c)	MC	Most distal point of the medial condyle
Tibia and fibula		
(c)	IE	Intercondyle eminence
(a)	TT	Prominence of the tibial tuberosity
(a)	HF	Apex of the head of the fibula
(a)	MM	Distal apex of the medial malleolus
(a)	LM	Distal apex of the lateral malleolus
(b) (c)	MMP	Most medial point of the ridge of the medial tibial plateau
(b) (c)	MLP	Most lateral point of the ridge of the medial tibial plateau
Foot		
(a)	CA	Upper ridge of the calcaneus posterior surface
(a)	FM	Dorsal aspect of the first metatarsal head
(a)	SM	Dorsal aspect of the second metatarsal head
(a)	VM	Dorsal aspect of the fifth metatarsal head

(a) Anatomic frames.
(b) Used to determine (a)-type landmarks.
(c) Supplementary landmarks.

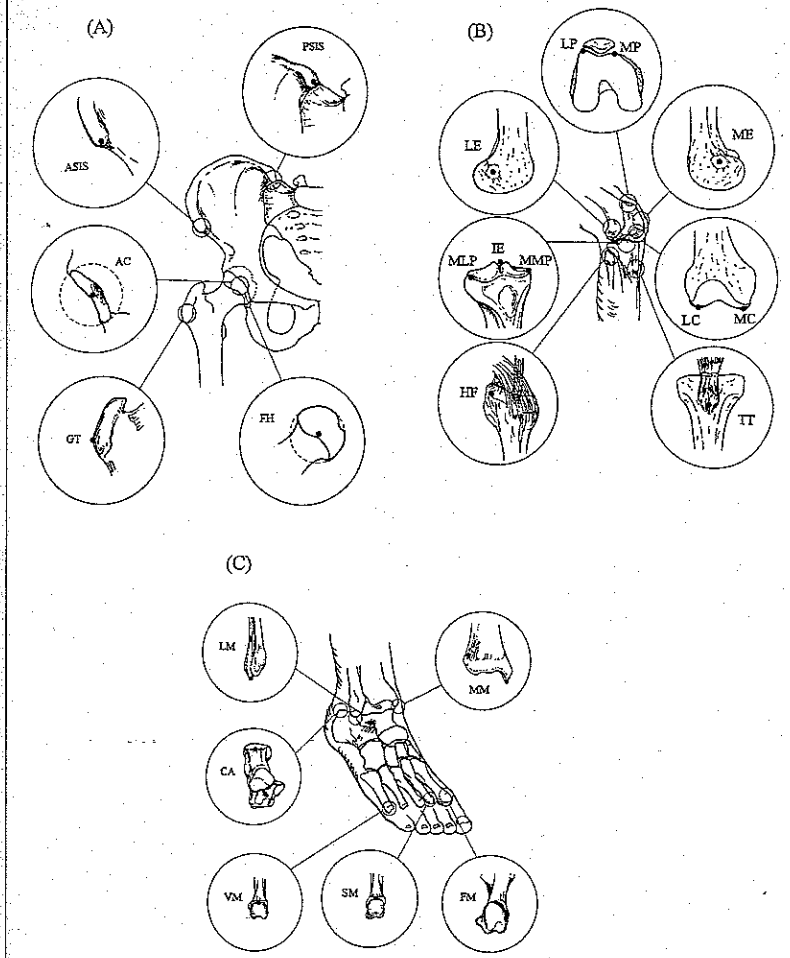

Figure 2.8 Anatomic landmarks in (A) the pelvis and proximal femur, (B) the distal femur and proximal tibia and fibula, and (C) the distal tibia and fibula and in the foot. See Table 2.1 for key to abbreviations.
Adapted from *Clinical Biomechanics, 10,* Cappozzo, A., et al., Position and orientation of bones during movement: Anatomical frame definition and determination, 171-178, 1995, with kind permission from Elsevier Science Ltd, The Boulevard, Langford Lane, Kidlington 0X5 1GB, UK.

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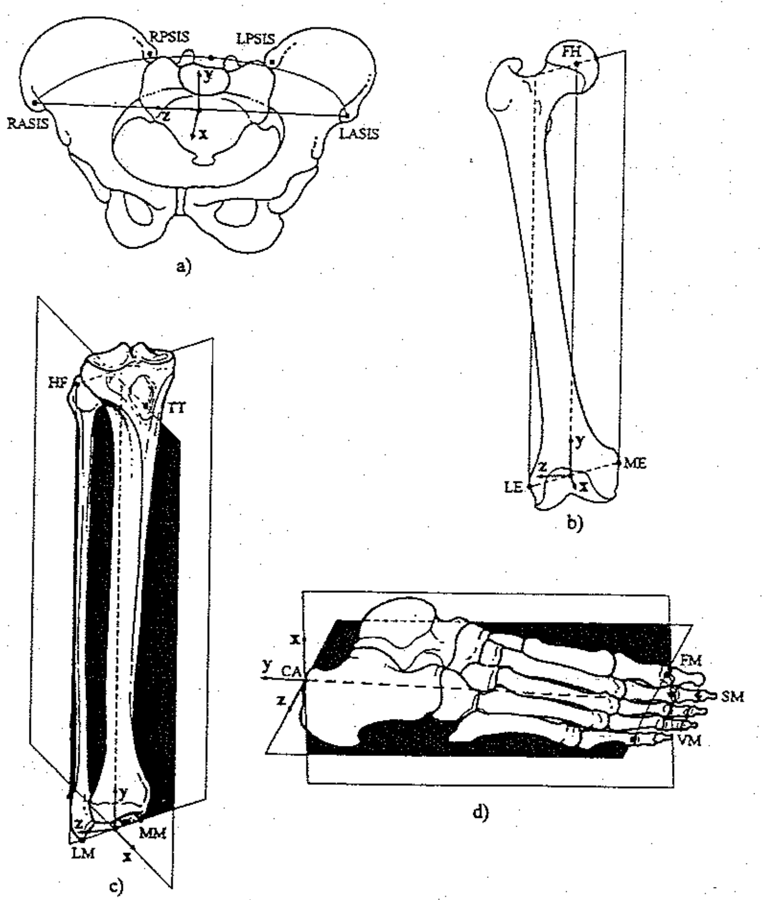

Figure 2.9 Bone-embedded anatomic frames.

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Right and left shank

O_s – The origin is at the midpoint of the line joining the lower ends of the medial and lateral malleoli (MM and LM).

y_s – The malleoli and the head of the fibula (HF) landmarks define a plane that is quasifrontal. A quasisagittal plane, orthogonal to the quasifrontal plane, is defined by the midpoint between the malleoli and the tibial tuberosity (TT). The y axis is defined by the intersection between the above-mentioned planes and its positive direction is proximal.

z_s – The z axis is in the quasifrontal plane and its positive direction is from left to right.

x_s – The x axis is orthogonal to the y-z plane and its positive direction is forward.

Right and left foot

O_f – The origin is at the calcaneus landmark (CA).

y_f – The calcaneus and the first and fifth metatarsal heads (FM and VM) define a quasitransverse plane. A quasisagittal plane, orthogonal to this latter plane, is defined by the CA and second metatarsal head (SM). The y axis is defined by the intersection of these two planes, and its positive direction is proximal.

z_f – The z axis lies in the quasitransverse plane and its positive direction is from left to right.

x_f – The x axis is orthogonal to the y-z plane and its positive direction is dorsal.

■ ■ ■ FROM THE SCIENTIFIC LITERATURE ■ ■ ■

Three-Dimensional Rotation of the Elbow Joint

Source: Chao, E.Y., & Morrey, B.F. (1978). Three-dimensional rotation of the elbow. *Journal of Biomechanics, 11,* 57-73.

The global, humeral, and forearm systems were defined (Figure 2.10, page 96). In the anatomic posture, the X and x axes corresponding to the humeral and the forearm systems were pointing in the anterior direction, the

See **ELBOW,** *p. 96*

■ ■ ■ ELBOW, *continued from p. 95*

Y and y axes were along the bones, and the Z and z axes were pointing laterally. The adopted rotational sequence was (1) flexion-extension, ϕ; (2) abduction-adduction, θ, or the carrying angle; and (3) axial rotation, ψ (Figure 2.10, page 97).

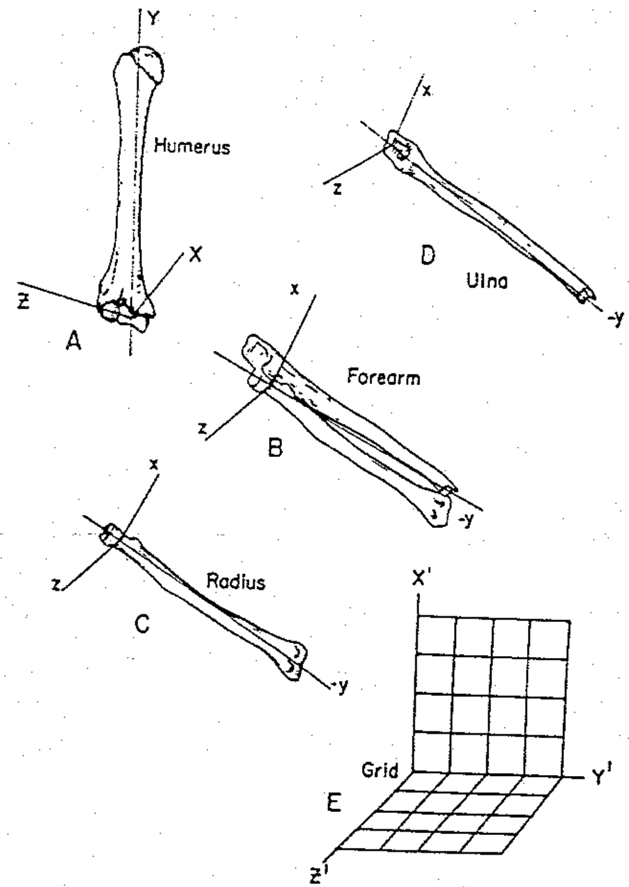

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Figure 2.10 Reference coordinate systems (this page) and the definition of the Euler's angles (next page).
Adapted from *Journal of Biomechanics, 11,* Chao, E.Y. & Morrey, B.F., Three-dimensional rotation of the elbow, 57-73, 1978, with kind permission from Elsevier Science Ltd, The Boulevard, Langford Lane, Kidlington 0X5 1GB, UK.

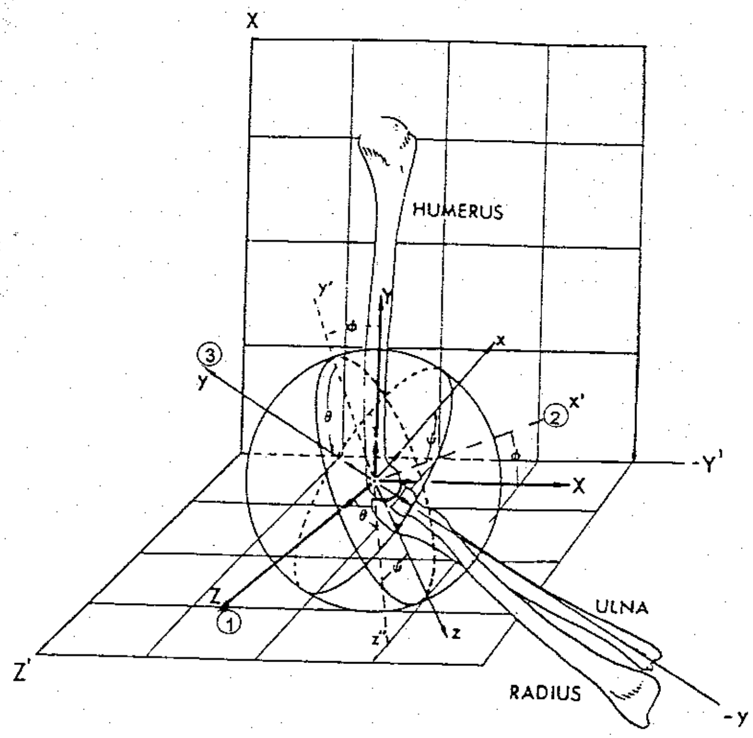

Figure 2.10 *(continued)*

The unit vectors of the forearm-fixed and the humerus-fixed systems are related by the rotation matrix

$$\begin{bmatrix} \vec{x} \\ \vec{y} \\ \vec{z} \end{bmatrix} = \begin{bmatrix} \cos\phi\cos\psi - \sin\phi\sin\theta\sin\phi & \sin\phi\sin\psi - \sin\psi\sin\theta\cos\phi & -\sin\psi\cos\theta \\ -\cos\theta\sin\phi & \cos\theta\cos\phi & \sin\theta \\ \sin\psi\cos\phi + \cos\psi\sin\theta\cos\phi & \sin\psi\sin\phi - \cos\psi\cos\theta\cos\phi & \cos\psi\cos\theta \end{bmatrix} \begin{bmatrix} \vec{X} \\ \vec{Y} \\ \vec{Z} \end{bmatrix}$$

See **ELBOW,** *p. 98*

■ ■ ■ ELBOW, *continued from p. 97*

where $\vec{x}$, $\vec{y}$, $\vec{z}$ and $\vec{X}$, $\vec{Y}$, $\vec{Z}$ are unit vectors along the forearm axes and the humeral axes, correspondingly. Note that in this equation the rows of the rotational matrix correspond to the axes of the system fixed at the distal segment, the forearm; the columns correspond to the axes of the proximal system, the humerus. The matrix is a transposition of the rotation matrices adopted in this book (see equation 1.2). Because of this difference and, also, because of the changed order of rotation, the rotational matrix is different from that presented in equation 1.32. In equation 1.32, the rotation sequence was Zy'x", in this example the sequence is Zx'y". If $\vec{x}$, $\vec{y}$, $\vec{z}$ and $\vec{X}$, $\vec{Y}$, $\vec{Z}$ are defined in the global reference system, the individual Euler's angles can be determined.

$$\text{Flexion angle, } \cos^{-1}\left(\frac{\vec{y}\cdot\vec{Y}}{\cos\theta}\right)$$

$$\text{Carrying angle, } \theta = \sin^{-1}\left(\vec{y}\cdot\vec{Z}\right)$$

$$\text{Axial rotation, } \psi = \left(\frac{\vec{z}\cdot\vec{Z}}{\cos\theta}\right)$$

Using equation 1.21 we can write in matrix form

$$[T_{PD}] = [T_{PF}][T_{FM}][T_{MD}] \qquad (2.7)$$

The transformation matrices $[T_{PF}]$ and $[T_{MD}]$, as well as the transformations between the two-joint reference systems, proximal and distal, affixed to the same body segment are constant; they do not depend on the joint configuration. These transformations are known as *link transformations*. The transformation matrix $[T_{FM}]$ is called the *joint transformation*.

