Fundamental Electrical Concepts

Electric Charge (Q)

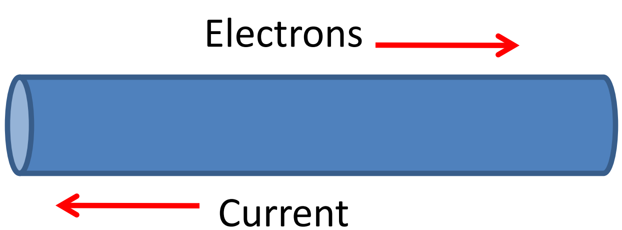
- Characteristic of subatomic particles that determines their electromagnetic interactions

- An electron has a $-1.602 \cdot 10^{-19}$ Coulomb charge

- The rate of flow of charged particles is called current

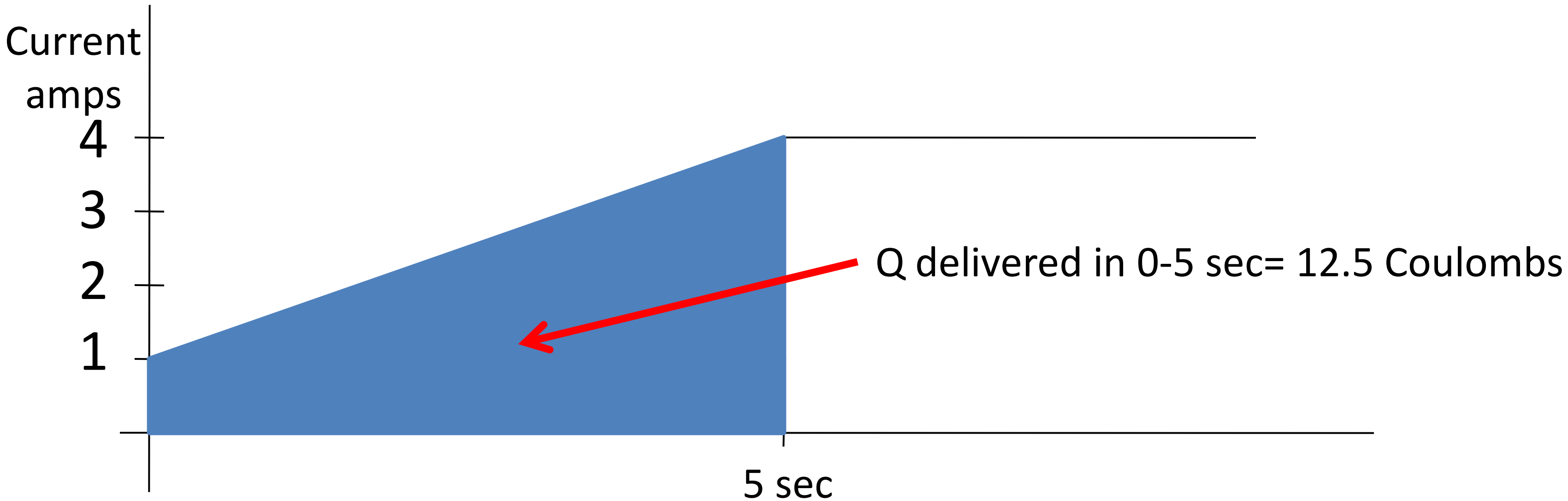
Current

- Current = (Number of electrons that pass in one second) · (charge/electron)
  - -1 ampere = $(6.242 \cdot 10^{18}$ e/sec$) \cdot (-1.602\ 10^{-19}$Coulomb/e)
  - Notice that an ampere = Coulomb/second
- The negative sign indicates that the current inside is actually flowing in the opposite direction of the electron flow



Current

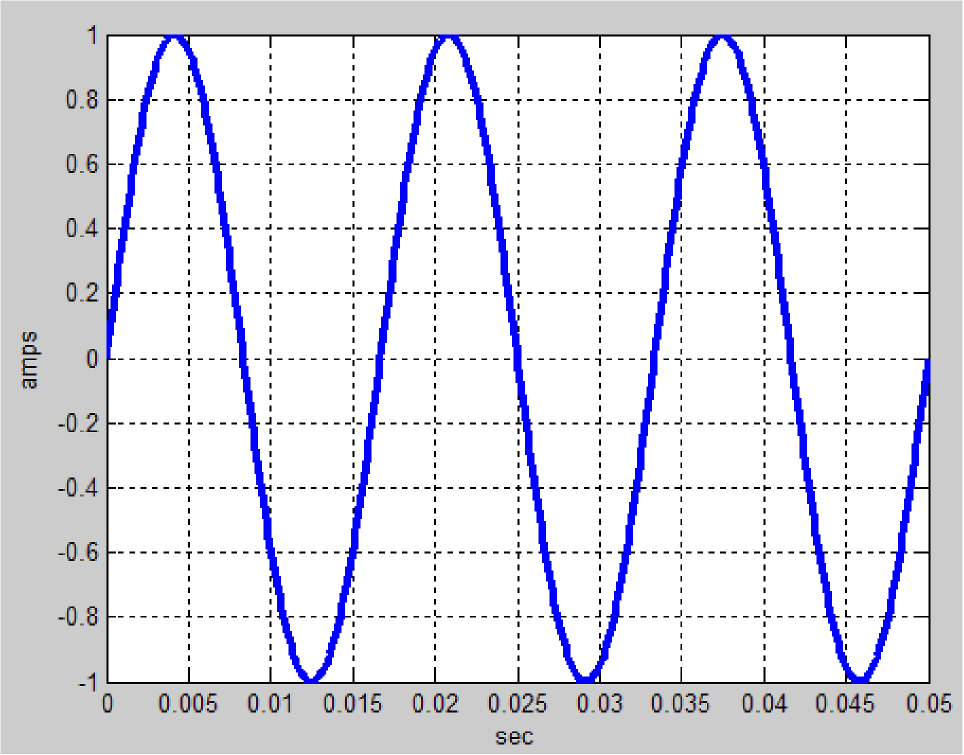
- $i = dq/dt$ – the derivitive or slope of the charge when plotted against time in seconds
- $Q = \int i \cdot dt$ – the integral or area under the current when plotted against time in seconds



AC and DC Current

- DC Current has a constant value

- AC Current has a value that changes sinusoidally



- ➢Notice that AC current changes in value and direction

- ➢No net charge is transferred

Why Does Current Flow?

- A voltage source provides the energy (or work) required to produce a current
  - ➢Volts = joules/Coulomb = dW/dQ
- A source takes charged particles (usually electrons) and raises their potential so they flow out of one terminal into and through a transducer (light bulb or motor) on their way back to the source's other terminal

Terms to Remember

- The ***Source*** can be any source of electrical energy. In practice, there are three general possibilities: it can be a battery, an electrical generator, or some sort of electronic power supply.
- The ***Load*** is any device or circuit powered by electricity. It can be as simple as a light bulb or as complex as a modern high-speed computer.
- (***Path***) a wire or pathway which will allow electron to flow throughout a circuit.
- ***Electricity*** can be described as the flow of charged particles. If the particles accumulate on an object, we term this static electricity.
- (*Direct Current*) An electrical current that travels in one direction and used within the computer's electronic circuits.
- (*Alternating Current*) The common form of electricity from power plant to home/office. Its direction is reversed 60 times per second.
- ***Circuit*** is a conducting path for electrons.

Voltage

- Voltage is a measure of the potential energy that causes a current to flow through a transducer in a circuit
- Voltage is always measured as a difference with respect to an arbitrary common point called ground
- Voltage is also known as electromotive force or EMF outside engineering

What is Ohm's Law?

Ohm's Law states that, at constant temperature, the electric current flowing in a conducting material is directly proportional to the applied voltage, and inversely proportional to the Resistance.

Why is Ohms Law important?

Ohm's Law is the relationship between power, voltage, current and resistance. These are the very basic electrical units we work with. The principles apply to alternating current (ac), direct current (dc), or radio frequency (rf) .

Georg Simon Ohm (1787-1854)

Ohm's Law

$$I = V / R$$

I	= Current (Amperes) (amps)
V	= Voltage (Volts)
R	= Resistance (ohms)

Characteristics of Ohm's Law

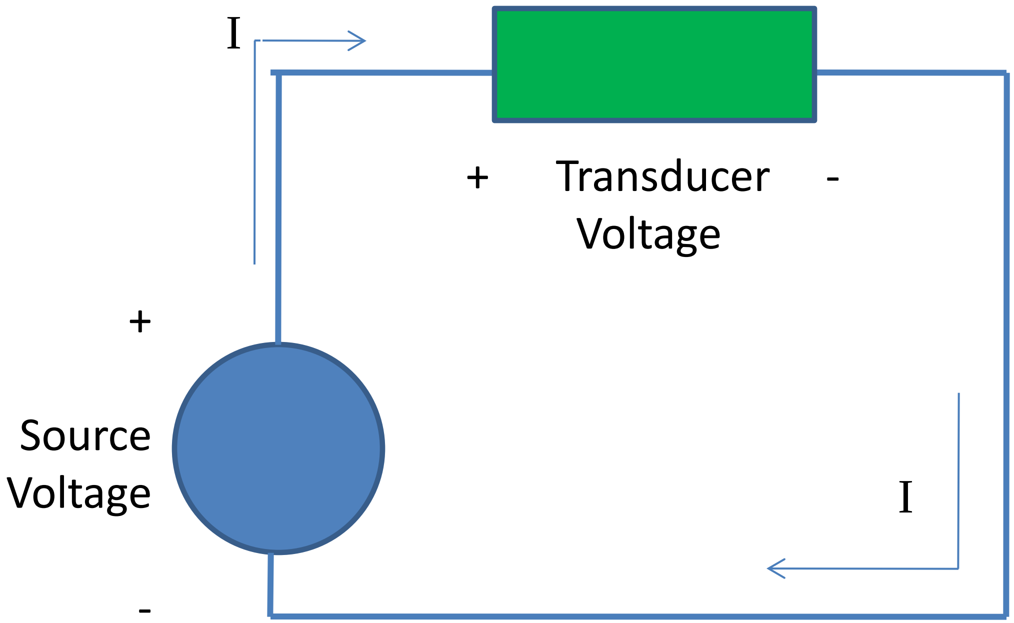
Voltage: Difference of potential, electromotive force, ability to do work.

Unit of measure Volt Symbol **V** (**Current:** Flow of electrons **Unit of measure Ampere** Symbol **I**

Resistance: Opposition to current flow Unit of measure Ohm often seen as the Greek letter Omega Symbol **R**

A Circuit

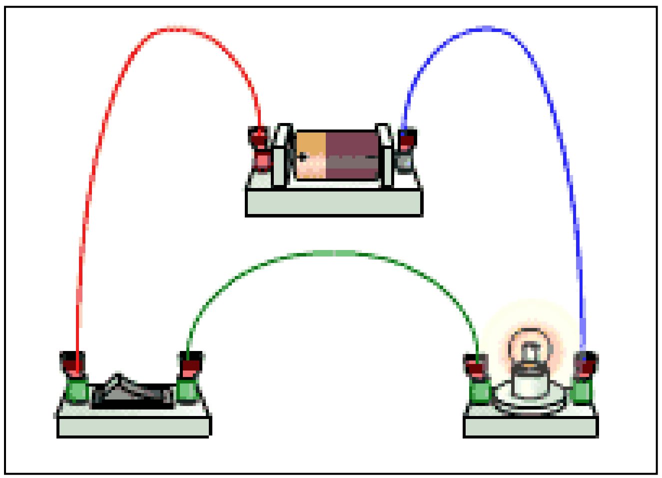
- Current flows from the higher voltage terminal of the source into the higher voltage terminal of the transducer before returning to the source

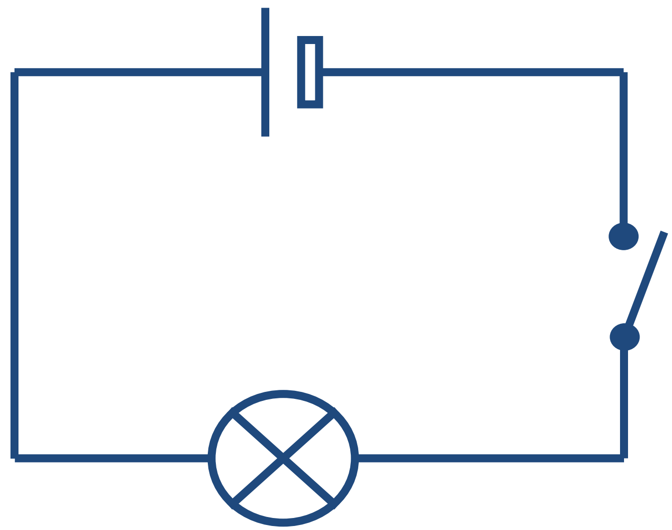


➢The source expends energy & the transducer converts it into something useful

circuit diagram

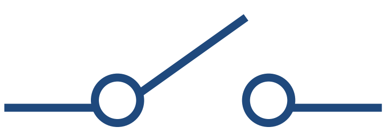
Scientists usually draw electric circuits using symbols;





cell

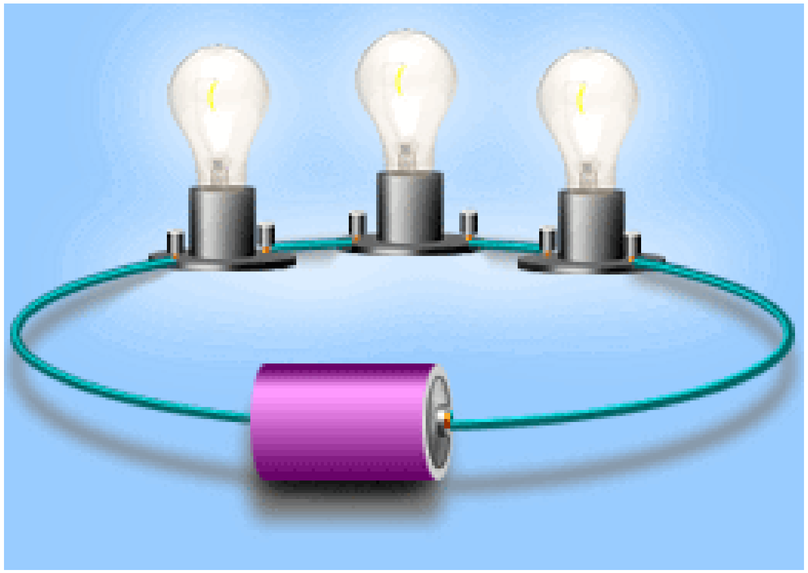
lamp



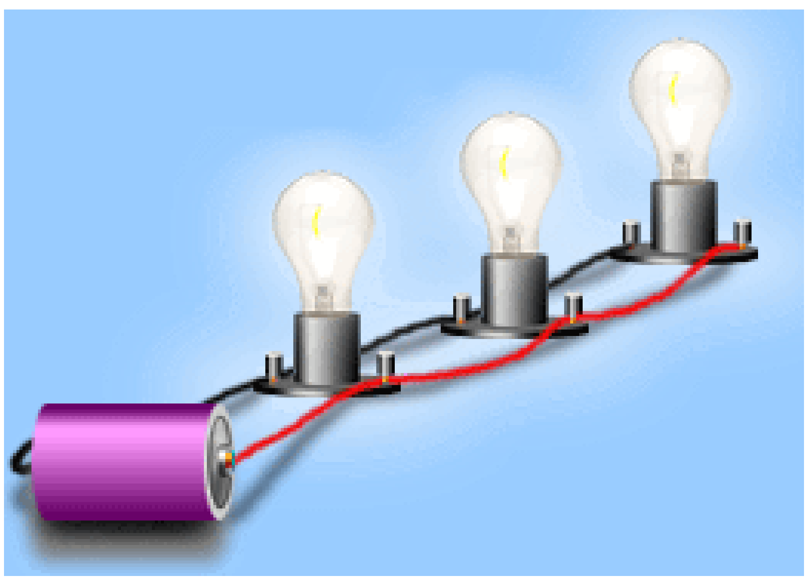
switch

wires

Simple Circuits



- Series circuit
  - All in a row
  - 1 path for electricity
  - 1 light goes out and the circuit is broken