2.1.5 Joint Rotation Convention

Representative papers: Chao, 1980; Grood & Suntay, 1983; Woltring, 1994.

To identify the relative attitude of two body segments, three spatial axes should be specified. In the joint rotation convention (JRC), two axes are fixed with the body segments, proximal and distal, and one is a floating axis. The following procedure is recommended:

1. The first axis is the first fixed body axis and is perpendicular to the sagittal plane of the proximal segment.
2. The second axis is the floating axis (the cross product of the first and third axes).
3. The third, or the second fixed-body axis, is the long axis of the distal segment (Figure 2.11).

Also, two reference lines are defined. They are embedded in each body perpendicular to the fixed axis.

To visualize the JRC consider the human knee joint as an example (Figure 2.11). The flexion-extension axis (axis 1-1) is anchored to the distal femur. Consequently, the plane of flexion-extension does not change its orientation

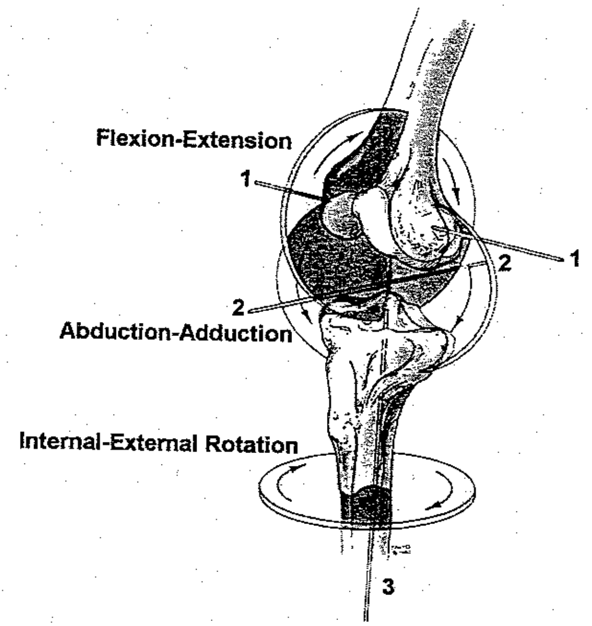

Figure 2.11 Joint rotation convention for a knee joint. Axis 1-1 is fixed to the femur and describes the flexion-extension motion. Axis 3-3 is fixed to the tibia along its longitudinal anatomic axis and defines internal-external rotation. The floating axis 2-2 is orthogonal to both axes 1-1 and 3-3 and is used to measure abduction and adduction.
From Chao, E.Y.S. (1990). Goniometry, accelerometry, and other methods. In N. Berme & A. Cappozzo (Eds.), *Biomechanics of human movement: Applications in rehabilitation, sports and ergonomics* (pp. 130-139). Worthington, Ohio: Bertec Corporation. Reprinted by permission.

throughout the motion history; it is always fixed with the femur. The axis of external-internal rotation (axis 3-3) changes its orientation with regard to the femur during the motion. This axis is along the long axis of the tibia. Consequently, the plane of the axial rotation always remains perpendicular to the longitudinal axis of the lower leg. The floating axis, which is the abduction-adduction axis in our example, is identical to the line of nodes. Although axis 2-2 is orthogonal to both axis 1-1 and axis 3-3, the abduction-adduction plane is fixed to neither the femur nor to the tibia. The orientation of this plane relies on the flexion-extension angle and does not depend on the internal-external rotation angle.

In all "traditional" angular conventions discussed in the preceding sections, the existence of two embedded reference frames was assumed. The relative position of the bodies was identified with the relative position of the two frames. In the JRC, however, one reference frame and two reference lines are defined. The magnitude of rotation around the fixed axis is measured by the angle formed between the reference line and the floating axis. For example, internal-external rotation in the knee joint is measured as the angle formed by the reference axis perpendicular to the long axis of the tibia and the floating, anteroposterior, axis.

Because the long axis of the distal segment is not perpendicular to the frontal axis of the proximal segment in all of the joints, the two terminal fixed axes are usually not orthogonal. Also, the fixed axes move with the body segments and their relative spatial orientation changes with motion. Thus, the JRC is not a Cartesian system of coordinates. The JRC angles are the Cardan's angles defined geometrically. The floating axis is parallel to the line of nodes and coincides with it when the three axes intersect at one point.

The JRC angles are anatomically meaningful. When the recommended procedure is followed, independent rotations around the defined axes match the clinical definitions for relative movement between segments. In addition, rotations about the JRC axes are time sequence–independent. The final displacement between the two bodies is independent of the temporal order in which the rotations are performed. It does depend, however, on the choice of the fixed and floating axes. The JRC angles should be defined in a certain sequence. After that, when the axes are defined, the rotations around these axes are sequence independent.

Because the JRC system is simply a variant of Euler's-Cardan's angles, the adverse effects seen in that system, particularly singularity, continue to occur. The JRC angles cannot be defined for some joint postures. When the longitudinal axis of a distal segment is collinear with the frontal axis of the proximal segment, the second axis of the reference frame is not defined (a singular position). The singularity appears because the cross product of two vectors cannot be calculated when the vectors are collinear. The magnitude of vector **V**, which

is the cross product of vectors **P** and **Q**, equals PQ sin α. When α is 0° or 180°, the cross product is sin $\alpha = 0$. Such a configuration can be seen when the arm is abducted 90° and the long axis of the arm is collinear with the shoulder frontal axis. The position of the second axis of the local frame is not defined for such a joint configuration. The JRC is convenient for identifying the instantaneous axis of rotation in the joint. However, as it was mentioned previously, the JRC is not an orthogonal system. Nonorthogonality can present a serious problem when joint forces and moments are going to be determined.

■ ■ ■ FROM THE SCIENTIFIC LITERATURE ■ ■ ■

Knee Movement During Walking

Source: Lafortune, M.A., Cavanagh, P.R., Sommer, H.J., III, & Kalenak, A. (1992). Three-dimensional kinematics of the human knee during walking. *Journal of Biomechanics, 25,* 347-357.

Metallic pins with the attached target markers were inserted into the tibia and femur of the subjects. Radiographs of the lower limbs were taken and then the subjects were filmed during walking at a mean speed of 1.2 m/s.

Six reference frames were defined: (1) the global reference frame for gait analysis; (2) the radiographic reference frame; (3) the tibial anatomic frame; (4) the tibial marker reference frame (the first technical frame); (5) the femoral anatomic reference frame; and (6) the femoral marker reference frame (the second technical frame). The 4 × 4 transformation matrices were used, and the following steps were taken:

1. The position vectors of target markers in the tibial and femoral anatomic frames were computed from values measured on the radiographs $[x_{TA}] = [T_{R/TA}][x_R]$ and $[x_{FA}] = [T_{R/FA}][x_R]$, where $[x_{TA}]$ and $[x_{FA}]$ are position vectors of point x in the tibial and femoral anatomic frames, $[x_R]$ is the position vector of point x in the radiographic reference frame, and $[T_{R/TA}]$ and $[T_{R/FA}]$ are corresponding transformation matrices.
2. The position vector of target markers in the tibial marker frame, $[x_{TM}]$, was computed from the known values of their position in the tibial anatomic frame, $[x_{TA}]$, $[x_{TM}] = [T_{TA/TM}][x_{TA}]$ and the position vector of target markers in the femoral anatomic frame, $[x_{FA}]$,

See **WALKING,** *p. 102*

■ ■ ■ WALKING, *continued from p. 101*

was computed from their location in the femoral marker reference frame $[x_{FA}] = [T_{FM/FA}][x_{FM}]$ where $[T_{TA/TM}]$ and $[T_{FM/FA}]$ are transformation matrices. Note that these two equations perform different transformations.

3. The two following transformations transform the position vector given in the tibial marker reference frame into the global reference frame $[x_G] = [T_{TM/G}][x_{TM}]$ and the position vector in the global frame into the vector in the femoral marker reference frame $[x_{FM}] = [T_{G/FM}][x_G]$.
4. The final computation of the location and attitude of the tibial anatomic reference frame with respect to the femoral anatomic reference frame was achieved according to the following equation:

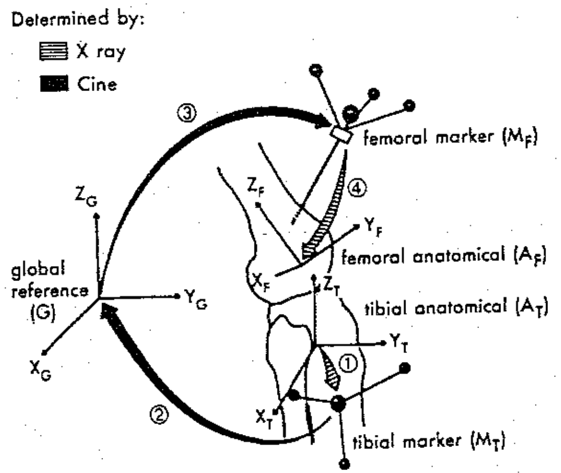

Figure 2.12 Transformations used to obtain the position of the tibial anatomic reference frame with respect to the femoral anatomic reference frame. Positions of the triads in the global reference frames were determined through cinematography, and their positions in the respective anatomic frames of reference were obtained through radiographs.
Reprinted from *Journal of Biomechanics, 25,* Lafortune, M.A., et al, Three-dimensional kinematics of the human knee during walking, 347-357, 1992, with kind permission from Elsevier Science Ltd, The Boulevard, Langford Lane, Kidlington 0X5 1GB, UK.

$[x_{FA}] = [T_{FM/FA}][T_{G/FM}][T_{TM/G}][T_{TA/TM}][x_{TA}]$. The sequence of matrix multiplication in the equation is shown in Figure 2.12.

Because $[x_{FA}] = [T_{TA/FA}][x_{TA}]$, it follows that

$$[T_{TA/FA}] = [T_{FM/FA}][T_{G/FM}][T_{TM/G}][T_{TA/TM}]$$

The matrix $[T_{TA/FA}]$ contains the kinematic information required to compute the locations of the markers in the femoral anatomic reference frame from their locations in the tibial anatomic reference frame. Although the general sequence of calculations does not depend on the chosen system of coordinates, the elements of the matrices do depend on them. In this study, the JRC system was used. Hence, the elements of matrix $[T_{TA/FA}]$ are the direction cosines of certain joint angles (flexion-extension, abduction-adduction, and internal-external rotation) and the translations along the lateromedial, longitudinal, and floating axes (shift, distraction, and drawer).

2.2 KINEMATIC CHAINS

A human body can be modeled as a multilink system comprising several body segments connected by joints. A linkage of rigid bodies is referred to as a *kinematic chain.* The simplest kinematic chain consists of two links connected by a single joint. A set of two adjacent links connected with one joint is called a *kinematic pair.* The fastening of bones in a kinematic pair is maintained either by their shape (a *form-closed pair*) or by externally applied forces (a *force-closed pair*). The hip joint comes nearest to the form-closed type, whereas the shoulder joint is considered force-closed because it is maintained by the surrounding muscles and ligaments.

Kinematic chains are either *serial* (simple) or *branched* (complex). In serial chains, each of the links is part of no more than two kinematic pairs. A branched chain contains at least one link that is part of more than two kinematic pairs. A human arm or leg can be considered a serial kinematic chain. When the trunk is also considered and articulations within the trunk are ignored, the chain is branched. The trunk enters five kinematic pairs: the two hip joints, the two shoulder joints, and the trunk-neck articulation. Kinematic chains are further classified as *open* or *closed.* The kinematic chain is referred to as open if one end of the chain (e.g., the distal segment) is free to move. In closed chains, constraints are imposed on both ends of the chain. Nonstationary (temporary) constraints are typical for many human movements (e.g., constraints imposed during the support periods in walking). Examples of open and closed kinematic chains in human movements are given in Figure 2.13.

2.2.1 Degrees of Freedom. Mobility of Kinematic Chains

The term "degrees of freedom" (DOF) refers to the independent coordinates required to completely characterize a body, or system, position. A single DOF can also be defined as an independent way the body can move. A rigid body, freely suspended in the air, has six DOF. It can translate along and rotate about three independent axes (longitudinal, vertical, frontal). When planar movement is considered, the body has three DOF: it can translate from one place to another in two directions and rotate.

The independent coordinates are any set of quantities that completely specify the state of a system. They are called *generalized coordinates*. The generalized coordinates are customarily written as $q_1, q_2, \ldots, q_n$, or simply as the q_i. The choice of a set of generalized coordinates to describe a system is not unique; there are many sets of quantities that completely specify the state of a given system.

A single point in space has three DOF. A system of n points has 3n DOF. However, if all the n points belong to a solid body they will maintain a constant distance from each other and the rigid body in space has only six DOF. A system of N rigid bodies, if not constrained, has 6N DOF. If some of the 6N coordinates are not independent (as would be the case if bodies were connected by joints or if the motion were confined along some path or on a surface), then the number of DOF decreases. If there are m equations of constraint, then only (6N − m) coordinates are independent, and the system possesses (6N − m) DOF. The constraints may be *complete* or they may be *one-sided*. For example, during a stance period the foot is free to move up but it cannot move down. When the equations of constraint connect only the coordinates, the constraints are termed *holonomic*. Constraints imposed on the velocities, accelerations, and so on are called *nonholonomic*. If the equations do not explicitly contain the time, the constraints are said to be *fixed* or *scleronomic*. Constraints changing in time are *rheonomic*.

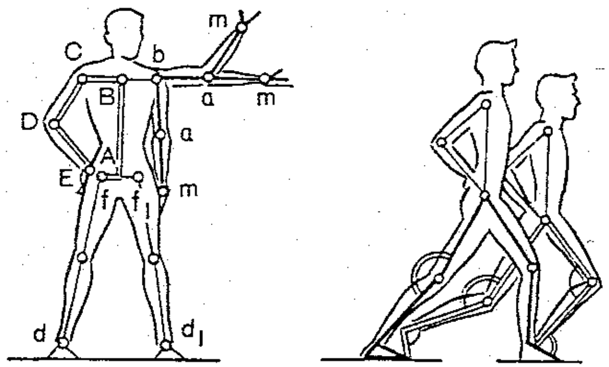

Figure 2.13 Kinematic chains in human movements. The left panel shows bam, an open chain, and ABCDEA and dff_1d_1, closed chains. The right panel shows a movement of the closed chain. In any closed chain, the joint angular motions are coupled.

The maximum number of DOF for each joint is six. If the translational motion within a joint (normally assumed to be small) is ignored, the maximal number of DOF is three. The number of DOF of a joint can be represented as a contrast; i.e., *six minus the number of constraints imposed*. Instead of the DOF, the *class of joint* may be defined. The class of joint is determined by the number of imposed constraints. Human joint geometry is complex. Joint surfaces are innately irregular in shape, or idiosyncratic, and therefore cannot be described as simple geometric surfaces. Consequently, when modeling joints there is a trade-off between accuracy and simplicity.

To simplify the study of human motion, the human body joints are classified as having one, two, or three rotational DOF. The joints with one DOF are called *hinge*, or *revolute*, joints. Joints with three DOF are referred to as *ball-and-socket*, or *spherical*, joints. Finger (interphalangeal) joints are examples of hinge joints; the hip and shoulder (glenohumeral) joints are ball-and-socket joints. Some joints (such as the sternoclavicular, which is used to shrug the shoulders, or the temporomandibular) have two DOF.

In mechanical models, when a twisting motion (e.g., pronation-supination or internal-external rotation) is considered, the twist is usually assigned to one of two joints, proximal or distal. For example, pronation-supination can be associated with either the elbow or the wrist. Depending on whether pronation-supination is assigned to either the elbow or the wrist joint, the elbow joint has one or two DOF. Note that joints having one DOF can also be called joints of the 5th class (because five constraints are imposed); joints with three DOF are joints of the 3rd class, and so on.

The total number of DOF in a kinematic chain is called the *mobility* of the chain. To calculate the mobility of a kinematic chain, the following convention is usually adopted: the external reference frame, which is fixed with the immovable environment, is counted as an additional link of the chain. With this convention, the number of rigid bodies included in the chain increases to N + 1. "Joint" 1 (between the external reference and the first link of the system) may have up to six DOF. The same is accepted for the last link of a closed kinematic chain. When using this convention, the mobility of a kinematic chain in space is described by the following formula:

$$F = 6(N - k) + \sum_{i=1}^{k} f_i \tag{2.8}$$

where N is the number of links, k is the number of joints, and f_i is the number of DOF in the *i*th joint. This equation is known as *Gruebler's formula*. For an open chain, the number of joints equals the number of links. This equality allows equation 2.8 to be simplified to

$$F\ (\text{for an open chain}) = \sum_{i=1}^{k} f_i \tag{2.9}$$

If the chain is closed, it has one more joint than links, k = N + 1. The mobility of the chain is

$$F\ (\text{for a closed chain}) = \sum_{i=1}^{k} f_i - 6 \tag{2.10}$$

When the kinetic chain is described using the planar system, Gruebler's formula is modified as

$$F = 3(N - k) + \sum_{i=1}^{k} f_i \tag{2.11}$$

For a closed chain in a planar system, the mobility is calculated as

$$F\ (\text{for a closed planar chain}) = \sum_{i=1}^{k} f_i - 3 \tag{2.12}$$

According to equation 2.12, a closed chain with four revolute joints in a planar system has four DOF with regard to the environment; three DOF are due to the mobility of the fictitious "joint" 1 and one DOF is due to the relative movement of the links. The chain has only one DOF when the relative movement of the links is considered exclusively.

Kinematic chains having only one DOF are called *mechanisms*. In mechanisms, at least one of the members is attached to an unmovable base (frame), which is considered an integral part of the mechanism. The position or movement of any one link of the mechanism prescribes the position or movement of all the other links. When such a situation takes place in human or animal movements the chain is called the *biomechanism*. For example, the rib cage during breathing and the legs of a fencer in the lunge position (if both the feet are on the support and the posterior leg is extended) have only one DOF; the chains are considered biomechanisms.

2.2.2 Open Kinematic Chains: The End-Effector Mobility

In many human movements, the last link of an open kinematic chain, typically the hand (a working tool) or the foot, needs to be positioned in a specified place with a specific orientation. This link is termed the *end effector.* The end effector may have its own mobility, for example, the human hand. The gripping function of the human hand and the positioning function of the human arm can be regarded as separate functions. To be positioned at an arbitrary point with an arbitrary orientation in space, the end effector must have at least six DOF. The number of DOF of the end effector equals the mobility of the kinematic chain. If the chain has less than six DOF, the end effector cannot be arbitrarily positioned within the reach area. When the chain has exactly six DOF, there exists exactly one joint configuration that places the end effector in a required position, i.e., in a required place with a required orientation. When the chain has more than six DOF, the end effector can be positioned at a required point with a required orientation in an infinite number of ways. Open kinematic chains with more than six DOF are termed *redundant chains*. For redundant chains, an infinite set of joint positions leads the end effector to the same location.

Maneuverability of the chain is described as M = F − 6, where F is the number of DOF of the chain. The maneuverability, M, equals the number of extra parameters that can be used to guide the end effector in a desired way; for example, to avoid obstacles or to minimize energy expenditure.

When studying the position of a kinematic chain, two main problems exist:

1. The joint coordinates are known and the end effector position is sought. This is called the *direct kinematic problem*.
2. The position of the end effector is known and the joint coordinates are sought. This problem is termed the *inverse kinematic problem*.

Note that the end-effector position is sought, or given, in absolute coordinates. The joint coordinates are the joint angles between adjacent segments.

2.2.3 Kinematics Models and Mobility of the Human Body

Kinematic models of the human body are those that represent its mobility and neglect all other aspects (e.g., the mass distribution). The models are classified as *anthropomorphic*, also called *skeletal*, or *functional*.

Skeletal models visually resemble the construction of the human body; the body segments are (typically) modeled as solid links and the human joints as the joints of the model. In functional models, the body segments are modeled as nodes of a graph (of a tree) and the joints as arcs connecting the nodes

(Figure 2.14). This representation has certain advantages for computer modeling and data structures; the joints are innately binary (they connect exactly two segments). Those connections are conveniently represented by the arcs having, not surprisingly, two ends. The segments may have several joints. For example, the trunk, if considered as a whole, provides a relationship between the positions of the two hip joints, the two shoulder joints, and the trunk-neck articulation.

In this book, only anthropomorphic models are used. One of them, in which the body is rendered as a stick figure, is shown in Figure 2.15.

The model represented in Figure 2.15 consists maximally of 18 rigid segments and 17 joints and possesses 41 DOF. If the sternoclavicular joints are ignored and the spine is considered one solid segment, the number of DOF decreases to 31. This is called a *gross body model,* in which many small joints (e.g., the interphalangeal joints) are not included. In the gross body model, the arm has seven DOF. Thus, its kinematic chain is redundant. This is easily demonstrated. When the hand is fixed on a table and the shoulder is fixed too, one can move the elbow without changing the hand or the shoulder position. The serious problem with gross body models is how to model the trunk segment. To consider the trunk as one rigid body is too unrealistic. Incorporation of fictitious trunk joints improves the quality of modeling but does not completely solve the problem. For example, when the model is presented on a screen, the trunk segments are separate from each other at some body angles. Although it is more realistic to represent the torso as a nonrigid (bending and twisting)

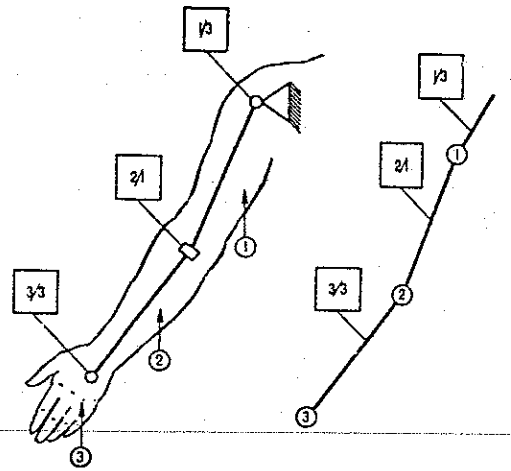

Figure 2.14 Two models of the arm: left, an anthropomorphic model, and right, a functional model. The "ratios" in the squares are the joint number over the number of DOF in the joint. The segment numbers are shown in the circles.

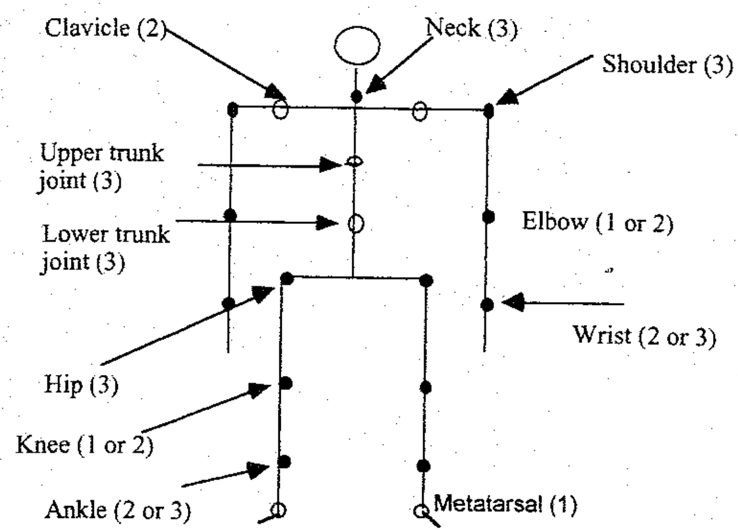

Figure 2.15 Kinematic model of human body. Filled circles designate the joints that are usually included in the model. Open circles are for the joints that are included only in some models.

segment, mathematical difficulties associated with this approach are, as of yet, unsolved.

To estimate the total mobility of the body all joints and body segments must be considered. According to estimations, there are 148 movable bones and 147 joints in the human body. Such a model is shown in Figure 2.16. With this approach the arm has not 7 but 30 DOF.

The total mobility can be estimated using a slightly modified Gruebler's formula, in which classes of joints instead of the number of DOF are used:

$$F = 6N - \sum_{i=3}^{5} i \cdot j_i \tag{2.13}$$

where F is the mobility of the body (the total number of DOF), N is the number of movable bones, i is the class of the joint (based on the number of imposed constraints, $i = 6 - f$, where f is the number of DOF), and j_i is the number of joints of the class i. It has been estimated that the human body has 148 movable bones connected by the joints, 29 joints of the 3rd class (with three DOF), 33 joints of the 4th class (with two DOF), and 85 joints of the 5th class (with one DOF). The total mobility of the human body is

$$F = (6 \cdot 148) - (3 \cdot 29) - (4 \cdot 33) - (5 \cdot 85) = 888 - 87 - 132 - 425 = 244 \tag{2.14}$$

Thus, the human skeletal system is highly redundant. It has 244 DOF and its maneuverability is 238. To position an end effector in space, the brain must specify not 6 but 244 variables, of which 238 are redundant and may be used to perform the motor task in an optimal way. In comparison, the arms of contemporary robots usually have no more than five or six DOF. In effect, only nonredundant robotic manipulators are used today.

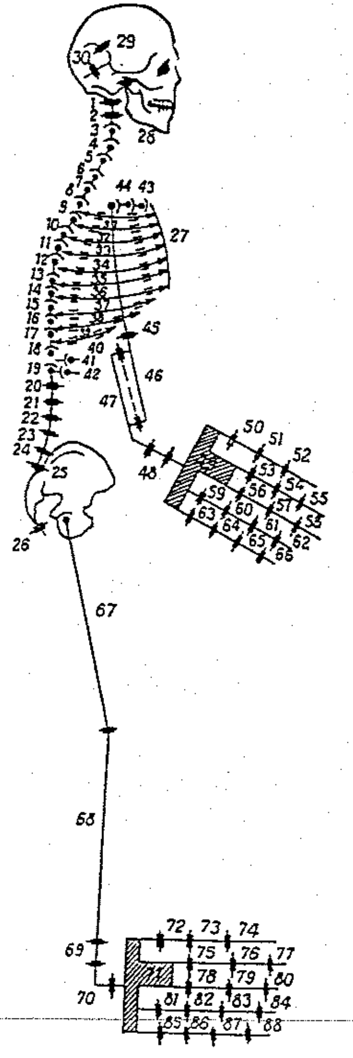

Figure 2.16 Mobility of the human body. The numbers correspond to the joints. Note that only one side of the body is shown.
From Morecki, A., Ekiel, J., & Fidelus, K. (1971). *Bionika ruchu*, Warsaw. (In Polish.) Reprinted by permission.

2.2.4 Constraints on Human Movements

Representative paper: Saltzman, 1979.

Holonomic (geometric) constraints in human movements can be classified into several groups: anatomic, actual, mechanical, and motor task constraints.

Anatomic constraints are those imposed by the structure of the musculoskeletal system. All joints are constrained in at least in two respects: first, body segments forming the joint contact each other (the requirement of the joint integrity), and second, the range of joint motion is limited. Constraints can also be put on movements about different joint axes. The movements about different axes are called *adjunct* movements when they can occur independently, and they are called *conjunct*, or *coupled*, movements when they have to occur together. If the joint movements are coupled, the number of axes of rotation in the joint exceeds the number of DOF. An example of this is the first carpometacarpal joint in which thumb flexion-extension occurs coupled with pronation-supination. Other examples of the coupled joint movements will be discussed in Chapter **5**.

Actual constraints are tangible physical obstacles to movement, such as supporting surfaces or elements of the surrounding environment. These constraints decrease human mobility. For example, during pedaling the leg can be modeled as a planar kinematic chain with three revolute joints (the hip, knee, and ankle; the location of the hip is assumed stationary). However during bicycling, one additional constraint is imposed by the constant contact with the pedal. Thus, only two DOF are left. One DOF is used to rotate the pedal; thus, the last unconstrained DOF is used to vary the pedaling technique. The technique variants are rather limited and only two exist: fixed ankle (or "toe" pedaling) and movable ankle (or "heel" pedaling).