- Parallel circuit
  - Many paths for electricity
  - 1 light goes out and the others stay on

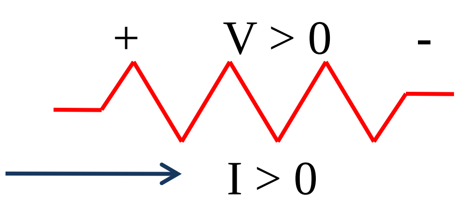
Series and Parallel Circuits

- Series Circuits
  - only one end of each component is connected
  - e.g. Christmas tree lights

- Parallel Circuits
  - both ends of a component are connected
  - e.g. household lighting

Passive Devices

- A passive transducer device functions only when energized by a source in a circuit
  - Passive devices can be modeled by a resistance
- Passive devices always draw current so that the highest voltage is present on the terminal where the current enters the passive device



- **Notice that the voltage is measured across the device**
- **Current is measured through the device**

Active Devices

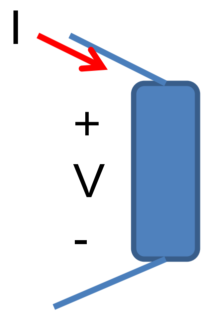
- Sources expend energy and are considered active devices
- Their current normally flows out of their highest voltage terminal
- Sometimes, when there are multiple sources in a circuit, one overpowers another, forcing the other to behave in a passive manner

Power

- The rate at which energy is transferred from an active source or used by a passive device
- P in watts = dW/dt = joules/second
- P= V·I = dW/dQ · dQ/dt = volts · amps = watts
- W = ∫ P · dt – so the energy (work in joules) is equal to the area under the power in watts plotted against time in seconds

Conservation of Power

- Power is conserved in a circuit - $\sum P = 0$
- We associate a positive number for power as power absorbed or used by a passive device
- A negative power is associated with an active device delivering power



If I=1 amp
V=5 volts
Then passive
P=+5 watts
(absorbed)

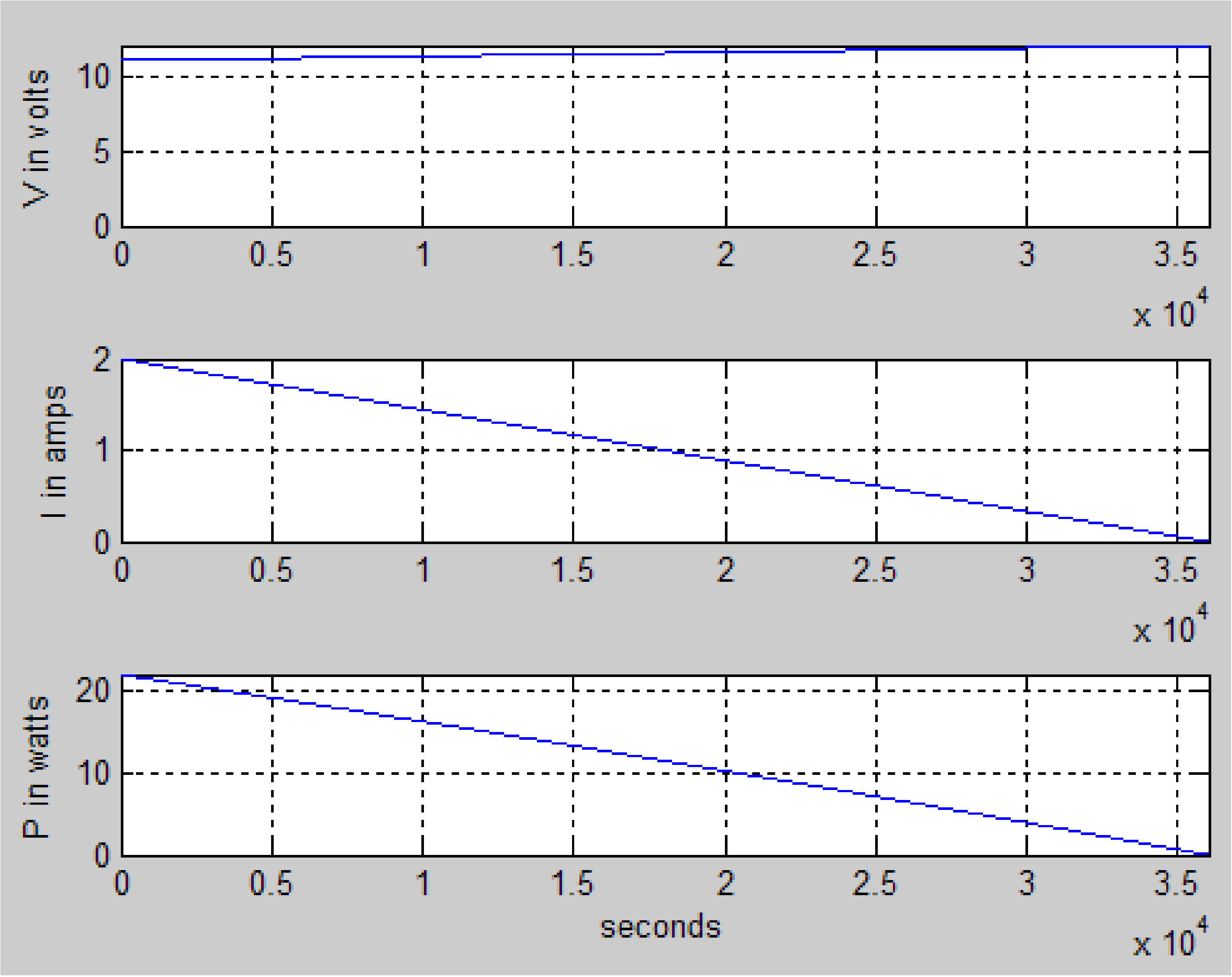
If I= -1 amp
V=5 volts
Then active
P= -5 watts
(delivered)

If I= -1 amp
V= -5 volts
Then passive
P=+5 watts
(absorbed)

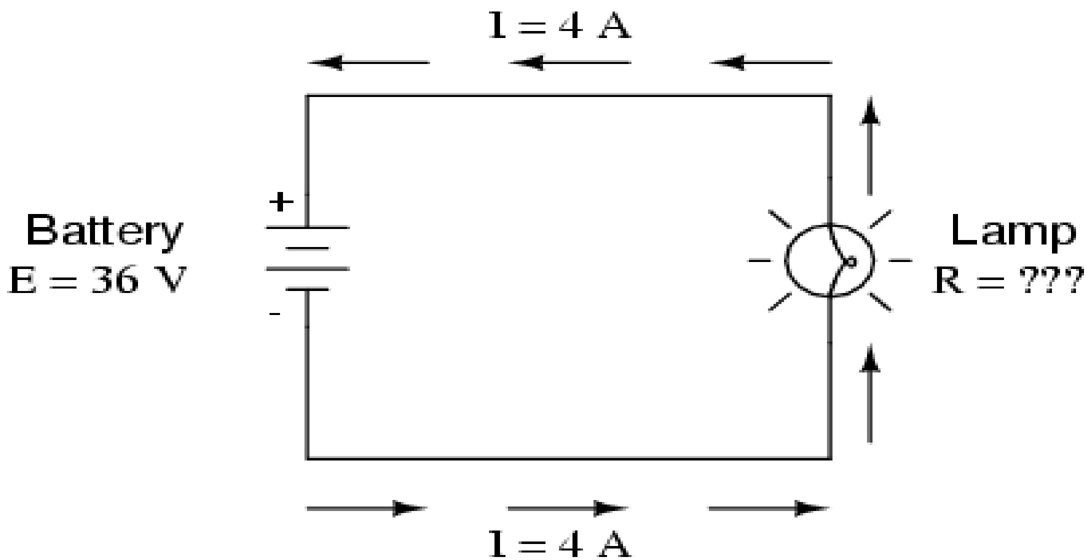
Example

- A battery is 11 volts and as it is charged, it increases to 12 volts, by a current that starts at 2 amps and slowly drops to 0 amps in 10 hours (36000 seconds)
- The power is found by multiplying the current and voltage together at each instant in time
- In this case, the battery (a source) is acting like a passive device (absorbing energy)

Voltage, Current & Power



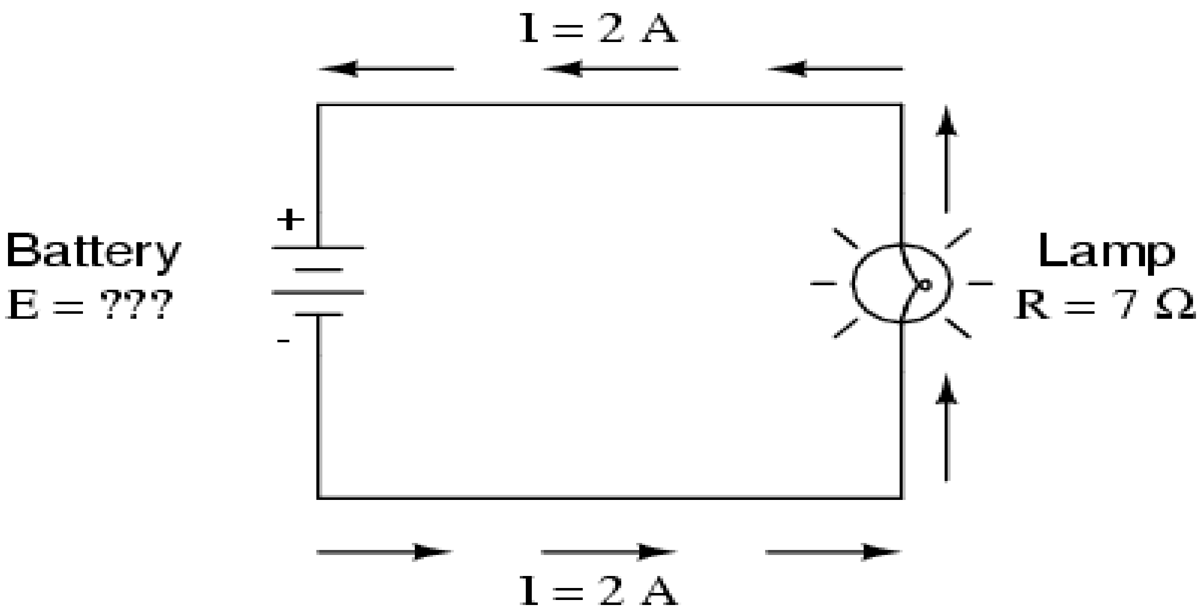
In this second example, we will calculate the amount of resistance (R) in a circuit, given values of voltage (E) and current (I):



What is the amount of resistance (R) offered by the lamp?

$$R = \frac{E}{I} = \frac{36\ V}{4\ A} = 9\ \Omega$$

In the last example, we will calculate the amount of voltage supplied by a battery, given values of current (I) and resistance (R):



What is the amount of voltage provided by the battery?

$$E = IR = (2\ A)(7\ \Omega) = 14\ V$$

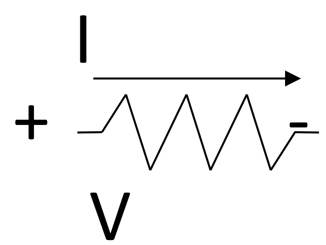
Resistance

- Property of a **device** that indicates how freely it will allow current to flow given a specific voltage applied. If low in value, current flows more freely.
- Measured in ohms (Ω = V/A or KΩ = V/mA)

$$R = \frac{\text{Voltage}}{\text{Current}} \quad \text{or } V = I \cdot R$$

This is Ohm's Law!