Both anatomic and actual constraints are real; they are caused by material bodies that prevent motion beyond a given border. In human movements, however, other requirements must also be satisfied to perform a motor task. It is convenient to consider these requirements as constraints.

Mechanical constraints are those that define movement geometry in an indirect way; a human must rely on them to prevent an accident, such as falling down. Two mechanical constraints are most typical: constraints necessitated by limited friction and constraints necessitated by balance demands. An example of the first is that to prevent slipping accidents people should perform takeoffs at angles that are larger than the so-called angle of friction, because small takeoff angles lead to slipping and falling. (The takeoff angle is the angle formed by the ground force vector and the support surface.) An example of the second is seen if we again consider the legs as chains with three rotational DOF. In a standing posture, one constraint is imposed because the vertical projection of the general center of mass must be inside a narrow area. Thus, just two DOF are left. As a result, during landing only two distinct strategies are possible: "trunk bending" and "knee bending" (Figure 2.17). When knee bending is prohibited, as in a pure "trunk-bending" strategy, the system has only one DOF; movements in the hip and the ankle joints are coupled. Mechanically unconstrained DOF are called the *permitted DOF*, or *mechanically permitted DOF*.

Motor task constraints are imposed both voluntarily and involuntarily by the performer to execute the desired movement or to fulfill the planned motor task. These constraints are classified as *instructional* (defined by an instruction of the experimenter, competition rules, or the like) and *intentional* (imposed by the performer himself or herself). Because the human motor system is extremely redundant, the number of these constraints is very high. The control of human and animal movements can be seen as the elimination of the redundant DOF. To accomplish a motor task, excessive mechanical DOF must somehow be conquered. This is called *Bernstein's problem*, after Nicholai A. Bernstein (1896-1966), a Russian biomechanist and physiologist.

The CNS reduces the number of DOF in several ways. For example, some joints are *frozen* during the fulfillment of a motor task; that is, their angular values do not change. Also, the movement of some joints is coupled over the duration of some actions. These couplings are called *functional synergies*. The

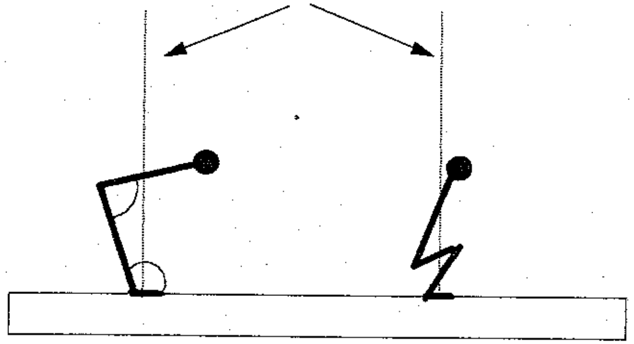

Figure 2.17 Planar kinematic chains with one permitted DOF (left, "trunk-bending" strategy) and two permitted DOF (right, "knee-bending" strategy). In the figure on the left, the hip and ankle angles are coupled. In the figure on the right, three angles (the hip, knee, and ankle) are interrelated; their values cannot be chosen arbitrarily.

concept of joint synergy suggests that several joints are controlled as a unit. Functional synergies and frozen joints decrease the number of DOF controlled by the nervous system. Also, only some DOF, called *essential*, are controlled unconditionally throughout the performance. Other variables, called *nonessential*, are under rather loose control. If a movement is repeated several times, the essential variables show small variability. The variability of nonessential movement parameters is much greater. For example, when hammering a nail, the variability of the hammer position is much less at the instant when the hammer touches the nail than during the wind-up phase. As it happens, this observation, made by Bernstein as early as 1920, was the starting point of his studies in biomechanics and motor control.

FROM THE SCIENTIFIC LITERATURE

Classification of Closed Kinematic Chains in Human Movement

Source: Vaughan, C.L., Hay, J.G., & Andrews, J.G. (1982). Closed loop problems in biomechanics. Part 1. A classification system. *Journal of Biomechanics, 15,* 197-200.

Kinematic analysis of human motion has often been used as a starting point for solving the *inverse dynamics problem*—calculating the forces and torques acting on the joints from the kinematic recordings. As in most mathematics, a solution is feasible when the number of unknowns does not exceed the number of equations; when one equation contains two unknowns it cannot be solved in a unique way. In the framework of the inverse dynamics problem, the number of equations is equal to the number of DOF in the chain. Closed chains have a decreased number of DOF, e.g., a planar closed four-link chain with revolute joints has only one DOF. Four unknown joint torques cannot be calculated from a single equation. Hence, the difference between the number of equations and the number of unknowns, Δ, is important. The difference defines whether the system of equations is overdetermined, determined, or underdetermined.

Body positions are classified according to the number of

- extremities in contact with a fixed external reference system: nonsupport (NS), single support (SS), double support (DS), triple support (TS), and quadruple support (QS), and

See **CHAINS**, *p. 114*

■ ■ ■ CHAINS, *continued from p. 113*

- closed loops formed by the extremities that are not in contact with the fixed external system: open loop (OL), single closed loop (SCL), and double closed loop (DCL).

As a result, five groups of body positions are defined (Figure 2.18 and Table 2.2). Among the groups, Δ varies from two to negative six. In the

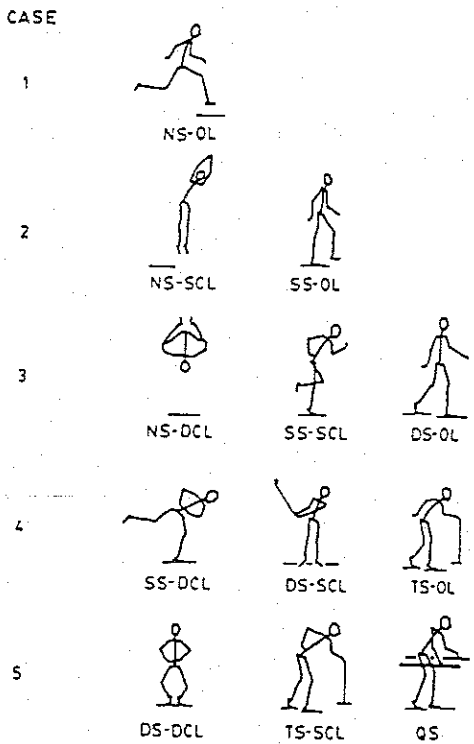

Figure 2.18 Classification system of whole body activities. See Table 2.2 for details.

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first group, the number of equations exceeds the number of unknowns by two—the system is overdetermined. The last three groups are underdetermined—the number of unknowns surpasses the number of equations.

Indeterminate problems can be made determinate by the use of appropriately placed force measuring devices, e.g., using two force platforms instead of one.

Table 2.2 Classification of Body Positions

Case	Δ	System of equations	Names[1]	Examples
1	2	Overdetermined	NS-OL	Airborne phase in running
2	0	Determined	NS-SCL	Diving, hands clasped together
			SS-OL	Walking, single support period
3	−2	Underdetermined	NS-DCL	Tucked position in somersaulting
			SS-SCL	Sprint relay exchange
			DS-OL	Walking, double support period
4	−4	Underdetermined	SS-DCL	Ice skating, hands on hips
			DS-SCL	Golf swing
			TS-OL	Walking with a cane
5	−6	Underdetermined	DS-DCL	Takeoff, hands on hips
			TS-SCL	Walking with cane, a hand on hip
			QS	Walking, hands holding parallel bars

[1]NS, nonsupport; OL, open loop; SCL, single closed loop; SS, single support; DCL, double closed loop; DS, double support; TS, triple support; QS, quadruple support.

2.2.5 Position Analysis of Kinematic Chains

In this section, unless stated otherwise, I am referring to open chains.

2.2.5.1 Two Simple Chains

Consider two simple chains, a two-link and a three-link. The problems to be addressed are direct kinematics, inverse kinematics, and a way of representing the chains. These chains will be analyzed repeatedly throughout the textbook from different perspectives.

2.2.5.1.1 Two-Link Planar Chain

The link lengths are ℓ_1 and ℓ_2 (Figure 2.19). The X axis is counted as an additional link of the chain, ℓ_0. The proximal end of link ℓ is constrained to the origin and, thus, only rotations in the joints are permitted. The external (or anatomic) angles in joints 1 and 2 are α_1 and α_2. The terminal point of the distal segment is designated P. The two-link planar chain is the simplest possible model of a human extremity. Oftentimes, the proximal link of this chain will be further referred to as the "upper arm" or "thigh" and the distal link as the "forearm" or "shank." Joint 2 in this case is the "elbow" or "knee" and joint 1, at the origin O, is the "shoulder" or "hip." The chain represents a human arm or leg without motion at the wrist or ankle.

Direct kinematics. When angles α_1 and α_2 are known, the position of point P, the tip of the chain, can be easily found. The projections of joint 2 and point P on the X and Y axes are

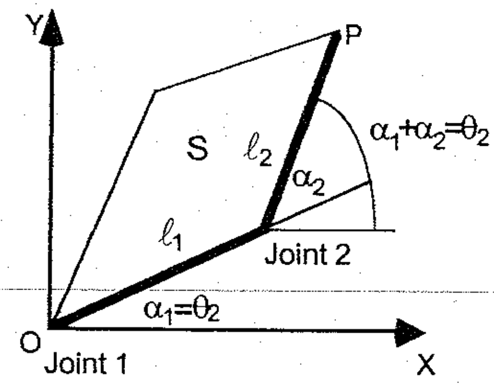

Figure 2.19 Model of a two-link planar chain.

$$
\begin{aligned}
X_2 &= \ell_1 \cos\alpha_1 \\
Y_2 &= \ell_1 \sin\alpha_1 \\
X_P &= \ell_1 \cos\alpha_1 + \ell_2 \cos(\alpha_1+\alpha_2) \\
Y_P &= \ell_1 \sin\alpha_1 + \ell_2 \sin(\alpha_1+\alpha_2)
\end{aligned}
\tag{2.15}
$$

Thus, the direct kinematics problem has a unique solution. The solution can also be written in a matrix form:

$$
\begin{bmatrix} X_P \\ Y_P \end{bmatrix} = \begin{bmatrix} \cos\alpha_1 & \cos(\alpha_1+\alpha_2) \\ \sin\alpha_1 & \sin(\alpha_1+\alpha_2) \end{bmatrix} \cdot \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} \tag{2.16}
$$

Inverse kinematics. The set of points that can be reached by the terminal point of the distal segment when the terminal point of the proximal link is affixed to an unmovable body is called the *reach area*. If the position of point P is given within the reach area, two configurations of the chain can be used to locate the tip P in the position X_P, Y_P. The second configuration (the "elbow-up" position; thin lines) is a reflection of the first one in the line through O and P. So, even in the simplest case, the inverse solution can be not unique.

Representation. A two-link chain can be conveniently described by the position and length of the radius-vector drawn from the origin O to the terminal point of the distal segment P. The distance between the origin of system O and the tip position P (X_P, Y_P) can readily be obtained from the law of cosines. The elbow, or knee, angle is $(\pi - \alpha_2)$ and the side OP of the triangle is

$$
OP = \left[\ell_1^2 + \ell_2^2 - 2\ell_1\ell_2 \cos(\pi - \alpha_2)\right]^{0.5} \tag{2.17}
$$

This distance will be called the *extremity stretch*. To designate the stretch, the symbol S will be used. A movement that decreases or increases the stretch is called extremity flexion or extension. A change in orientation of the radius-vector is called extremity rotation. Planar movement of the extremities is a combination of rotation and flexion-extension. The described representation of the two-link chain is equivalent to its representation through joint angles. In both cases the system with two DOF is explicitly defined by two independent parameters. Note that arm or leg flexion-extension depends on the changes in the second joint only; extremity rotation is a function of two joint angles. When a pure flexion-extension takes place (without extremity rotation), the terminal point moves along the radius, which maintains the same orientation in space. During such a pushing or pulling motion, links 1 and 2 (the upper arm and forearm or the thigh and shank) concurrently rotate in opposite directions in a coupled way. When planar arm movements are discussed, simultaneous flexion-extension of the elbow and shoulder joints in the same direction is oftentimes called the *whipping* action. A *reaching* action involves flexion-extension of the elbow and shoulder joints in opposite directions.

2.2.5.1.2 Three-Link Planar Chain

The link lengths are ℓ_1, ℓ_2 and ℓ_3 (Figure 2.20). The angles in the joints 1, 2, and 3 (the numbers are not shown in the picture) are α_1, α_2, and α_3. The distal segment, ℓ_3 is the end effector and should be placed in the plane in such a way that its distal terminal P is located at a point with the coordinates (X_P, Y_P). Let us try first to solve the direct problem of kinematics, and then the inverse problem. After that, representation of the three-link planar models will be discussed.