- Obeys the passive sign convention

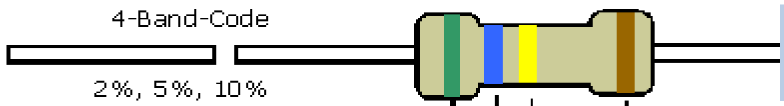


if I > 0 then V > 0

Resistors

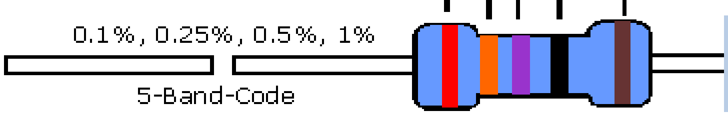
- Resistors are devices that are used in a variety of circuits to make the currents and voltages what you desire
- They are color-coded and come in various tolerances, $\pm$ 1%, , $\pm$ 5% and , $\pm$ 10% are most common

Resistor Color Code



56 * 10 kΩ = 560 kΩ

COLOR	1st BAND	2nd BAND	3rd BAND	MULTIPLIER	TOLERANCE
Black	0	0	0	1Ω	
Brown	1	1	1	10Ω	± 1% (F)
Red	2	2	2	100Ω	± 2% (G)
Orange	3	3	3	1KΩ	
Yellow	4	4	4	10KΩ	
Green	5	5	5	100KΩ	±0.5% (D)
Blue	6	6	6	1MΩ	±0.25% (C)
Violet	7	7	7	10MΩ	±0.10% (B)
Grey	8	8	8		±0.05%
White	9	9	9		
Gold				0.1	± 5% (J)
Silver				0.01	± 10% (K)



237 * 1 Ω = 237 Ω

Electronix Express / RSR
http://www.elexp.com

1-800-972-2225
In NJ 732-381-8020

http://www.elexp.com/t_resist.htm

Conductance (G)

- Conductance is the reciprocal of resistance
- Its unit is the siemen = amps/volts = $1/\Omega$
- $G = I / V = 1 / R$
- An equivalent measure of how freely a current is allowed to flow in a device

Resistivity (ρ)

- Property of a **material** that indicates how much it will oppose current flow
- $R = (\rho \cdot \text{length}) / (\text{cross sectional area})$
- Units are ohms · meters ($\Omega \cdot m$)
- As the wire gets bigger, so does the cross section making the resistance smaller
- As the length gets longer, the resistance goes up proportionately

Conductors

- Materials with electrons that are loosely bound to the nucleus and move easily (usually one electron in the outer shell)
- Their **low resistance goes up as the material is heated**, due to the vibration of the atoms interfering with the movement of the electrons
- The best conductors are superconductors at temperatures near 0^o Kelvin

Semiconductors

- Materials with electrons that are bound more tightly than conductors (usually 4 electrons in the outer shell)
- Impurities are added in controlled amounts to adjust the resistivity down
- **Semiconductors become better conductors at higher temperatures** because the added energy frees up more electrons (even though the electron flow is impeded by the increased atomic vibration)

Insulators

- Materials that have all 8 electrons in the outer shell tightly bound to the nucleus
- It takes high temperatures or very high electric fields to break the atomic bonds to free up electrons to conduct a current
- Very **high resistivities and resistances**

Resistivity of Materials

- Glass
- Gold
- Silver
- Silicon
- Aluminum
- Copper

Put a number 1 beside the material that is the best conductor, a 2 next to the material next most conductive, etc.

Resistivity of Materials

1 Silver - A conductor - $\rho=1.64 \cdot 10^{-8}$ ohm-m

2 Copper - A conductor - $\rho= 1.72 \cdot 10^{-8}$ ohm-m

3 Gold – A conductor - $\rho=2.45 \cdot 10^{-8}$ ohm-m

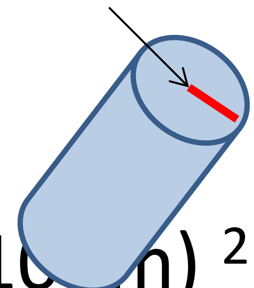
4 Aluminum - A conductor - $\rho= 2.8 \cdot 10^{-8}$ ohm-m

5 Silicon - A semiconductor - $\rho=6.4 \cdot 10^{2}$ ohm-m

6 Glass – An insulator – $\rho=10^{12}$ ohm-m

Example

- A round wire has a radius of 1 mm = 10^{-3}m
- The wire is 10 meters long
- The wire is made of copper
- $R = (1.72 \cdot 10^{-8}\ \Omega{\cdot}m \cdot 10\ m) / (\pi \cdot (10^{-3}\ m)^{2})$
- $R = 0.0547\ \Omega$
- This is negligible in most circuits where resistances are often thousands of ohms



Power used by Resistors

- We saw that $P = I \cdot V$
- If by Ohm's law, $V = I \cdot R$, then $P = I^2 \cdot R$
- Or since $I = V / R$, then $P = V^2 / R$
- Since resistance is always positive for passive devices, then power is always positive (meaning that power is always absorbed or used)

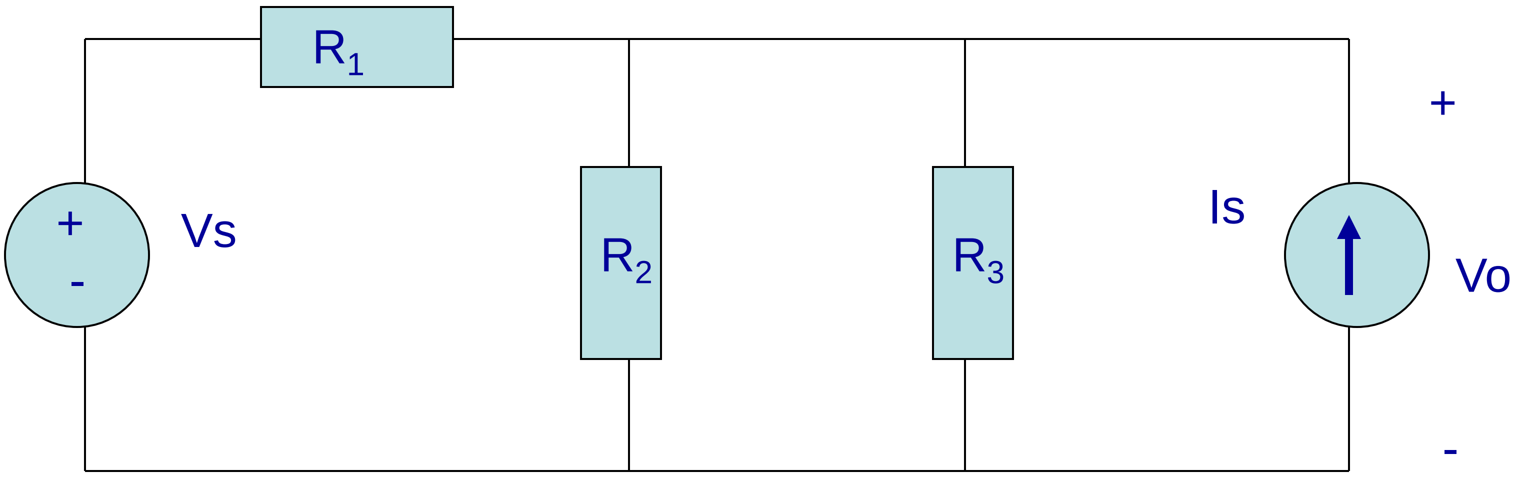
Kirchoff’s Laws

Circuit Definitions

- Node – any point where 2 or more circuit elements are connected together
  - Wires usually have negligible resistance
  - Each node has one voltage (w.r.t. ground)
- Branch – a circuit element between two nodes
- Loop – a collection of branches that form a closed path returning to the same node without going through any other nodes or branches twice

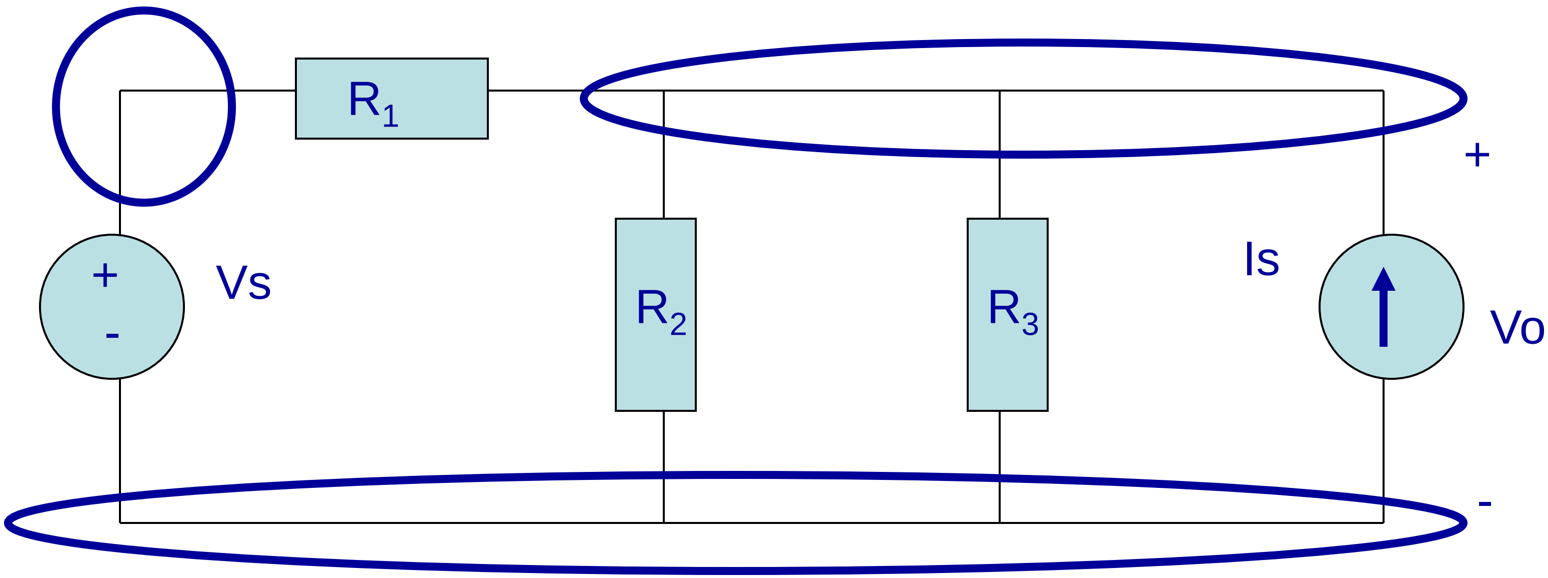
Example

- How many nodes, branches & loops?



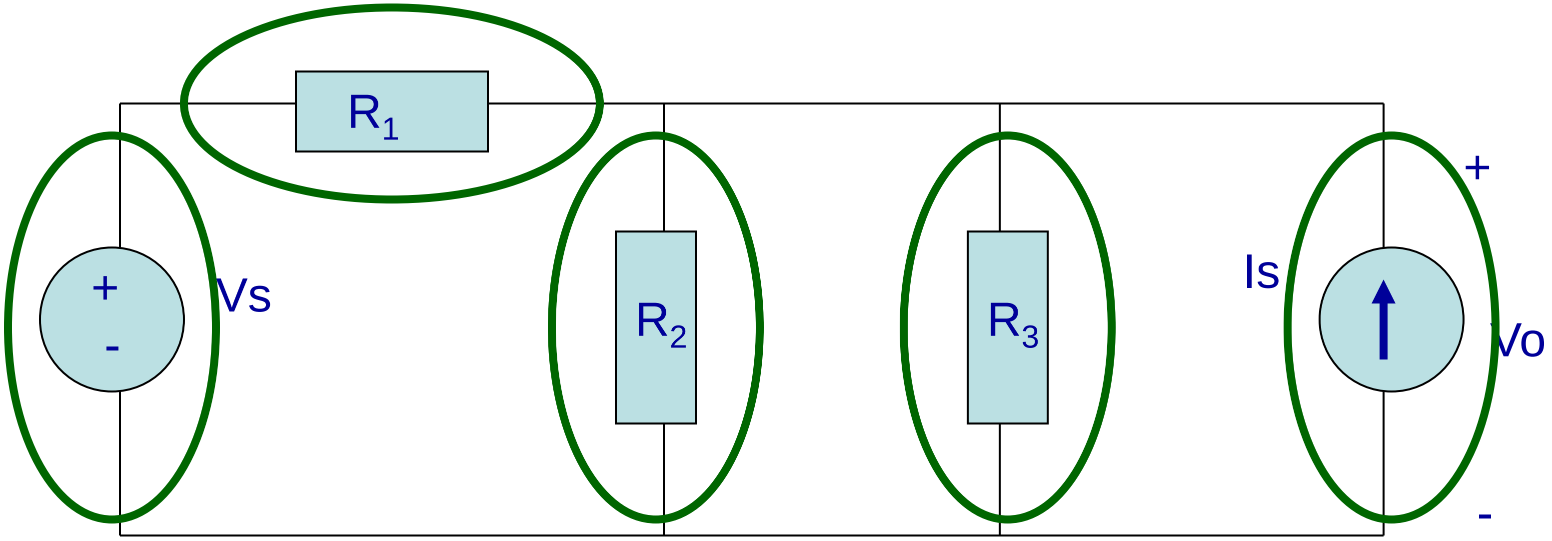
Example

- Three nodes



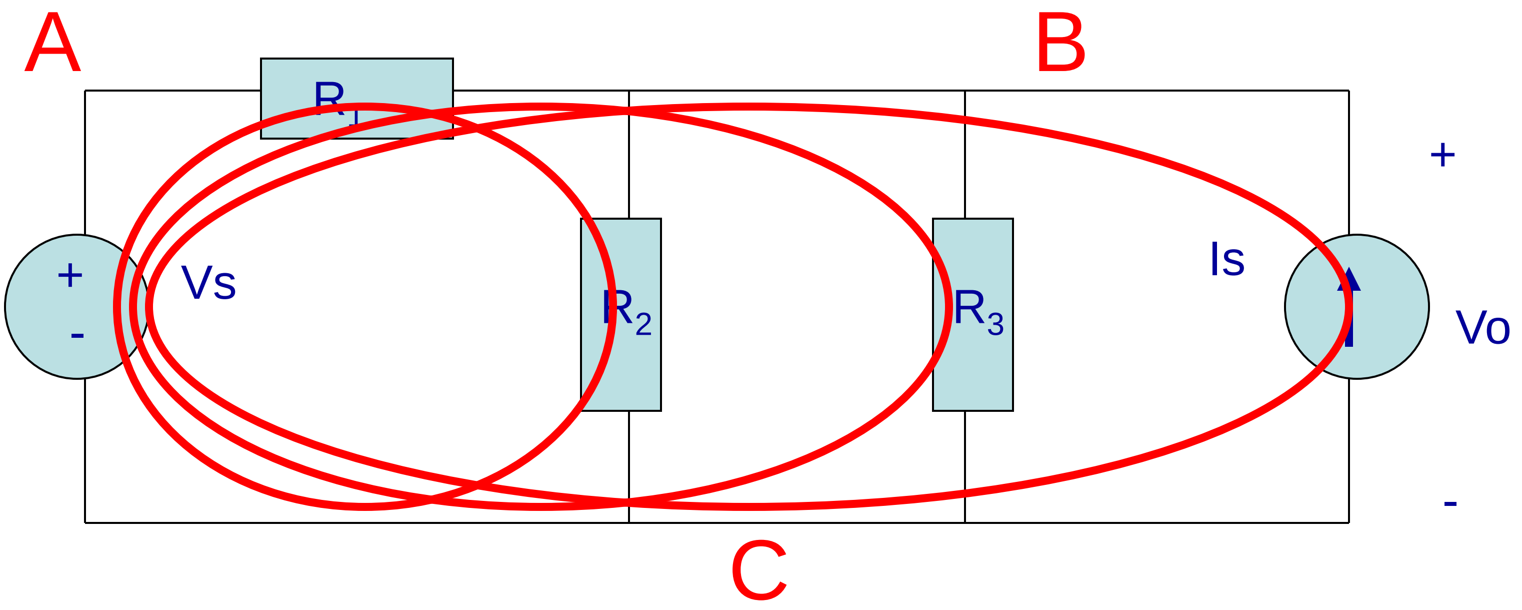
Example

- 5 Branches



Example

- Three Loops, if starting at node A

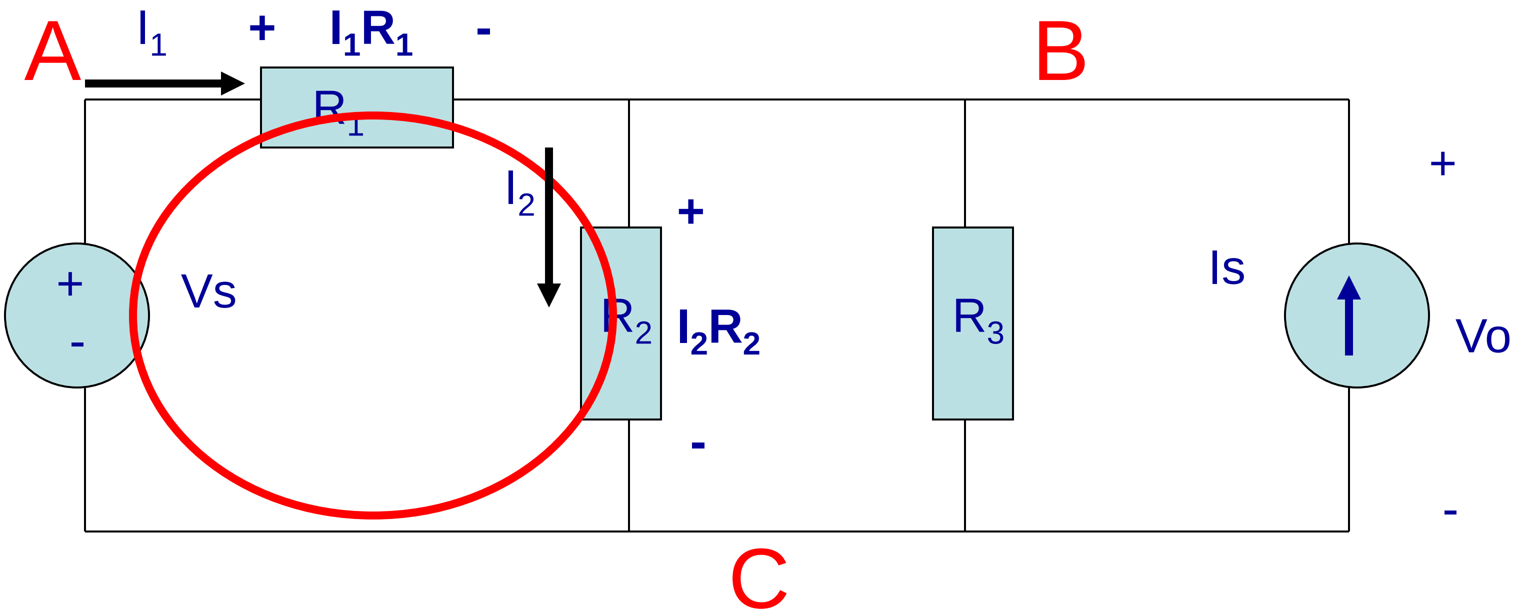


Kirchoff’s Voltage Law (KVL)

- The algebraic sum of voltages around each loop is zero
  - Beginning with one node, add voltages across each branch in the loop (if you encounter a + sign first) and subtract voltages (if you encounter a – sign first)
- Σ voltage drops - Σ voltage rises = 0
- Or Σ voltage drops = Σ voltage rises

Example

- Kirchoff's Voltage Law around 1st Loop

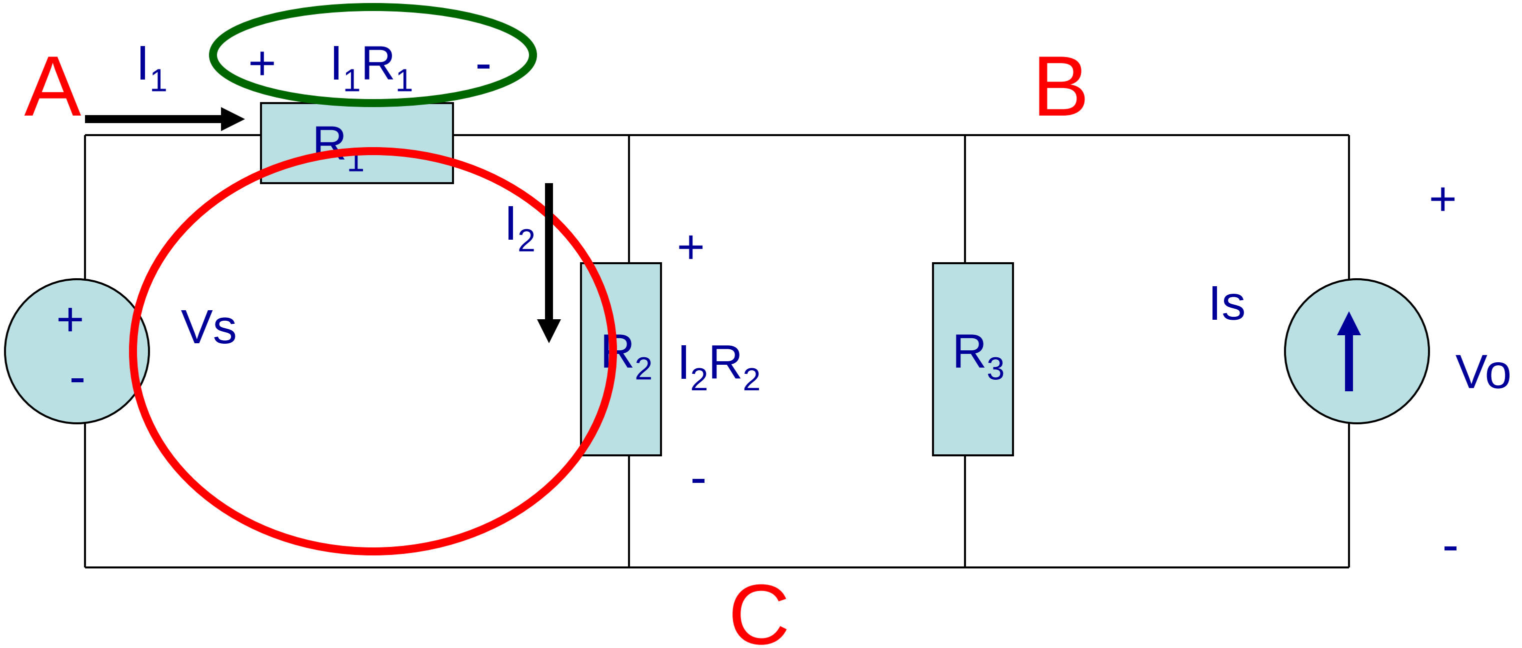


Assign current variables and directions

Use Ohm's law to assign voltages and polarities consistent with passive devices (current enters at the + side)

Example

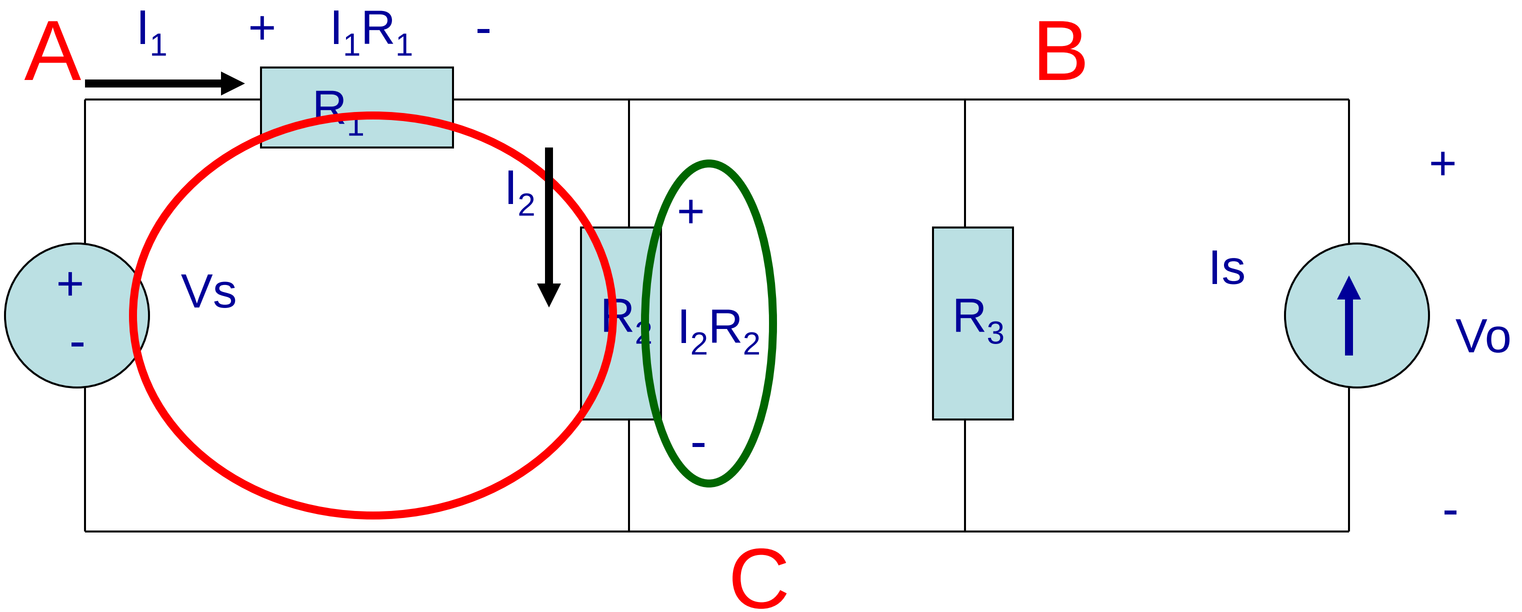
- Kirchoff's Voltage Law around 1st Loop



Starting at node A, add the 1st voltage drop: + I_1R_1

Example

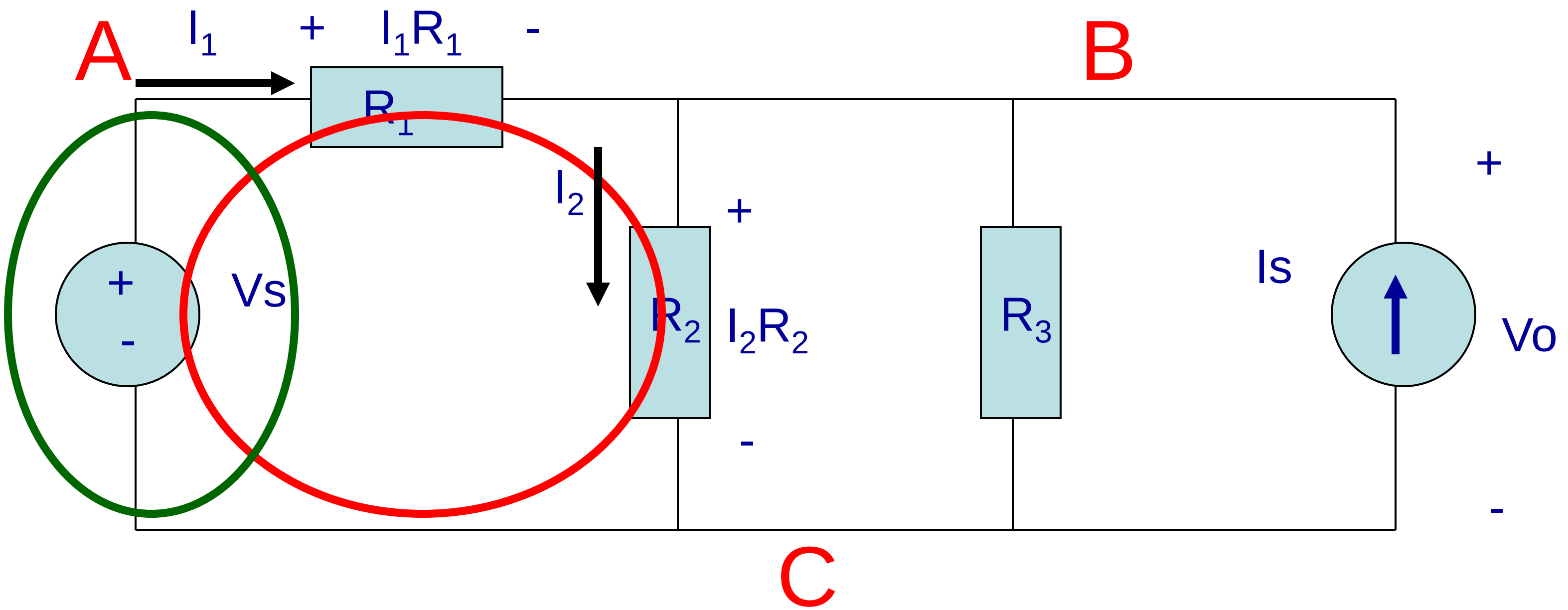
- Kirchoff's Voltage Law around 1st Loop