Direct kinematics. The joint coordinates, α_1, α_2 and α_3, are given; the position of the end effector, ℓ_3, or its terminal point P (X_P, Y_P), need to be found. It is easily seen from the figure that orientation of link ℓ_i in the absolute reference frame, θ_i, is the sum of the more proximal joint angles: $\theta_1 = \alpha_1$, $\theta_2 = \alpha_1 + \alpha_2$, and $\theta_3 = \alpha_1 + \alpha_2 + \alpha_3$. Note that such a simple representation is possible because the external (or anatomic) rather than internal (or included) joint angles were used. The projections of joints 2 and 3 and of the terminal point P on the X and Y axes are

$$
\begin{aligned}
X_2 &= \ell_1 \cos\theta_1 = \ell_1 \cos\alpha_1 \\
Y_2 &= \ell_1 \sin\theta_1 = \ell_1 \sin\alpha_1 \\
X_3 &= \ell_1 \cos\theta_1 + \ell_2 \cos\theta_2 = \ell_1 \cos\alpha_1 + \ell_2 \cos(\alpha_1 + \alpha_2) \\
Y_3 &= \ell_1 \sin\theta_1 + \ell_2 \sin\theta_2 = \ell_1 \sin\alpha_1 + \ell_2 \sin(\alpha_1 + \alpha_2) \\
X_P &= \ell_1 \cos\theta_1 + \ell_2 \cos\theta_2 + \ell_3 \cos\theta_3 \\
&= \ell_1 \cos\alpha_1 + \ell_2 \cos(\alpha_1 + \alpha_2) + \ell_3 \cos(\alpha_1 + \alpha_2 + \alpha_3) \\
X_P &= \ell_1 \sin\theta_1 + \ell_2 \sin\theta_2 + \ell_3 \sin\theta_3 \\
&= \ell_1 \sin\alpha_1 + \ell_2 \sin(\alpha_1 + \alpha_2) + \ell_3 \sin(\alpha_1 + \alpha_2 + \alpha_3)
\end{aligned}
\tag{2.18}
$$

Again, the direct problem is easily solved. In matrix form the equation for the point P is

$$
\begin{bmatrix} X_P \\ Y_P \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \cos\theta_2 & \cos\theta_3 \\ \sin\theta_1 & \sin\theta_2 & \sin\theta_3 \end{bmatrix} \cdot \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{bmatrix}
\tag{2.19}
$$

Inverse kinematics. The coordinates of the end effector point, X_P, Y_P, are given. The joint coordinates, α_1, α_2, and α_3, need to be found. The problem has an

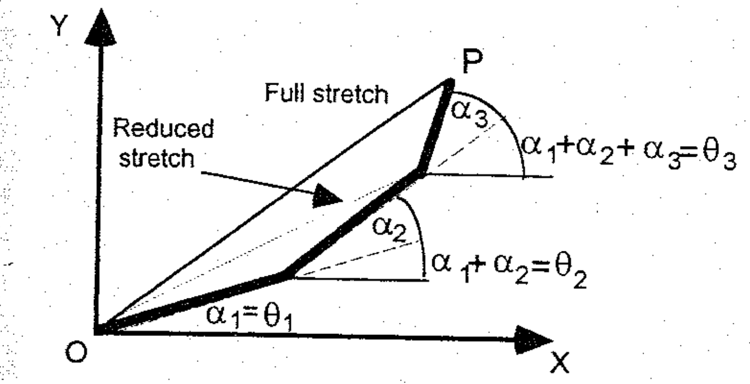

Figure 2.20 Model of a three-link planar chain. The terminal point of the distal segment P has the coordinates (X_P, Y_P) in the absolute reference frame.

infinite number of solutions: two equations in three unknowns cannot be solved in a unique way. Because of the redundancy of the kinematic chain, the same pointing task may be performed in many different ways.

Representation. When radius-vectors are drawn from the origin O to the tip P or to joint 3 (i.e., the wrist or ankle), the corresponding distances are called *full stretch* and *reduced stretch* (see Figure 2.20). The arm or leg rotation and flexion-extension are as previously described. The three-link extremity defined by its radius-vector (stretch and position) has one redundant DOF. When the terminal point is fixed, the chain can still change its configuration; however, all its joint angles are coupled.

2.2.5.2 Multilink Chains and Transformation Analysis

Even a simple three-link chain, when analyzed in space, is a rather complex object, almost impossible to visualize and geometrically analyze. Geometric representation is becoming useless for complex systems; analytic methods should be used. Consider an open kinematic chain of N rigid bodies where N > 2 (Figure 2.21). The links are numbered from zero (for an imaginary unmovable object connected with link 1) to N. Except for the last link, each link of the chain has two joints connecting it to adjacent bodies. The joints are numbered from 1, between the axis X and link 1, to N, between links (N – 1) and N. Local coordinate systems, x_i, y_i, z_i, are attached to each link i. The position of each link can also be represented in the global frame X,Y,Z. In the ensuing paragraphs, the direct kinematic problem is discussed. A single solution to the inverse problem does not exist because of the chain's redundancy.

The position of frame i relative to the previous frame (i – 1) can be described by the 4 × 4 transformation matrix $[T_{(i-1),i}]$, where (i – 1) and i stand

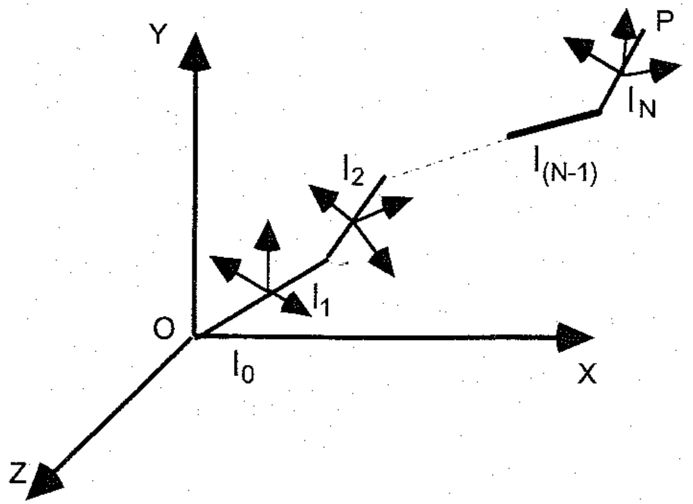

Figure 2.21 Model of a multilink open chain in space.

for the link numbers. The matrices of local transformation are assumed to be known. The end-effector position is then defined as the composition of sequential coordinate transformations $[T_{(i-1),i}]$, where i runs from 1 to N. Because the composition of several displacements is given by matrix multiplication of the corresponding transformation matrices (see equation 1.22), the end-effector position in the global space is defined by the homogeneous transformation

$$[T_{0,N}] = [T_{0,1}][T_{1,2}], \ldots, [T_{(N-1),N}] \tag{2.20}$$

where $[T_{0,N}]$ is the homogeneous transform. Equation 2.20 is termed the *structure equation* of the chain. It defines the position of the end effector, N, in terms of the relative positions of each link in the chain. The 4 × 4 matrix $[T_{0,N}]$ comprises a position vector of the origin of the local end-effector coordinate frame as well as a 3 × 3 rotation matrix.

The matrices $[T_{(i-1),i}]$ define both the *link* transformations, prescribed by the link anatomy, and the *joint* transformations determined by the joint position. To clarify an origin of transformation, the joint coordinate systems should be used in addition to the local frames fixed with the links (similar to the local systems described in Section **1.3.3**). After defining the link transformations as $[L_i]$ and the joint transformations as $[J_{(i-1),i}]$, the position of the end effector is given as

$$[T_{0,N}] = [J_{0,1}][L_1][J_{1,2}][L_2], \ldots, [J_{(N-1),N}] \tag{2.21}$$

and the position of the tip of the end effector as

$$[T_{0,N}] = [J_{0,1}][L_1][J_{1,2}][L_2], \ldots, [J_{(N-1),N}][L_N] \tag{2.22}$$

where $[L_N]$ is the position of the tip in the joint coordinate system of joint N. The transformations $[L_i]$ are constant; they are defined by link geometry. The transformations $[J_{(i-1),i}]$ contain the joint coordinates.

When a closed rather than an open chain is analyzed, the position of the end effector and the starting point is the same. Hence, for the closed chain, the

■ ■ ■ FROM THE SCIENTIFIC LITERATURE ■ ■ ■

Describing Arm Position With Respect to the Trunk

Source: van der Helm, F.C.T. & Pronk, G.M. (1995). Three-dimensional recording and description of motions of the shoulder mechanism. *Journal of Biomechanical Engineering, 117,* 27-40.

When reading this example from the literature the reader is advised to refresh the knowledge from Section **1.2.5.1.3,** in addition to that in Section **2.2.5.2.**

The arm is connected to the trunk through the shoulder mechanism, a constellation of four bones—the humerus, scapula, clavicle, and thorax. Upper arm motion involves three joints: (a) the sternoclavicular, between the thorax and clavicle; (b) the acromioclavicular, between the clavicle and scapula; and (c) the glenohumeral, between the scapula and humerus. Also, the thorax rotates with respect to the environment at the utmost humeral elevation angles. The following procedure was used to describe an arm movement.

1. A global reference system was selected with the origin at the suprasternal notch, with the X axis pointing laterally, the Y axis pointing cranially, and the Z axis pointing dorsally. Only right shoulders were analyzed. The local reference systems were fixed with the thorax, clavicle, scapula, and humerus (Figure 2.22).
2. The following matrices defining orientation of the local coordinate systems with respect to the global system were introduced: [Th] for the thorax, [C] for the clavicle, [S] for the scapula, and [H] for the humerus. If the notation system used in this book were employed, these matrices would have been labeled $[J_{01}]$, $[J_{02}]$, $[J_{03}]$,

See **ARM,** *p. 122*

■ ■ ■ ARM, *continued from p. 121*

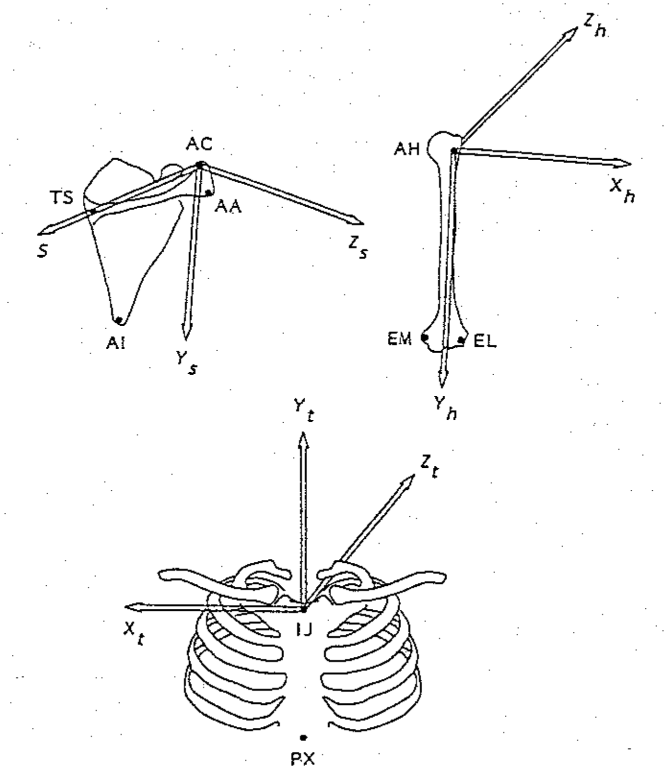

Figure 2.22 Local coordinate systems of the thorax, the right scapula, and the humerus. The scapula and humerus are drawn from the rear. AC, acromion; TS, trigonum spinae; AI, angulus inferior; AA, angulus acromialis; AH, the gap between acromion and humerus; EM and EL, medial and lateral epicondyli; IJ, incisura jugularis; PX, processus xyphoideus.

and $[J_{04}]$ or, using the notation from Chapter **1**, $[R_{G1}]$, $[R_{G2}]$, $[R_{G3}]$, and $[R_{G4}]$.

3. The rest positions were labeled with subscript 0 and the final position with subscript i. Hence, $[H_0]$ means "the rest position of the humerus with respect to the global system of coordinates," and $[H_i]$ indicates "the final position of the humerus in the global coordinate system."
4. The following matrices define *rotation* of the bone from its rest position to position i: $[Rt_{0i}]$ for the thorax, $[Rc_{0i}]$ for the clavicle, $[Rs_{0i}]$ for the scapula, and $[Rh_{0i}]$ for the humerus. With the notation system employed in Chapter **1**, these matrices would be labeled as $[R_{12}]$ or $[D_{12}]$, with an additional symbol for the thorax, clavicle, scapula, and humerus. Note that the authors of this paper did not use specific symbols for $[R_{12}]$ or $[D_{12}]$. Rather, they simply indicate whether the rotation is defined with respect to the global or local axes and use the same symbol for the different matrices.
5. The final positions of the bones as well as the global and local rotation matrices were represented as shown in Table 2.3. Note that because rotations only are discussed, the matrices are orthogonal, that is, their transposes are equal to their inverses.
6. The shoulder mechanism, as compared with other kinematic chains, is specific in that (a) movements in the sternoclavicular, acromioclavicular, and glenohumeral joints are not precisely defined with respect to the proximal bones, e.g., arm flexion is usually described as joint movement around the frontal axis of the trunk but not as a rotation around an axis fixed with the scapula, and (b) the clavicle and scapula are hidden underneath the skin and their instant positions are difficult to record. As a result, the joint rotations were defined with respect to the global axes (Table 2.4).

The left column of Table 2.4 gives the final position of the bone as a function of its initial position and joint angular displacements. The right column represents the joint angular displacements (rotation matrices) as a function of the initial position of the bone in the global frame, the final position of the bone, and the rotation matrices describing joint angular displacement in the proximal joints. Note how troublesome it is to describe the distal bone orientation when all the joint displacements (rotation matrices) are taken with respect to the global axes (compare with equation 2.20).

See **ARM,** *p. 124*

■ ■ ■ ARM, continued from p. 123

Table 2.3 Orientational Matrices Describing a Bone Orientation With Regard to Bone Reference Position

Bone	Bone rotation with regard to local axes: Final bone orientation	Bone rotation with regard to local axes: Local rotation matrix (transformation matrix)	Bone rotation with regard to global axes: Final bone orientation	Bone rotation with regard to global axes: Global rotation matrix (displacement matrix)
Thorax, $[Th_i]$	$[Th_i] = [Th_0][Rt_{0i}]$	$[Rt_{0i}] = [Th_0]^T[T_i]$	$[Th_i] = [Rt_{0i}][Th_0]$	$[Rt_{0i}] = [T_i][Th_0]^T$
Clavicle, $[C_i]$	$[C_i] = [C_0][Rc_{0i}]$	$[Rc_{0i}] = [C_0]^T[C_i]$	$[C_i] = [Rc_{0i}][C_0]$	$[Rc_{0i}] = [C_i][C_0]^T$
Scapula, $[S_i]$	$[S_i] = [S_0][Rs_{0i}]$	$[Rs_{0i}] = [S_0]^T[S_i]$	$[S_i] = [Rs_{0i}][S_0]$	$[Rs_{0i}] = [S_i][S_0]^T$
Humerus, $[H_i]$	$[H_i] = [H_0][Rh_{0i}]$	$[Rh_{0i}] = [H_0]^T[H_i]$	$[H_i] = [Rh_{0i}][H_0]$	$[Rh_{0i}] = [H_i][H_0]^T$

Table 2.4 Bone Orientation as a Result of Joint Rotations With Regard to Global Axes

Bone	Bone Orientation	Global Rotation Matrices
Thorax	$[Th_i] = [Rt_{0i}][Th_0]$	$[Rt_{0i}] = [T_i][Th_0]^T$
Clavicle	$[C_i] = [Rt_{0i}][Rc_{0i}][C_0]$	$[Rc_{0i}] = [Rt_{0i}]^T[C_i][C_0]^T$
Scapula	$[S_i] = [Rt_{0i}][Rc_{0i}][Rs_{0i}][S_0]$	$[Rs_{0i}] = [Rc_{0i}]^T[Rt_{0i}]^T[S_i][S_0]^T$
Humerus	$[H_i] = [Rt_{0i}][Rc_{0i}][Rs_{0i}][Rh_{0i}][H_0]$	$[Rh_{0i}] = [Rs_{0i}][Rc_{0i}]^T[Rt_{0i}]^T[H_i][H_0]^T$

successive transformation given by equation 2.22 is an identity transformation, $[T_{0,N}] = [I]$.

$$\left[J_{0,1}\right]\left[L_1\right]\left[J_{1,2}\right]\left[L_2\right], \ldots, \left[J_{(N-1),N}\right]\left[L_N\right] = [I] \tag{2.23}$$

This is a matrix loop equation. A vector loop approach can also be employed. The vector technique includes several consecutive steps:

1. Attach vectors to the links forming a closed loop. The magnitude and the direction of the vectors should correspond to the length and orientation of the links.
2. Write a vector loop position equation. The equation states that the sum of the vectors in the loop is zero.
3. Break the vector equation into scalar component equations. Solve the equations for the position unknowns.