Add the voltage drop from B to C through R_2: **+ I_1R_1 + I_2R_2**

Example

- Kirchoff's Voltage Law around 1st Loop

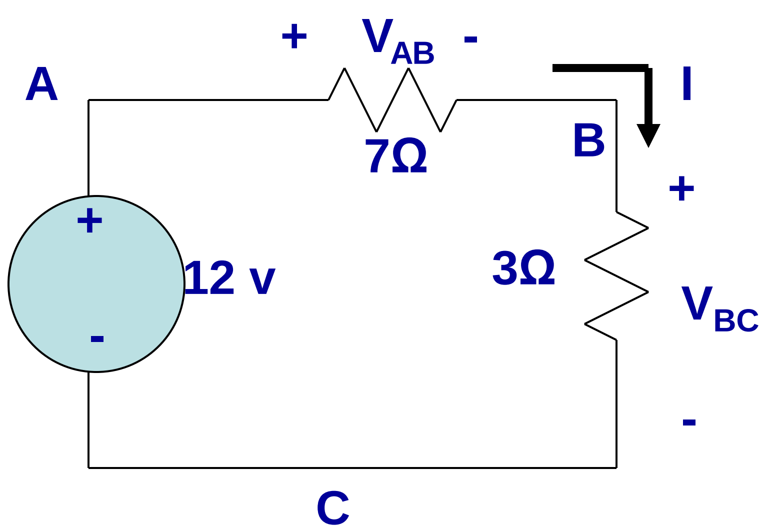


Subtract the voltage rise from C to A through Vs: $+ I_1R_1 + I_2R_2 - Vs = 0$

Notice that the sign of each term matches the polarity encountered 1st

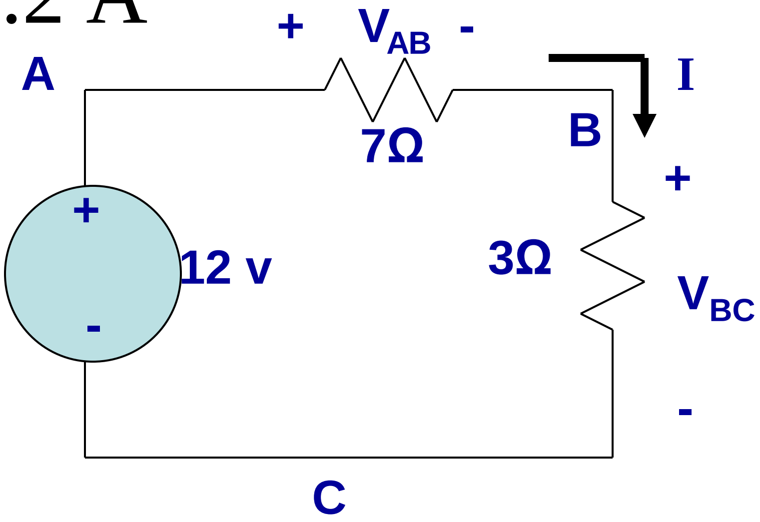
Circuit Analysis

- When given a circuit with sources and resistors having fixed values, you can use Kirchoff’s two laws and Ohm’s law to determine all branch voltages and currents



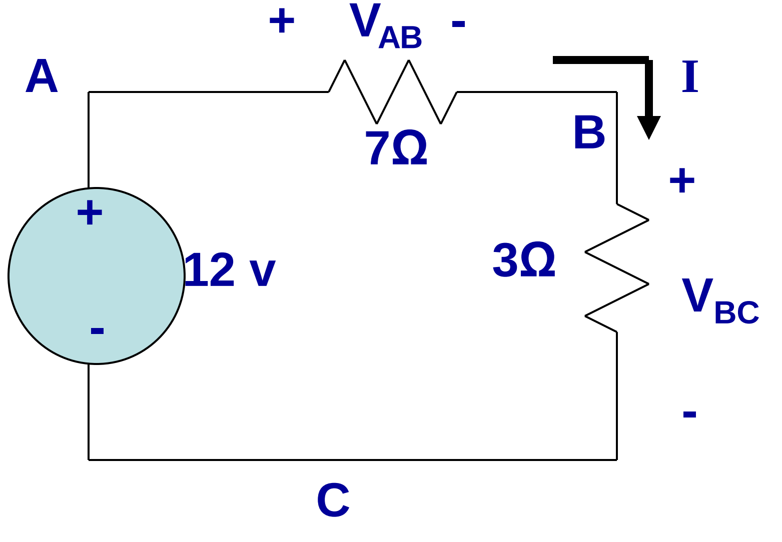
Circuit Analysis

- By Ohm's law: $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$
- By KVL: $V_{AB} + V_{BC} - 12\text{ v} = 0$
- Substituting: $I \cdot 7\Omega + I \cdot 3\Omega - 12\text{ v} = 0$
- Solving: $I = 1.2\text{ A}$



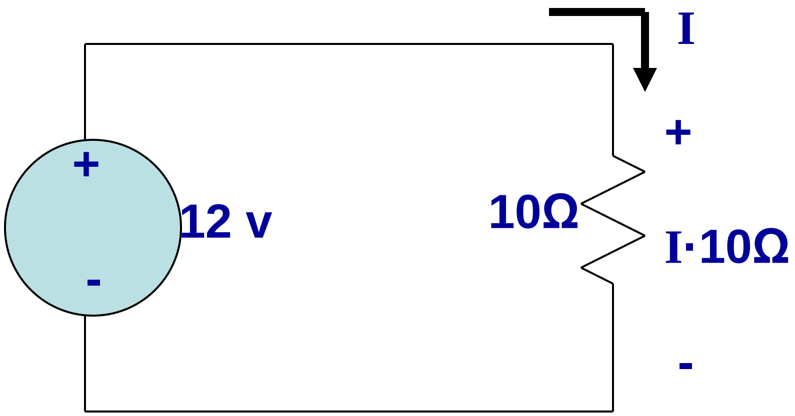
Circuit Analysis

- Since $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$
- And $I = 1.2$ A
- So $V_{AB} = 8.4$ v and $V_{BC} = 3.6$ v



Series Resistors

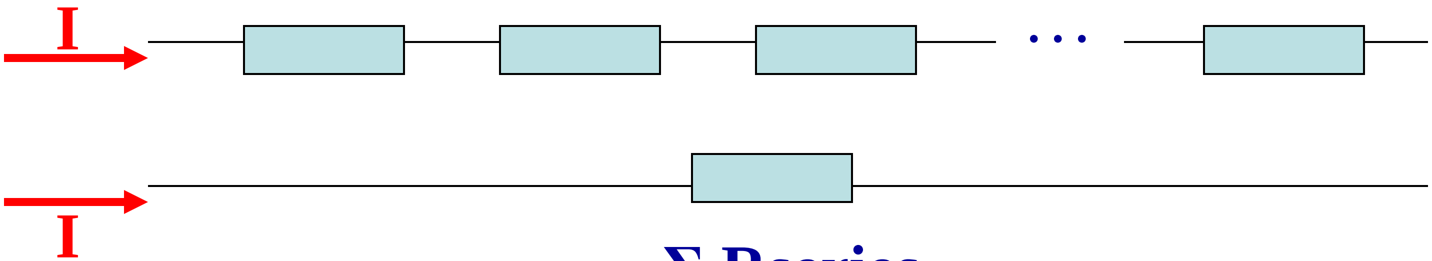
- KVL: $+I \cdot 10\Omega - 12\text{ v} = 0$,   So $I = 1.2$ A
- From the viewpoint of the source, the 7 and 3 ohm resistors in series are equivalent to the 10 ohms



Series Resistors

- To the rest of the circuit, series resistors can be replaced by an equivalent resistance equal to the sum of all resistors

Series resistors (same current through all)



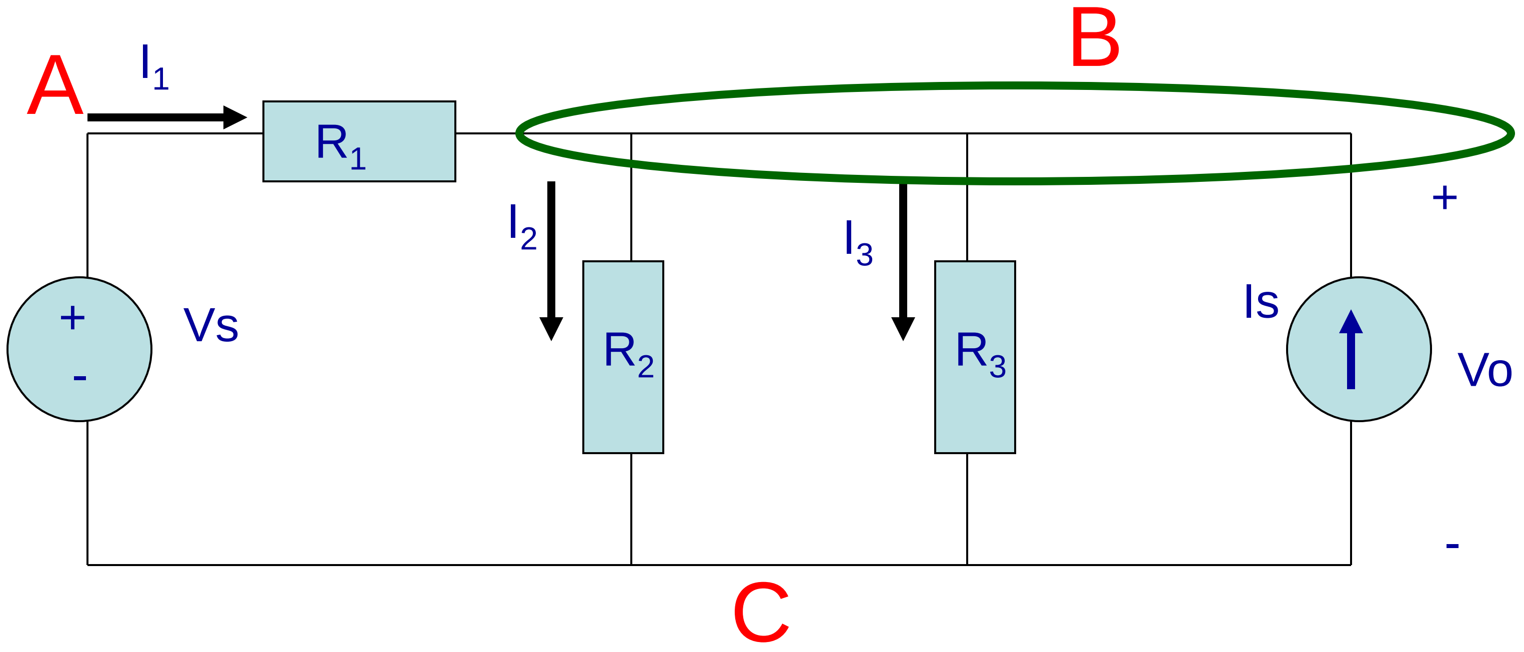
Σ Rseries

Kirchoff's Current Law (KCL)

- The algebraic sum of currents entering a node is zero
  - Add each branch current entering the node and subtract each branch current leaving the node
- Σ currents in - Σ currents out = 0
- Or Σ currents in = Σ currents out

Example

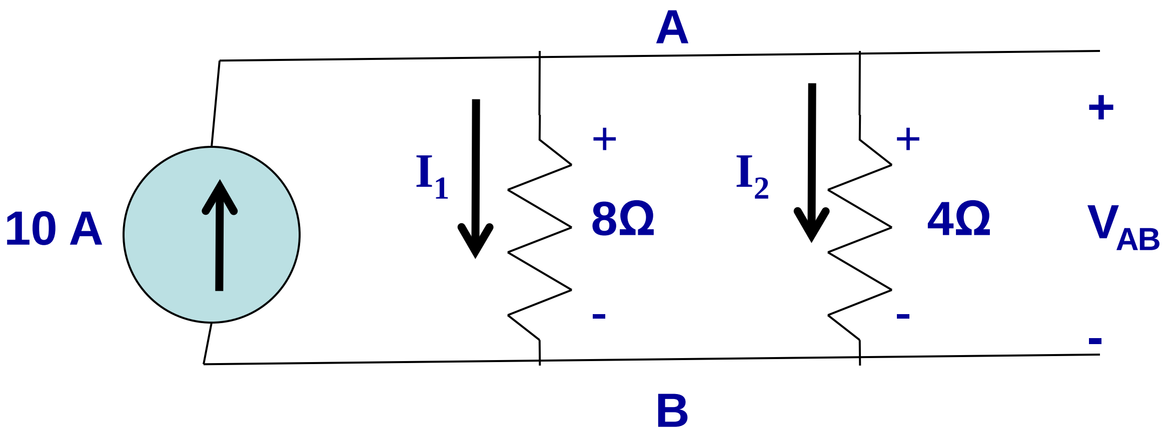
- Kirchoff's Current Law at B



Assign current variables and directions

Add currents in, subtract currents out: $\mathbf{I_1 - I_2 - I_3 + Is = 0}$

Circuit Analysis



By KVL: $-I_1 \cdot 8\Omega + I_2 \cdot 4\Omega = 0$

Solving: $I_2 = 2 \cdot I_1$

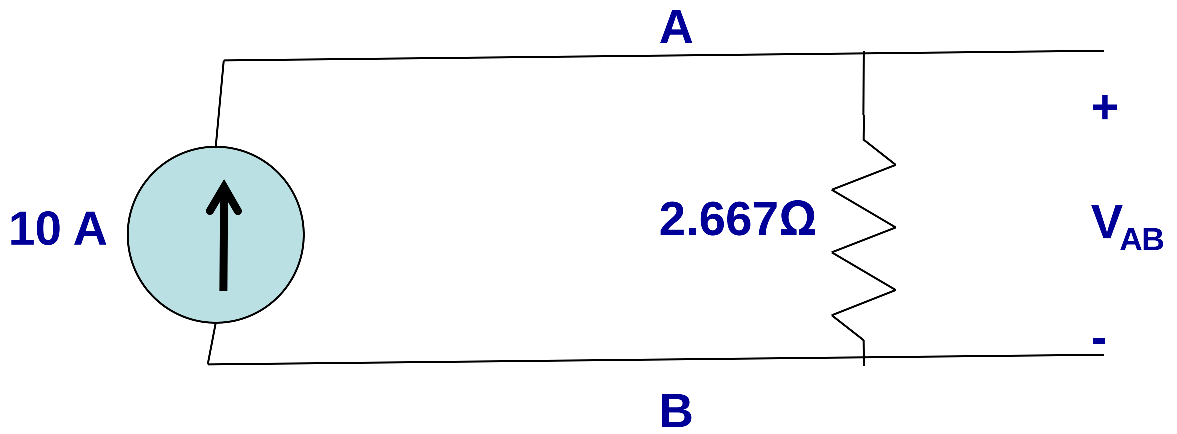
By KCL: $10A = I_1 + I_2$

Substituting: $10A = I_1 + 2 \cdot I_1 = 3 \cdot I_1$

So $I_1 = 3.33$ A and $I_2 = 6.67$ A

And $V_{AB} = 26.33$ volts

Circuit Analysis



By Ohm's Law: $V_{AB} = 10\ \text{A} \cdot 2.667\ \Omega$

So $V_{AB} = 26.67$ **volts**

Replacing two parallel resistors (8 and 4 Ω) by one equivalent one produces the same result from the viewpoint of the rest of the circuit.

Parallel Resistors

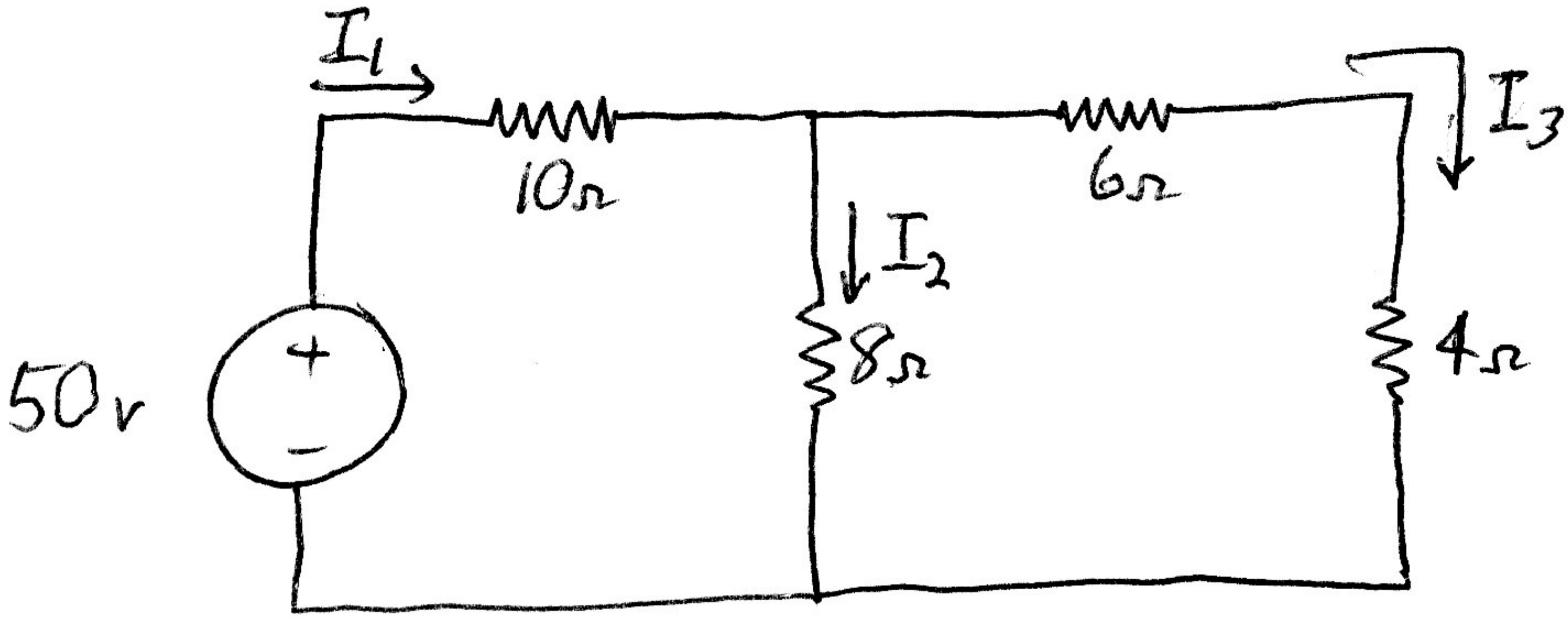
- The equivalent resistance for any number of resistors in parallel (i.e. they have the same voltage across each resistor):

$$Req = \frac{1}{1/R_1 + 1/R_2 + \cdots + 1/R_N}$$

- For two parallel resistors:

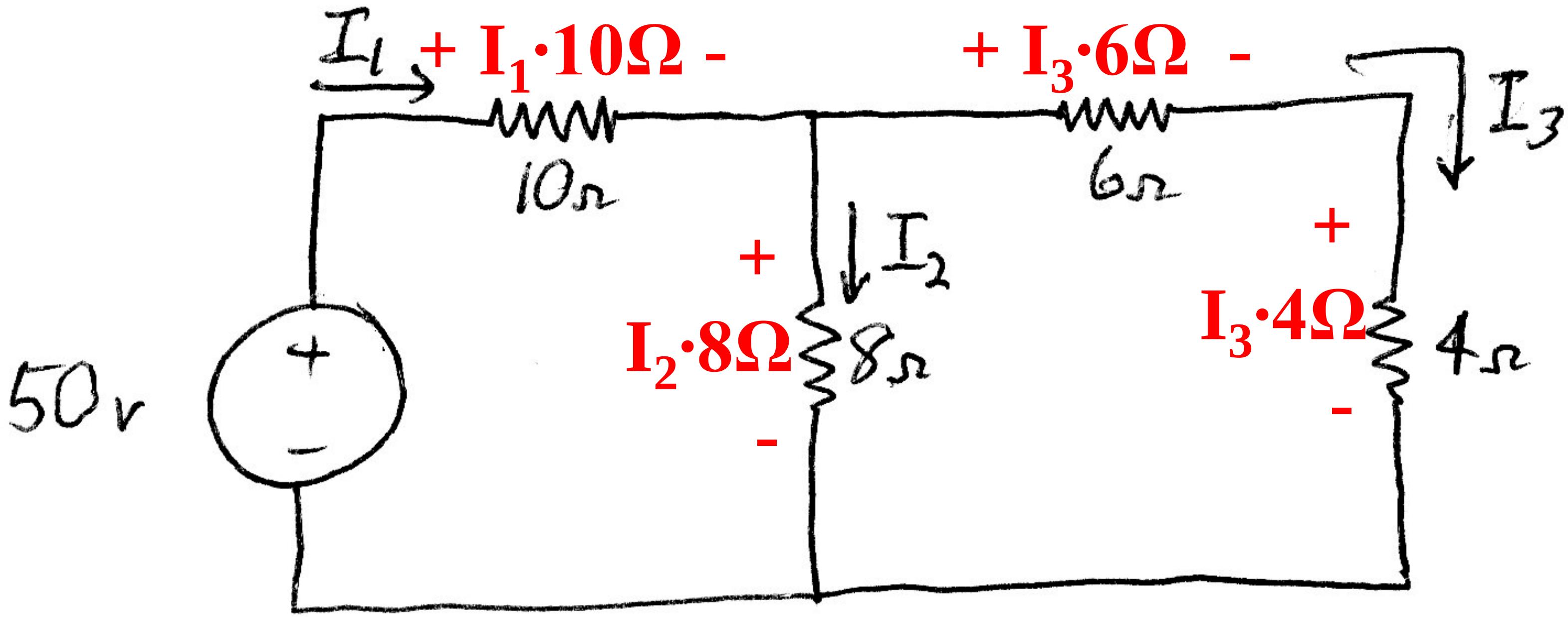
$$Req = R_1 \cdot R_2 / (R_1 + R_2)$$

Example Circuit



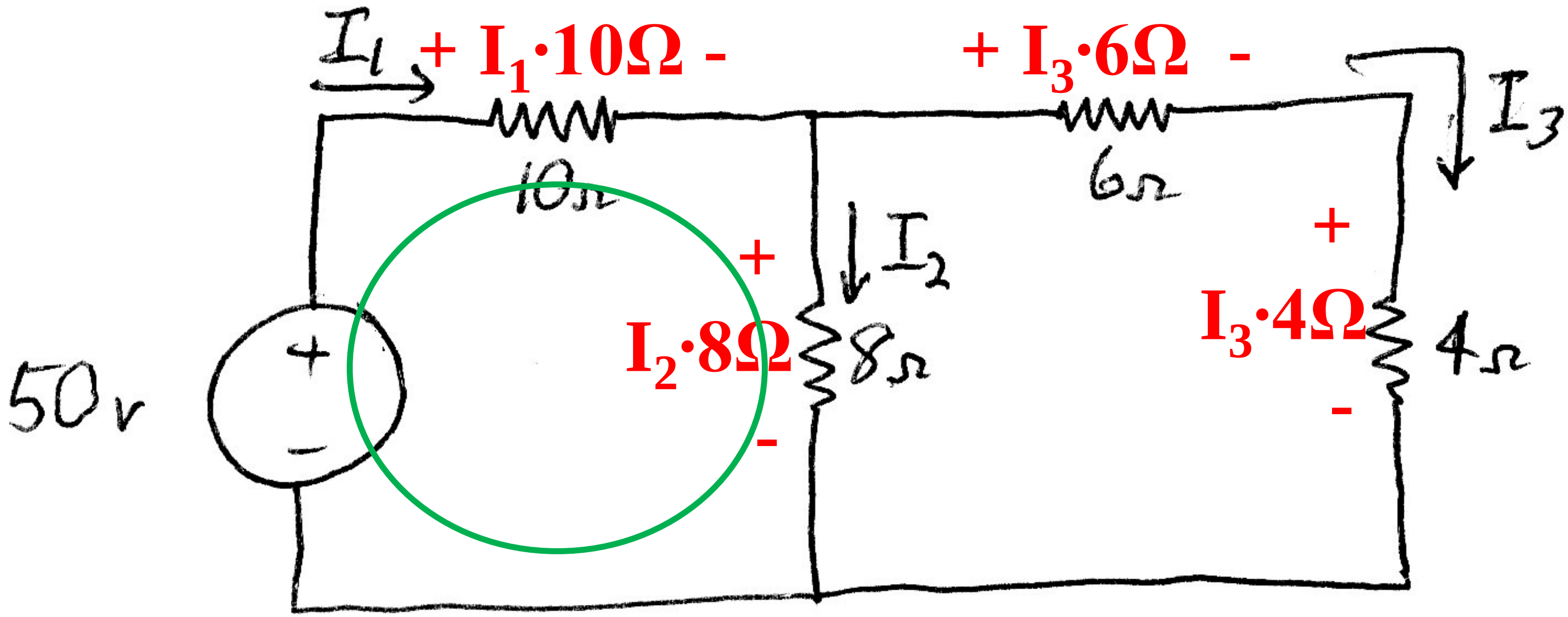
Solve for the currents through each resistor
And the voltages across each resistor