Unknown velocity and acceleration can also be found by differentiating the equations with regard to time.

2.2.5.3 Denavit-Hartenberg Convention

Representative papers: Chao, Rim, Smidt, & Johnston, 1970; Denavit & Hartenberg, 1955; Raikova, 1992.

Structure equations are the most universal tools for defining positions of serial link chains. The equations, however, are redundant and their parameters are difficult to visualize. This is because specific information about the chain (for example, joint constraints) is not used in the definition. The method called the

FROM THE SCIENTIFIC LITERATURE

Vector Loop Technique in Analysis of Closed Kinematic Chains

Source: You, H. (1996). A study on the determination of an optimal seat-pedal relationship using kinematic simulation. A student research project in the Advanced Biomechanics of Human Motion class. Department of Kinesiology, Penn State University.

The goal of the study was to find an optimal seat-pedal location for drivers of various body dimensions. The seat-pedal system and its vector model are shown in Figure 2.23. The system includes two closed kinematic chains. To show how the method works, we apply it to the first closed chain, which comprises five segments. The corresponding vectors represent the hip joint height ($\mathbf{r}_1$), the femur ($\mathbf{r}_2$), the shank ($\mathbf{r}_3$), the segment from the ankle joint to the heel contact point ($\mathbf{r}_4$), and the vector from the heel point to the vertical projection of the hip joint ($\mathbf{r}_5$). The following vector loop equation is valid

$$\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4 + \mathbf{r}_5 = 0$$

For the horizontal and vertical components the equations are

$$\mathbf{r}_1 \sin\theta_1 + \mathbf{r}_2 \sin\theta_2 + \mathbf{r}_3 \sin\theta_3 + \mathbf{r}_4 \sin\theta_4 + \mathbf{r}_5 \sin\theta_5 = 0$$

$$\mathbf{r}_1 \cos\theta_1 + \mathbf{r}_2 \cos\theta_2 + \mathbf{r}_3 \cos\theta_3 + \mathbf{r}_4 \cos\theta_4 + \mathbf{r}_5 \cos\theta_5 = 0$$

The length of the femur, r_2, the shank, r_3, and the ankle-to-heel distance, r_4, are constant. Also, $\theta_1 = \pi/2$ and $\theta_5 = 0$. The distances r_1 and r_5 are unknown or controlled variables. The angles θ_2, θ_3, and θ_4 are also unknown or controlled variables. The angles depend—among other parameters—on the knee included angle, α_1, and the ankle included angle, α_2. The chain consists of five links and five revolute joints. Hence, according to Gruebler's formula for closed chains in a planar system (equation 2.12) the chain possesses two DOF.

The loop vector equations of the chain were solved and the solutions were tabulated. Using the tables, a designer interested in an adjustment of a car seat to a driver of certain body dimensions (the lengths r_2, r_3, and r_4 are known) can use either one of two strategies: (1) select the values of r_1 and r_5 and then determine the joint angles, or (2) select the values of any two angles and determine the values of r_1 and r_5 and the third angle.

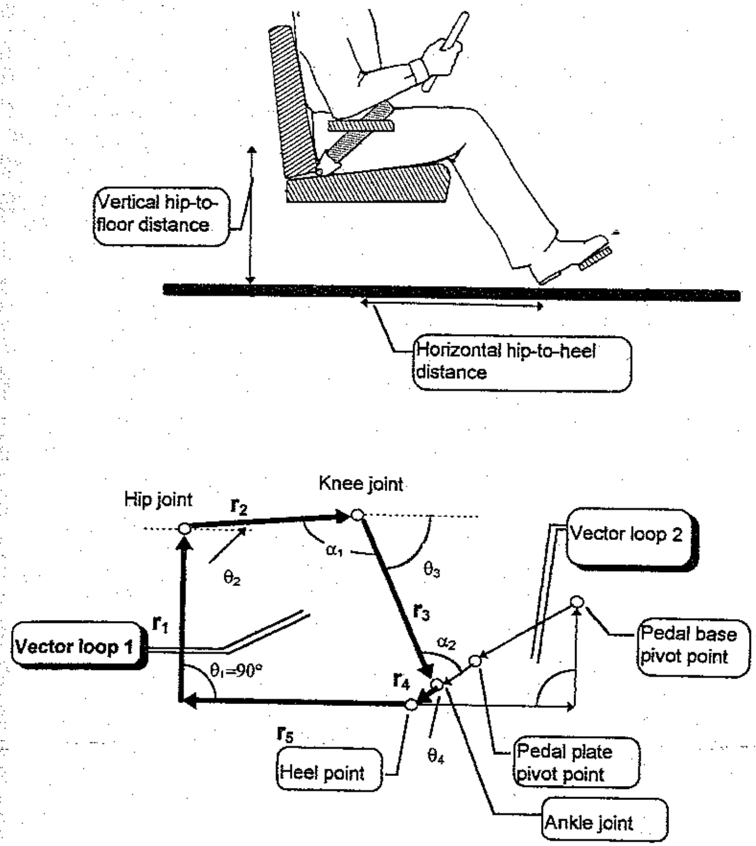

Figure 2.23 A seat-pedal system and its vector model.

Denavit-Hartenberg convention provides a de facto standard procedure for describing multilink chains, especially in robotics. The convention is based on local reference systems attached to the links but defined in the joints. The *i*th frame moves with the *i*th link. The Denavit-Hartenberg notation is introduced for chains with one-DOF joints, both revolute and prismatic (telescopic). We confine the discussion to the revolute joints only. The convention is further explained through an example (Figure 2.24).

Suppose one is interested in a two-link chain formed by the link (i – 1) and the link i. The motion of interest is rotation in joint i. The motion is performed

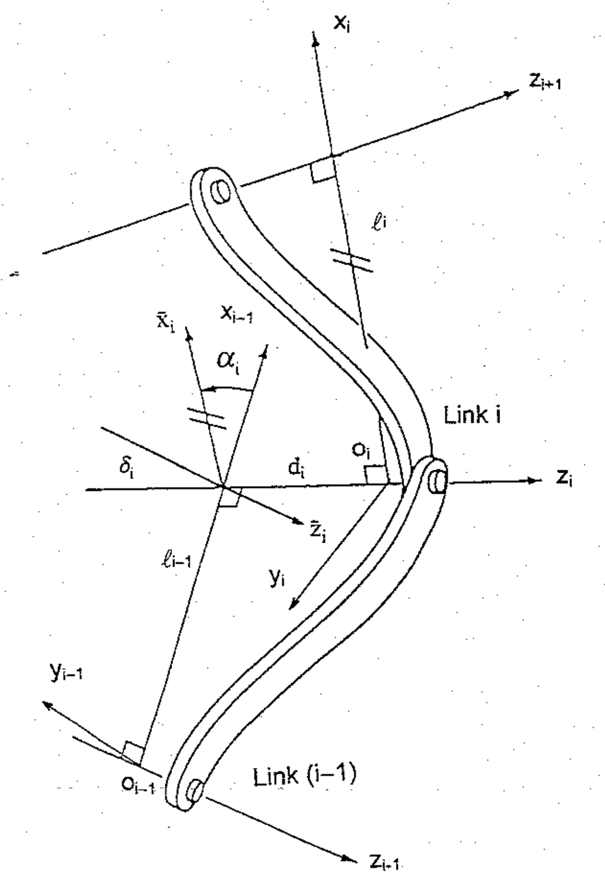

Figure 2.24 Parameters used in the Denavit-Hartenberg notation. The two back slashes (\\) through the lines x_i and $\hat{x}_i$ indicate that the lines are parallel. Rotation in the joint i is defined by the angle α_i formed by the axes x_{i-1} and $\hat{x}_i$.

about the axis z_i. For the sake of illustration, the axes of rotation in the joints (i – 1), i, and (i + 1) are not assumed to lie in one plane. In general, for a link i, the proximal joint is labeled joint i and the distal joint is joint (i + 1). The axes z_i and z_{i+1} define two lines in the link i. In a similar way, two lines are defined for each (intermediate) body segment. The problem is to assign local coordinate frames that permit the compact description of joint transformations. We define, as an example, the frame for the link i. Frames are determined by two rules:

1. The z_i axis lies along the axis of rotation of the *i*th joint.
2. The x_i axis is perpendicular to both the z_i axis and the z_{i+1} axis.

Rule 2 means that the x_i axis is a common normal (perpendicular) to the axes of rotation in the two neighboring joints, i and (i + 1). Trace out this relationship in Figure 2.19 to make sure this point is clear. The interception of the z_i and x_i axes defines the origin of the *i*th local frame, o_i, and y_i completes the right-handed coordinate system. Whenever z_i and z_{i+1} are not parallel, the common perpendicular is unique and o_i is uniquely specified. Note that the position of o_i along z_i might be outside of the joint i. Selection of the o_i location for the parallel axes of rotation in joints will be discussed later in the text.

Assuming that local coordinate systems in other joints are defined in the same way, the following parameters are used to determine the relative location of the two frames:

- The segment length, ℓ, which is defined as the shortest distance between the axes of flexion-extension in the two joints, i and (i + 1), i.e., the elbow and the wrist. The segment length defined in such a way is called the *biomechanical length of the body segment*. Recall that in anatomy and anthropology the segment length is defined differently, as a projected distance between anatomic points rather than the axes of rotation. Contrary to the *anatomic length*, the biomechanical length is measured along the common normal of the two axes of rotation.
- The interjoint offset, d_i, is defined as the distance between the points o_{i-1} and o_i measured along the elbow joint axis, z_i (in joint i). In the elbow joint, because of the valgus angulation of the forearm with the elbow fully extended and the forearm supinated, the joint offset is easily observed.
- The twist, or offset, angle, δ_i, which differs from zero when the axes of flexion-extension in two joints of the same link, for example, in the elbow and the wrist, are not exactly in the same (frontal) plane. For the elbow and wrist, this angle is small. In the ankle joint, the axis of flexion-extension does not rest in the frontal plane. Therefore, the twist angle formed by the axes of rotation in the knee joint and the ankle joint is rather large.
- The angle of rotation (flexion-extension) in the joint i, α_i.

The *segment length*, *joint offset*, and *twist angle* are anatomic parameters that are invariable. Joint configuration depends on the angle of rotation only. The four parameters of the Denavit-Hartenberg convention are used to define the local coordinate systems and the transformation matrices. The transformation matrices are homogeneous: they define both the rotation and translation of one local joint system with regard to another. A 4 × 4 matrix associated with joint (i – 1) transforms coordinates given in the local frame i into the coordinates in the frame i – 1. This transformation can be seen as the following sequence of operations:

1. rotation by $-\delta_i$ to align z_i and z_{i-1};
2. translation along x_{i-1} by ℓ_{i-1}, bringing z_i and z_{i-1} into coincidence;
3. translation along z_i by d_i aligning the xy_i plane (defined in the joint i) with the xy_{i-1} plane, defined in the joint (i – 1); and
4. rotation by $-\alpha_i$, bringing the two frames into the full alignment.

In matrix form this is expressed as

$$[T_{i-1,i}] = \underbrace{\begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{2nd rotation (flexion-extension)}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{2nd translation (joint offset alignment)}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & \ell_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{1st translation (along the common normal)}}$$

$$\times \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{1st rotation (twist angle)}} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & \ell_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & \ell_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.24}$$

Equation 2.24 represents a standard Denavit-Hartenberg transformation matrix. The last column of this matrix locates the origin of frame i in terms of the reference system (i – 1). The upper left 3 × 3 submatrix represents rotation of the *i*th frame relative to the frame (i – 1). There are n + 1 frames on an n-joint kinematic chain.

Several additional rules apply in the Denavit-Hartenberg notation. Ball-and-socket joints are described as three joints with the twist angle 90° and the zero values of the segment length and the joint offset. When two axes of an intermediate link are parallel (making the twist angle 0°), the common normal is usually chosen in such a way that the joint offset d_i also becomes zero. The first and the last bodies of the chain have no associated joints. To define the local frames for them, two fictitious "joints" are introduced: joint 1 is between the global frame counted as the zero link of the chain and the first link of the chain, and the joint (n + 1) is arbitrarily specified at the last link. Once the axes of rotation in these "joints" are defined, other parameters of the Denavit-

FROM THE SCIENTIFIC LITERATURE

Using Denavit-Hartenberg Convention for Studying Finger Movement

Source: Casolo, F., & Lorenzi, V. (1994). Finger mathematical modeling and rehabilitation. In: F. Schuind, K.N. An., W.P. Cooney III, & M. Garcia-Elias (Eds.). *Advances in the Biomechanics of the Hand and Wrist.* NATO ASI Series. Series A: Life Sciences Vol. 256. New York: Plenum Press, pp. 197-223.