Example Circuit



Using Ohm's law, add polarities and expressions for each resistor voltage

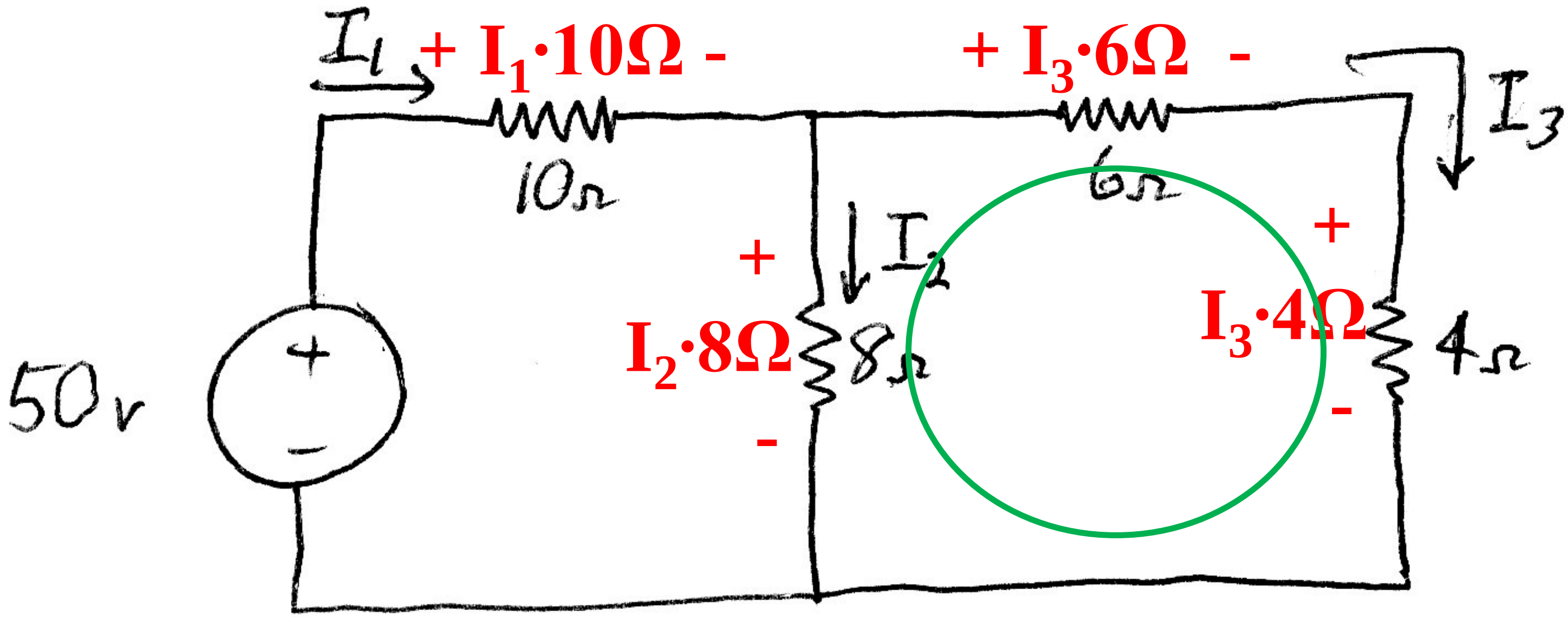
Example Circuit



Write 1st Kirchoff's voltage law equation

$-50\text{ v} + I_1 \cdot 10\Omega + I_2 \cdot 8\Omega = 0$

Example Circuit

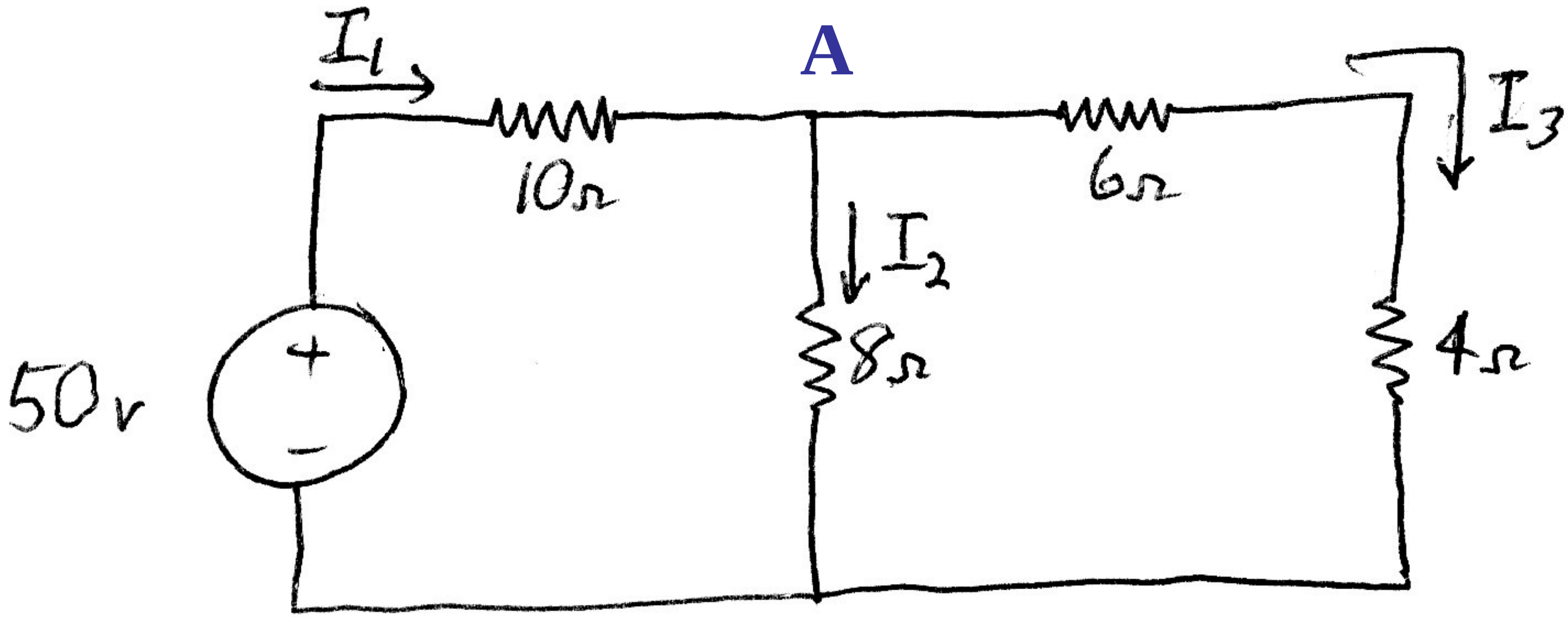


Write 2nd Kirchoff's voltage law equation

$$-I_2 \cdot 8\Omega + I_3 \cdot 6\Omega + I_3 \cdot 4\Omega = 0$$

$$\text{or } I_2 = I_3 \cdot (6+4)/8 = 1.25 \cdot I_3$$

Example Circuit



Write Kirchoff's current law equation at A

$$+I_1 - I_2 - I_3 = 0$$

Example Circuit

- We now have 3 equations in 3 unknowns, so we can solve for the currents through each resistor, that are used to find the voltage across each resistor
- Since $I_1 - I_2 - I_3 = 0$, $I_1 = I_2 + I_3$
- Substituting into the 1st KVL equation

$$-50\text{ v} + (I_2 + I_3)\cdot 10\Omega + I_2\cdot 8\Omega = 0$$

$$\text{or } I_2\cdot 18\ \Omega + I_3\cdot 10\ \Omega = 50\text{ volts}$$

Example Circuit

- But from the 2nd KVL equation, $I_2 = 1.25 \cdot I_3$
- Substituting into 1st KVL equation:

$(1.25 \cdot I_3) \cdot 18\ \Omega + I_3 \cdot 10\ \Omega = 50$ volts

Or: $I_3 \cdot 22.5\ \Omega + I_3 \cdot 10\ \Omega = 50$ volts

Or: $I_3 \cdot 32.5\ \Omega = 50$ volts

Or: $I_3 = 50$ volts$/32.5\ \Omega$

Or: $I_3 = 1.538$ amps

Example Circuit

- Since $I_3 = 1.538$ amps

  $I_2 = 1.25 \cdot I_3 = 1.923$ amps
- Since $I_1 = I_2 + I_3$, $I_1 = 3.461$ amps
- The voltages across the resistors:

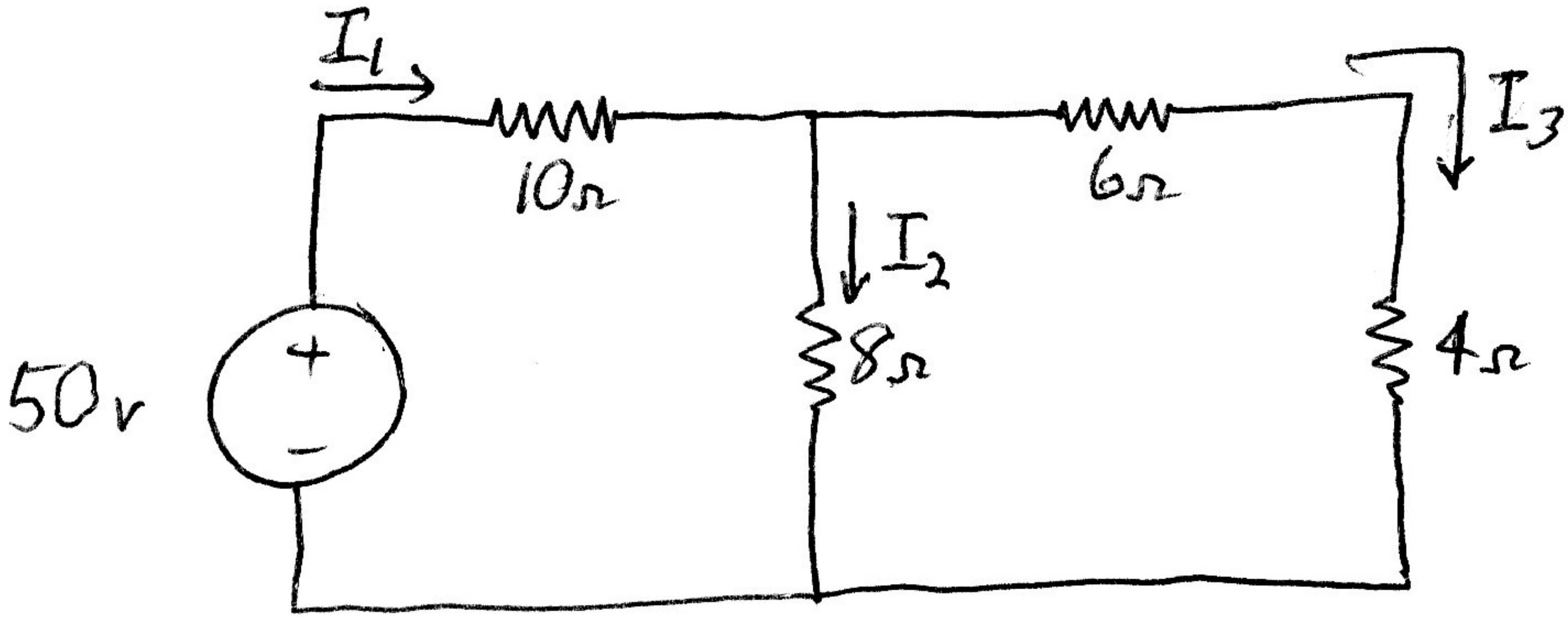
  $I_1 \cdot 10\Omega = 34.61$ volts

  $I_2 \cdot 8\Omega = 15.38$ volts

  $I_3 \cdot 6\Omega = 9.23$ volts

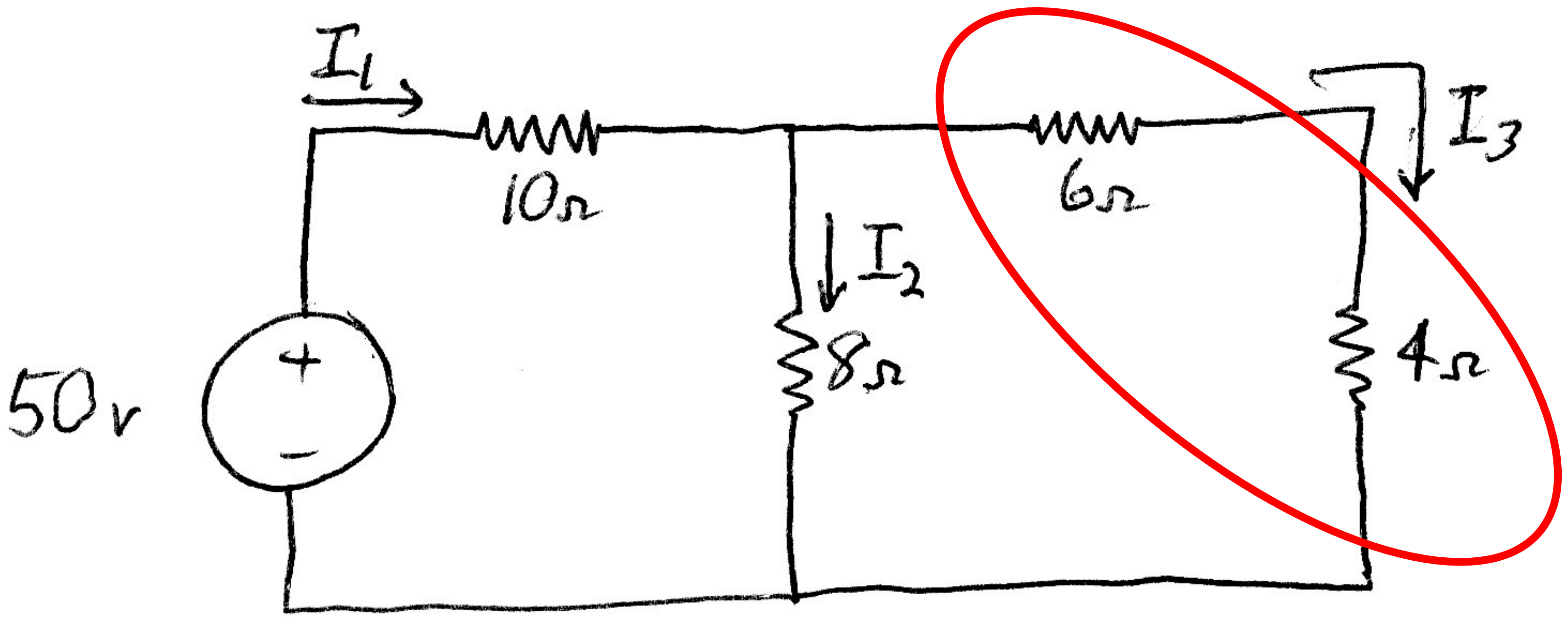
  $I_3 \cdot 4\Omega = 6.15$ volts

Example Circuit



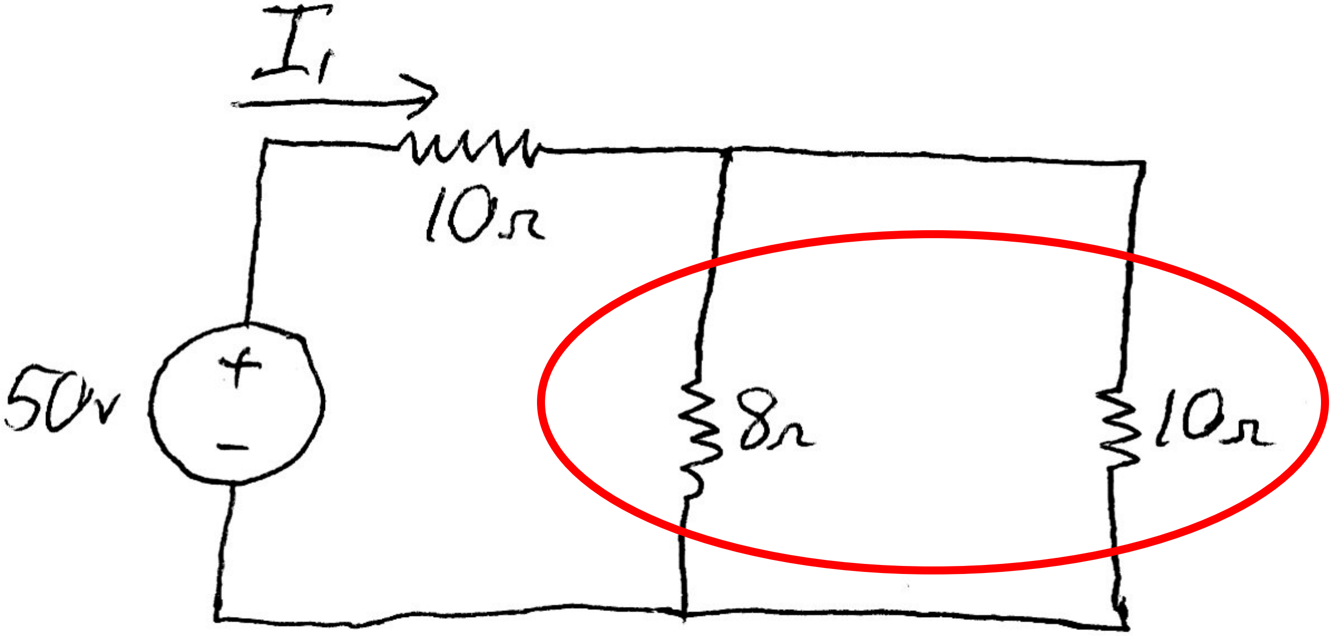
Solve for the currents through each resistor And the voltages across each resistor using Series and parallel simplification.