The finger was described as an open chain of three rigid bodies with the metacarpal bone as a reference frame (Figure 2.25). The interphalangeal

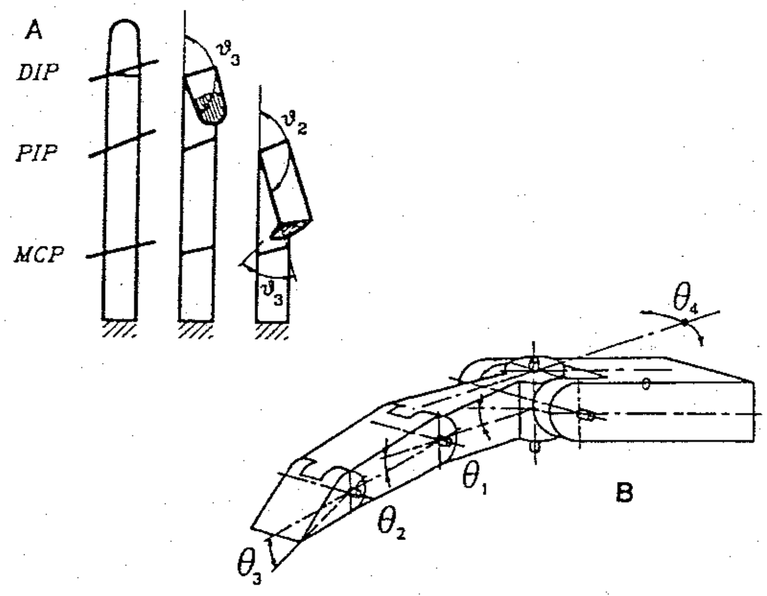

Figure 2.25 The Denavit-Hartenberg convention can be used for representing finger movement. A. Joint axes of rotation are generally not parallel. B. Scheme of the finger joint kinematics.

Adapted from Casolo, F. & Lorenzi, V. (1994). Finger mathematical modeling and rehabilitation. In F. Schuind, K.N. An., W.P. Cooney III, & M. Garcia-Elias (Eds.), *Advances in the biomechanics of the hand and wrist.* (pp. 197-223). NATO ASI Series. Series A: Life Sciences Vol. 256. New York: Plenum Press.

See **DENAVIT-HARTENBERG,** *p. 132*

■ ■ ■ DENAVIT-HARTENBERG, *continued from p. 131*

joints were modeled as hinges, i.e., revolute joints with one DOF. In accordance with experimental observations, the axes of the interphalangeal joints were not assumed to be parallel (see Figure 2.25A). The metacarpophalangeal joint was represented as a joint with two DOF, flexion-extension and abduction-adduction. Hence, the chain had four DOF (Figure 2.25B). To define the chain configuration, the following parameters should be known: (a) segment length, in total 3; (b) angles defining axes of orientation, e.g. Euler's angles, in total 9; and (c) angles associated with the finger movement, in total 4, flexion-extension angle in the distal interphalangeal joint, proximal interphalangeal joint, and metacarpophalangeal joint, and abduction-adduction angle in the metacarpophalangeal joint.

The coordinate systems were selected according to the Denavit-Hartenberg convention. Each phalanx and joint, from proximal to distal, was labeled with k = 1, 2, 3 (0 for the metacarpal bone). A reference frame k was assigned to the center of joint k. The reference axes were oriented as follows: x_k along the joint axis of rotation, directed from the ulna to the radius; y_k perpendicular to a plane defined by x_k and k + 1 joint center, directed dorsally; and z_k (for the right hand) coherent with the right-hand thumb rule.

With such a convention, the position matrix of the distal phalanx with respect to the metacarpal frame can be calculated by means of the simple expression:

$$[T_{0,3}] = [T_{0,1}]\,[T_{1,2}]\,[T_{2,3}]$$

Hartenberg notation can be measured in the manner discussed in the previous paragraphs.

With Denavit-Hartenberg matrices, the structure equation of the chain is defined as usual (see equation 2.20).

2.3 BIOLOGICAL SOLUTIONS TO KINEMATIC PROBLEMS

Representative papers: Abeele et al. 1993; Soechting & Flanders, 1989.

Fit humans move easily in an immediate environment, catch and manipulate objects, avoid obstacles, and adjust their motion to the changes in the extrapersonal space. Purposeful movement toward an object can be performed at a very early age. Four-month-old babies can grasp a toy. To move about efficiently in the physical environment and to manipulate objects, the CNS needs to integrate the sensory information about the location of the object relative to the body (or the body with regard to the object) on the one hand, and the body posture (joint configuration) on the other.

The methods of kinematic geometry described previously are commonly used to define the position of body parts with regard to each other and the environment. Together, they provide a powerful tool for describing body position, including its location, orientation, and posture. These tools are used efficiently by external observers, both biomechanists and engineers. This does not imply, however, that the CNS uses similar approaches to control body position. Most probably, it does not. The internal representation of body position is executed in another way, which still remains unsolved. In what follows, a brief description of the problem is provided.

2.3.1 Internal Representation of the Immediate Extrapersonal Space

Representative paper: Kosslyn et al., 1992.

Human beings use two approaches to judge the location of objects in space. In the first, which is called *categorical*, spatial relations of objects are described verbally by such expressions as "to the left" or "to the right" or by prepositions "below," "on," and "in front of." The precise location of the object is not made explicit in such terms. For example, when the object is located to the left of another, it can be at any place with regard to the horizontal and vertical axes. Some spatial categories, for instance, laterality, develop relatively late in life. Children begin to discern left and right at the age of 7 or 8. Neural structures underlying the perception of categorical relations are contained mainly within the left hemisphere of the brain. In general, the left hemisphere is important for conceptual thinking, including any category formation, and language. Patients with a left-hemisphere stroke have decreased ability to recognize categorical spatial transformations.

Representation of the precise location of objects is called *coordinate representation*. The neural substrate of the perception of coordinate relations is mainly within the right hemisphere of the brain. Patients with right-hemisphere strokes mistakenly estimate the coordinates of the object. Although the metric aspects, such as distances, angles, or volume, can be put into language, this is typically done only in technology and science and requires special education. When asked to estimate metric aspects verbally, most people do it rather imprecisely. However, to move successfully, the metric aspects must be perceived accurately.

To discuss the internal coordinate representation, it seems natural to preserve the main terminology used in kinematics and to speak about spatial reference frames used by the CNS. These (internal) reference frames, however, may be of any nature and type. They can be formed in the sensors (e.g., location of a point on the retina, direction of velocity sensed by semicircular canals in the vestibular organ), in the motor system (e.g., the location of the head with regard to the ground), or they can be constructed by the brain. Sensory messages initially expressed in various frames must be somehow integrated by the CNS. This is believed to be a multistage (hierarchical) process, a sequence of transformations (mappings) between successive reference frames. For example, to locate a visual target with regard to the performer's body, the following neural transformations should occur:

1. The visual object produces an image on the retina; the object's location is presented in the retinal reference frame.
2. The eye orientation within the head should be sensed. The object's coordinates are transformed from the retina-centered reference frame into the head-centered reference frame.
3. The head position with regard to the trunk should be taken into account. The transformation of the object's coordinates from the head-centered to the trunk-centered frame takes place. As a result, the position of the target with regard to the body (if the trunk and the body are synonymous) is established.

If the matrix method were used, it would result in multiplication of retinal coordinates by the two matrices describing the transformation "from the eyes to the head" and "from the head to the trunk." If the target position needs to be determined in relation to a particular body part (e.g., the shoulder, hand, finger) rather than to the trunk, the computational complexity increases.

2.3.2 Internal Representation of the Body Posture

For an illustration, first consider how the problem of end-effector positioning has been solved in robotics. This is usually done in two steps:

1. the (real or desired) position of the end effector is given in an external reference system, and then
2. the inverse kinematics solution is sought to define the appropriate joint angular coordinates.

Hence, the position of the robot's hand is given first in the extrinsic coordinates, and then the intrinsic coordinates of the joint angles are found. Evidently, for multilink chains with many DOF, there is no unique solution to the inverse kinematics problem. Many joint configurations can bring the end effector to the same position.

2.3.2.1 Frames of Reference and the "Body Scheme"

Representative papers: Berkenblit et al., 1986; Soechting & Flanders, 1992.

The frames used by the CNS for spatial representation of body posture (e.g., joint angles, limb angular orientation in the external space) are still objects of discussion and intensive research. The possibility also exists that various reference frames are used concurrently or in sequence. In the latter case, the movement is broken into parts for which different frames are used. In what follows, some considerations regarding the internal representation of body posture are provided.

The "robotics" approach, in which the end effector position is described by a composition of several joint transformations, inevitably leads to the accumulation of errors from various joints. The positioning accuracy of the terminal point in the chain is less than the accuracy of individual joint positioning. Surprisingly, joint errors are not accumulated in human movements: the accuracy of the end-effector positioning does not depend on the length of the chain. To illustrate this you can conduct a simple experiment. With any finger of one hand touch the same finger of the opposite hand behind your back. Most people can do this immediately or during a second attempt. Let us formally describe the motor task. Suppose that an internal global system is fixed somewhere in the shoulder girdle. In the experiment, each arm chain includes at least six joints (the glenohumeral, elbow, wrist, and the three finger joints). Therefore, for each arm the following structure equation can be written:

$$\mathbf{P} = [T_{sh}][T_{elb}][T_{wrist}][T_{carpometacarpal}][T_{metacarpophalangeal}][T_{interphalangeal}][L_N] \quad (2.25)$$

where **P** is a three-dimensional vector of the position of the terminal point in the global frame, the [T]s are transformation matrices, and $[L_N]$ is the position of the tip in the joint coordinate system of joint N. Each transformation matrix contains certain "errors of measurement" (people reproduce a joint angular position with some variation). Despite these errors, and the involvement of 12 joints totally, the final accuracy of the two chains is very high. When the same movement is performed several times, the variability of the end-effector trajectory is less than that of the individual joints.

The complexity and robustness of strategies used by animal species for positioning their limbs are confirmed by classic physiological experiments with spinal frogs (frogs with dissected brains and intact spinal cords). When a piece of paper soaked in acid is applied to a frog's skin, the frog wipes the paper away from the body. This multijoint movement is performed in a coordinated manner, disregarding almost all attempts to fool the spinal frog (e.g., to place the stimulus at different locations, like on the ipsilateral forelimb, to change the limb position, to load the limb, to introduce mechanical obstacles to the movement). If the task is solvable, the spinal frogs solve it, and usually on the first trial.

These experiments have led to the conclusion that a *body scheme* is somehow represented in the CNS. The body scheme can be viewed as a specific internal representation of the body's kinematic geometry. At present we know little about the internal representation. It is postulated that a body-centered frame of reference is somehow constructed and used.

It seems that the orientation of individual body segments in the gravity field is one of the parameters used in the body scheme. Gravity provides a stable vertical reference axis (a *geocentric reference frame*). The inclination of a body limb to the gravitational vertical is perceived with a higher accuracy than joint angles. It has been hypothesized that the CNS employs a geocentric reference system rather than a joint-based system. This hypothesis stems, in part, from the fact that people can easily maintain a joint angular position, regardless of the whole body's positioning in the gravity field. For instance, during trunk bending the head is easily maintained in the same position with regard to the trunk even though the head is differently oriented in the field of gravity. This is possible only because the activities of the neck muscles at each trunk position are adjusted to counterbalance the gravity force. Likewise, when a horizontally placed forearm is supinated or pronated, the muscles maintaining the forearm horizontally vary according to the orientation of the forearm relative to the upper arm. Thus, by assumption, the position of the body segments relative to the vertical, rather than to the neighboring segment only, is used in the internal representation of the body posture. This approach is used when studying the internal representation of kinematic geometry in human movements (Figure 2.26). The geocentric system, however, is not the only frame of reference used by the CNS; astronauts perform skilled maneuvers in the absence of gravity.

Evidence has been provided to show that errors in movement execution are due to mistakes in sensorimotor transformation from the extrinsic to the intrinsic coordinates: the subjects readily recognize small changes in both the location of an external target (thus, extrinsic representation was very accurate) and in limb position (intrinsic representation was accurate too), but they fail to precisely point out the target without visual control.

When a tool is used by a person, that person should locate and orient the tool in an extrinsic coordinate system (also called a *task space*). To do that, the

person should construct a local frame centered on the object being used. He or she should sense (or know) the dimensions of the instrument, its functional axes, inertial properties, and so on. The tool becomes an extension of oneself, a new end effector of the human body's kinematic chain.

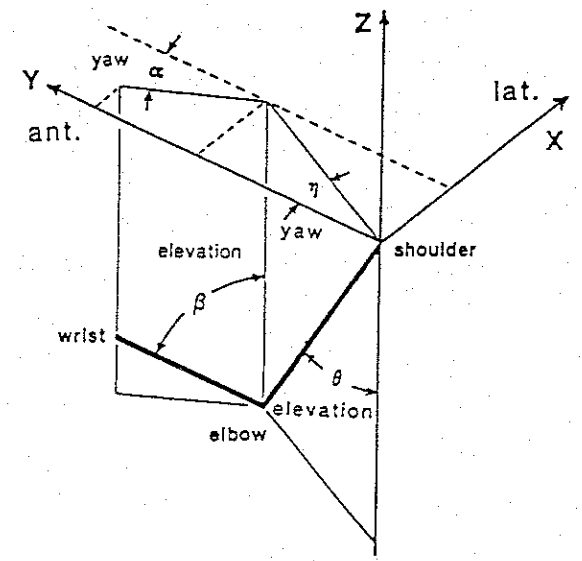

Figure 2.26 The relationship between target location (extrinsic coordinates) and arm orientation (intrinsic coordinates). Arm and forearm inclination are given with respect to the vertical, rather than as joint angles. Two angles are used: (a) elevation (pitch), which is the angle between the limb segment and the vertical axis, measured in the vertical plane; and (b) the yaw angle between the limb segment and the anterior axis, measured in the horizontal plane. The target location is described in Cartesian coordinates X,Y,Z. The relations between extrinsic and intrinsic coordinates are:

$$X = \ell_a \sin\theta \sin\eta + \ell_f \sin\beta \sin\alpha$$

$$Y = \ell_a \sin\theta \cos\eta + \ell_f \sin\beta \cos\alpha$$

$$Z = \ell_a \cos\theta + \ell_f \cos\beta$$

where ℓ_a is the length of the upper arm, ℓ_f is the length of the forearm, the angles θ and η are the vertical and horizontal angles of the upper arm, and β and α are the vertical and horizontal angles of the forearm. Note that the chain is redundant (three equations with four unknowns) and the equations are nonlinear.
From Soechting, J.F. & Flanders, M. (1989). Errors in pointing are due to approximations in sensorimotor transformations. *Journal of Neurophysiology, 62*, 595-608. Reprinted by permission.

2.3.2.2 Vector Field Representation of the Movement Geometry

Representative papers: Bizzi et al., 1991; Georgopulos et al., 1993; Mussa-Ivaldi and Giszter, 1992.

From neurophysiological experiments it follows that some populations of neural cells are tuned to a certain direction of movement. Two examples follow.

In the experiments with spinal frogs, a certain region of the spinal cord (the premotor layers in the lumbar gray matter) was stimulated with microelectrodes and the forces at the ipsilateral ankle were registered. The position of the limb was systematically changed (Figure 2.27A) while the same site was stimulated. The registered forces were directed to a single point within the limb reach. Considered together, all of the forces formed a field that converged at the equilibrium point (Figure 2.27B). When the limb was at the equilibrium point, the electrostimulation did not elicit force. Microstimulation of various sites generated different fields with different zero-force points. The fields were added vectorially. Two simultaneous microstimulations to two spinal regions elicited a vector field that was proportional to the vector sum of the fields produced by the independent stimulation of each region. These experiments suggested that the limb postures are represented in the form of convergent force fields acting on the limb's end point.

In another group of experiments, neural cells in motor and premotor cortices, whose activity depends on movement direction, have been discovered. Individual neurons from this population are tuned to the direction of movement and have a preferred direction. When movement is performed in the preferred direction, the activity of a specific neuron is maximal (Figure 2.28, top panel). Otherwise, the activity of the neuron decreases progressively with movement farther away from the preferred direction. The cell's discharge rate changes as a linear function of the cosine of the angle between the preferred direction of the cell and the movement direction (Figure 2.28, bottom panel):

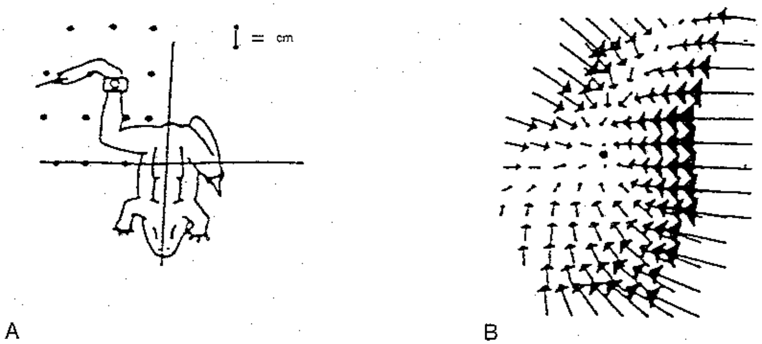

Figure 2.27 Force fields obtained from microstimulation of the frog's spinal cord. A. The positioning of the frog's extremity. The black dots indicate the locations at which the forces were measured. B. Vector field of forces (interpolated). The zero-force (equilibrium) point is indicated by a filled circle.
Adapted by permission from Mussa-Ivaldi, F.A. & Giszter, S.F. (1992). Vector field approximation: A computational paradigm for motor control and learning. *Biological Cybernetics, 67,* 491-500.

$$E_i = b + k \cos \theta_i \tag{2.26}$$

where E_i is a firing rate of the *i*th neuron, θ_i is the angle between the movement direction and the preferred direction for the *i*th neuron, and b and k are parameters.

When movement is performed in a certain direction, the large ensemble of directionally tuned cells is activated (Figure 2.29). The activity of the ensemble is represented by the neuronal population vector, which is calculated as a weighted sum of vector contributions of single cells:

$$\mathbf{P}(t) = \sum_i V_i(t)\mathbf{C}_i \tag{2.27}$$

where $\mathbf{P}(t)$ is the neuronal population vector, $\mathbf{C}_i$ is the unit preferred direction vector of the *i*th neuron, V_i is the weight proportional to the firing rate of the *i*th neuron, and t is time.

Neuronal population vectors accurately predict the direction of movement both during the periods at which the movement is being planned and when the movement is executed. The neuronal population vectors do not differ for movements of various amplitudes. It is important to mention that the movement direction in this set of experiments have been determined relative to the initial position of the hand. Hence, the direction angle θ is represented in the reference frame fixed with the hand. The preferred direction of the individual cells has not been affected by the changes in initial and intended arm position. This fact implies that the cells do not encode the location of the target itself but rather they encode the vectorial difference between the two hand positions, initial and intended.

The described findings suggest that the CNS represents movement geometry in a manner that hardly resembles the methods that have been developed in theoretical kinematics and realized in engineering.

2.4 SUMMARY

Various coordinate systems are used to describe a joint configuration, i.e., body posture. The following systems were discussed in the chapter: the clinical

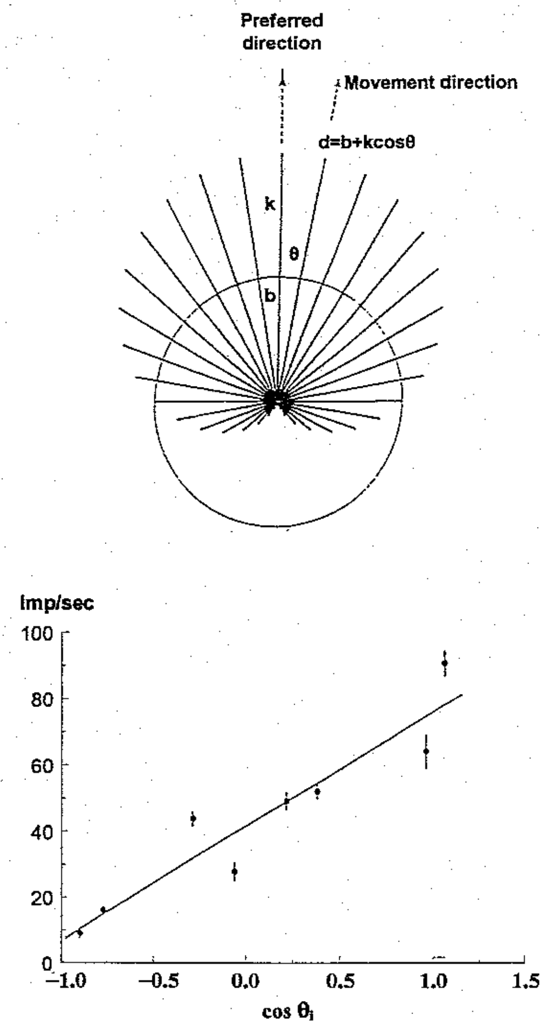

Figure 2.28 Relationship between single cell discharge and direction of movement. On the top panel, the relationship between the movement direction and the preferred direction of a cell is shown. The length of a line indicates the discharge rate of the cell with movement in the direction of the line. The bottom panel shows the mean discharge rate of the cell versus cos θ_i. Vertical bars are ±1 standard deviation.
Adapted by permission from Schwartz, A.B., Kettner, R.E., & Georgopulos, A.P. (1988). Primate motor cortex and free arm movements to visual targets in three-dimensional space. 1. Relations between single cell discharge and direction of movement. *Journal of Neuroscience, 8,* 2913-2927.

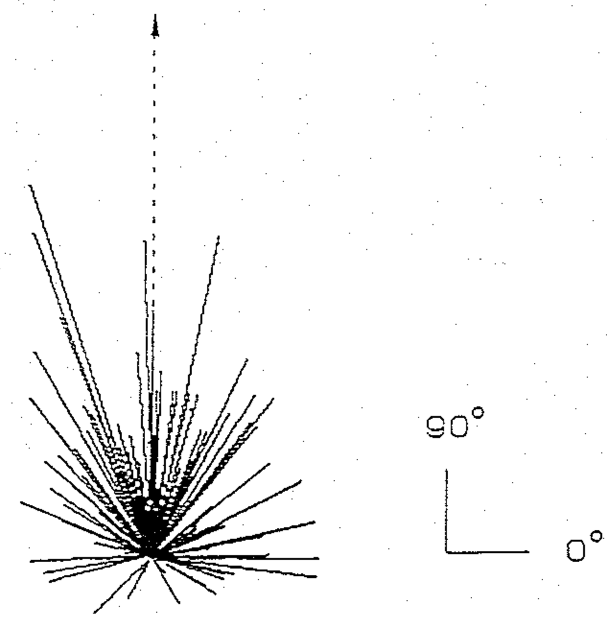

Figure 2.29 Vector contributions of 151 directionally tuned cells for movements directed at 90°. The spatial congruence between the movement direction and the direction of the population vector (dashed arrow) is evident.
Adapted by permission from Kalaska, J.F., Gaminiti, R., & Georgopulos, A.P. (1983). Cortical mechanisms related to the direction of two-dimensional arm movements: Relations in parietal area 5 and comparison with motor cortex. *Experimental Brain Research, 51,* 247-260.

system, globographic presentation, segment coordinate system, and the joint rotation convention.

Human body segments are combined in kinematic chains. The kinematic chain is open if one end of the chain (e.g., distal segment) is free to move. In closed chains, constraints are imposed on both ends of the chain. The term "degree of freedom" refers to an independent way in which the body can move. A free rigid body has six DOF, three translatory and three rotatory. The total number of DOF of a kinematic chain is called the mobility of the chain. Interest in the study of kinematic chains primarily revolves around two issues: (a) direct kinematics, when the joint coordinates are known and the end-effector position is sought; and (b) inverse kinematics, when the end-effector position is known and the joint coordinates are sought. The human skeletal system has many (about 245) DOF and is highly redundant. To perform a coordinated movement, excessive mechanical DOF should be overcome (Bernstein's problem).

The structure equation defines the position of the end effector (in the global system of coordinates) in terms of the relative position of each link in the chain. The Denavit-Hartenberg convention is often used when describing multilink chains.

The manner in which the geometries of the body and the immediate environment are represented in the CNS is currently the object of intensive research. At least three reference frames are used by the CNS to control body posture and movement: (1) the exocentric frame, in which immediate extrapersonal space is represented; (2) the geocentric frame, based on the gravity vector; and (3) the egocentric frame, representing the relative position of various body parts. Recent findings suggest that body posture, as well as movement geometry, is represented in the CNS in a vector form.

2.5 QUESTIONS FOR REVIEW

1. Give an example of three different technical coordinate systems.
2. What are the advantages and disadvantages of the clinical reference system?
3. Discuss the difference between the joint angles used in the globographic convention and Euler's angles.
4. Write down an equation relating coordinates of a point measured in two different segmental coordinate systems, one of which is fixed with a distal body segment and the second with the proximal segment. Use a homogeneous transform.
5. Using the joint rotation convention, define the coordinate axes for the hip joint. Which of the axes is the floating axis?
6. Give an example of singular joint configuration.
7. How many DOF does a joint of the 4th class have? Do rotational joints of the 1st class exist? What is the class of a hinge joint? How many DOF does a ball-and-socket joint have?
8. Define mobility and maneuverability of a kinematic chain.
9. Write down and explain Gruebler's formula for an open and closed kinematic chain.
10. How many DOF does a planar four-link model consisting of the foot, shank, thigh, and torso (the trunk, head and neck, and arms combined) have?
11. A driver pushes a pedal that rotates about a fixed axis. By assumption, the driver's pelvis does not move. How many DOF does a planar kinematic chain consisting of the hip, shank, foot, and the pedal have? Is this chain open or closed? How do various assumptions about relative motion of the foot with regard to the pedal influence the answer?
12. Discuss the direct and inverse problem of kinematics.
13. An end point of the two-link kinematic chain has coordinates X and Y (see Figure 2.19). The length of the links is ℓ_1 and ℓ_2. Determine the angles θ_1 and θ_2.
14. A four-link planar chain is either open or closed. Write down the structure equations for the chain assuming the two configurations.
15. Discuss the Denavit-Hartenberg convention.
16. Discuss the problem of the external representation of extrapersonal space and body posture.

Answer to Question 13

$$\theta_2 = \cos^{-1}\left(\frac{X^2 + Y^2 - \ell_1^2 - \ell_2^2}{2\ell_1\ell_2}\right) \qquad \theta_1 = \tan^{-1}\left(\frac{Y}{X}\right) - \tan^{-1}\left(\frac{\ell_2 \sin\theta_2}{\ell_1 + \ell_2\cos\theta_2}\right)$$

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3
CHAPTER

Differential Kinematics of Human Movement

In Chapters **1** and **2**, the position of the human body and its parts have been discussed. This chapter is about movement of the body, specifically the movement of biokinematic chains. Throughout this chapter, joint motion is considered pure rotation. Translational motion within a joint is ignored. Unless stated otherwise, the reader should assume that one end of the chain does not move (for clarity, let it be a proximal terminal point). Only open kinematic chains are analyzed.

The chapter starts with discussing the velocity of kinematic chains (Section **3.1**). By reason of didactics, the text advances from simple to complex models, from planar movement to movement in three dimensions and from two-link chains to multilink chains. Movement of planar two-link chains is discussed in detail. It serves as the simplest example of various phenomena of human limb kinematics. The following concepts are addressed in this subsection: joint versus segment velocity, joint velocity versus the end-point velocity (the important notion of a Jacobian is introduced here), direct and inverse kinematic problems, singularity of the kinematic chain, degeneracy in human motion, inverse kinematics of the two-link planar chain, kinematics of pushing motion, maximizing end-point velocity, and—last but not least—the concept of instantaneous center of rotation and how to locate it. The chapter then progresses to planar three-link chains. Three-link chains, as well as multilink chains, are redundant; the position of the end point does not dictate angular positions at the joints. The issue of the chain redundancy is considered and instant centers of rotation are discussed. The main part of the discussion on instant centers of rotation is Kennedy's theorem, which defines the location of the instant centers of rotation in multilink chains. It is typical for higher animals to have a zigzag arrangement of the three-link extremities. Subsection **3.1.1.3** is devoted to multilink chains. The use of the Moore-Penrose pseudoinverse matrices for analysis of these chains is briefly highlighted here.