Example Circuit



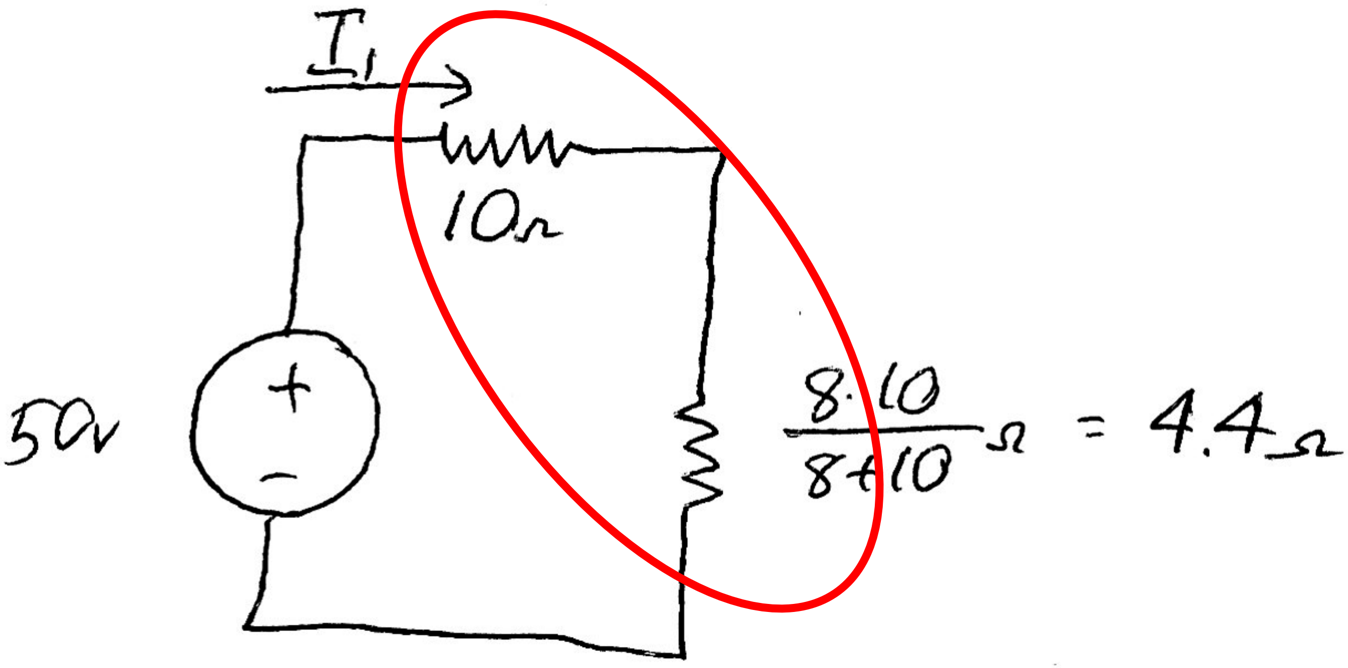
The 6 and 4 ohm resistors are in series, so are combined into 6+4 = 10Ω

Example Circuit



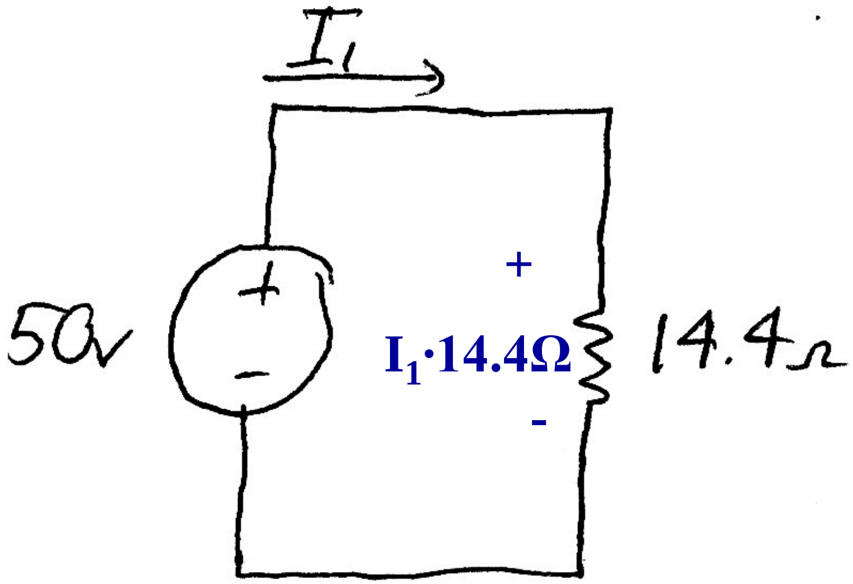
The 8 and 10 ohm resistors are in parallel, so are combined into 8·10/(8+10) =14.4 Ω

Example Circuit



The 10 and 4.4 ohm resistors are in series, so are combined into 10+4 = 14.4Ω

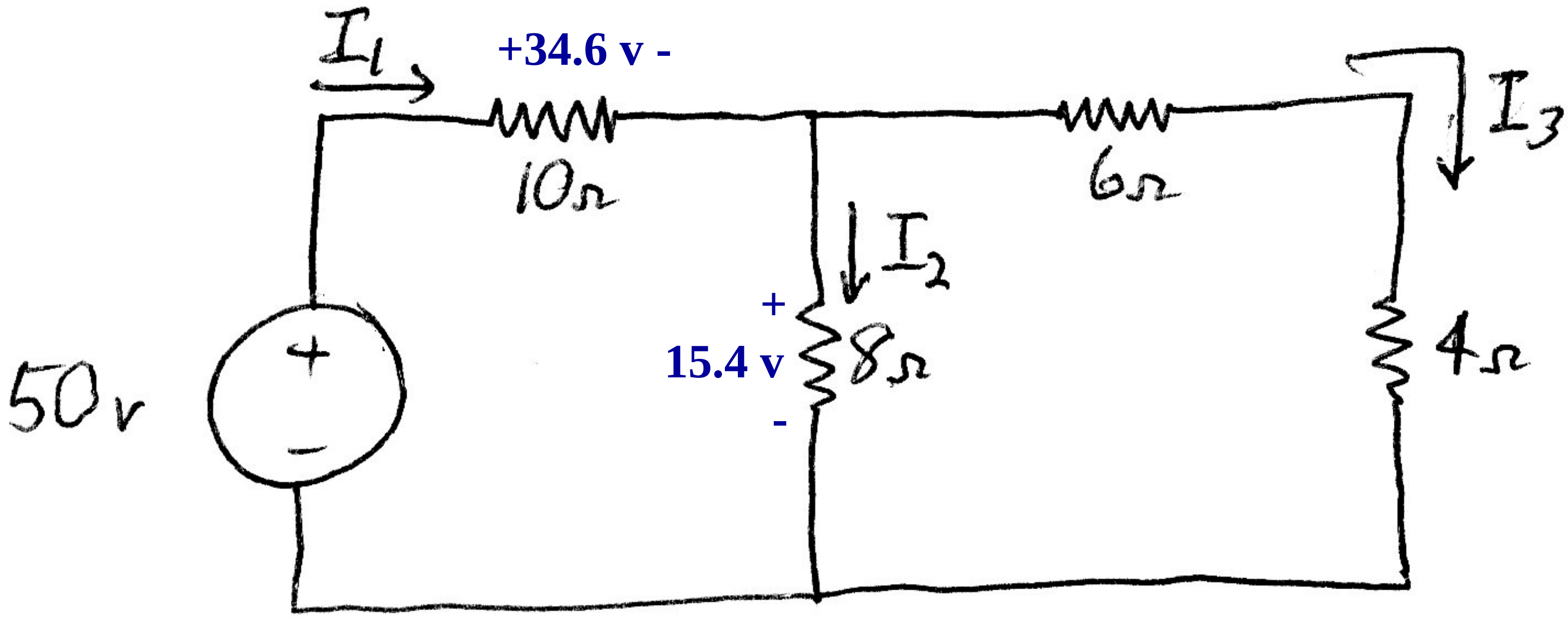
Example Circuit



Writing KVL, $I_1 \cdot 14.4\Omega - 50\text{ v} = 0$

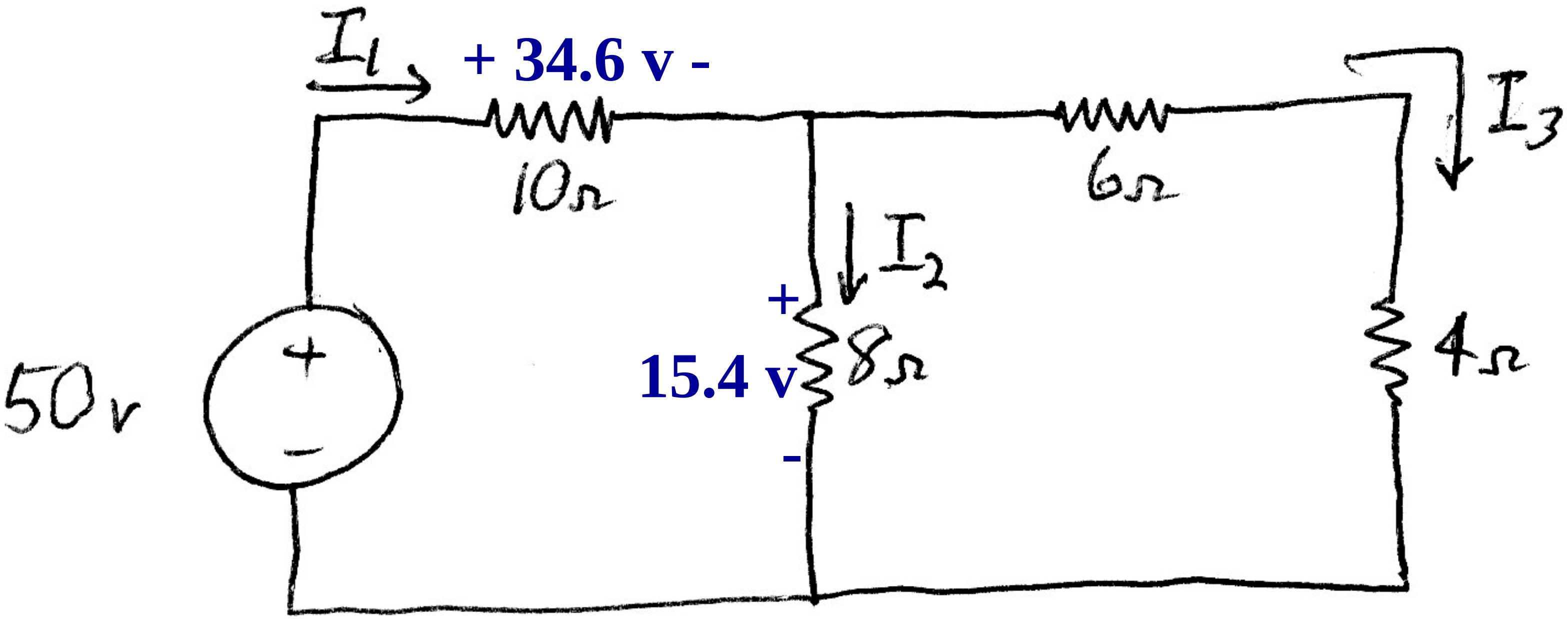
Or $I_1 = 50\text{ v} / 14.4\Omega = 3.46\text{ A}$

Example Circuit



If I_1 = 3.46 A, then $I_1 \cdot 10\ \Omega$ = 34.6 v
So the voltage across the 8 Ω = 15.4 v

Example Circuit

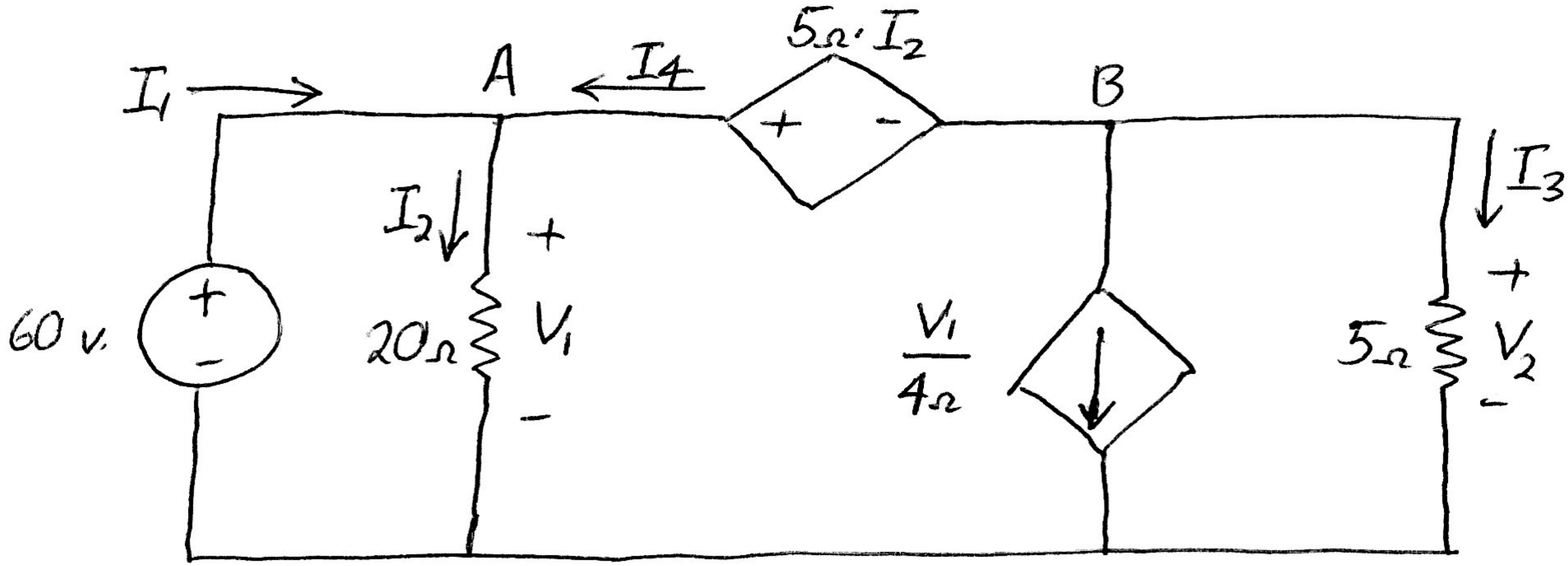


If $I_2 \cdot 8\ \Omega = 15.4$ v, then $I_2 = 15.4/8 = 1.93$ A

By KCL, $I_1 - I_2 - I_3 = 0$, so $I_3 = I_1 – I_2 = 1.53$ A

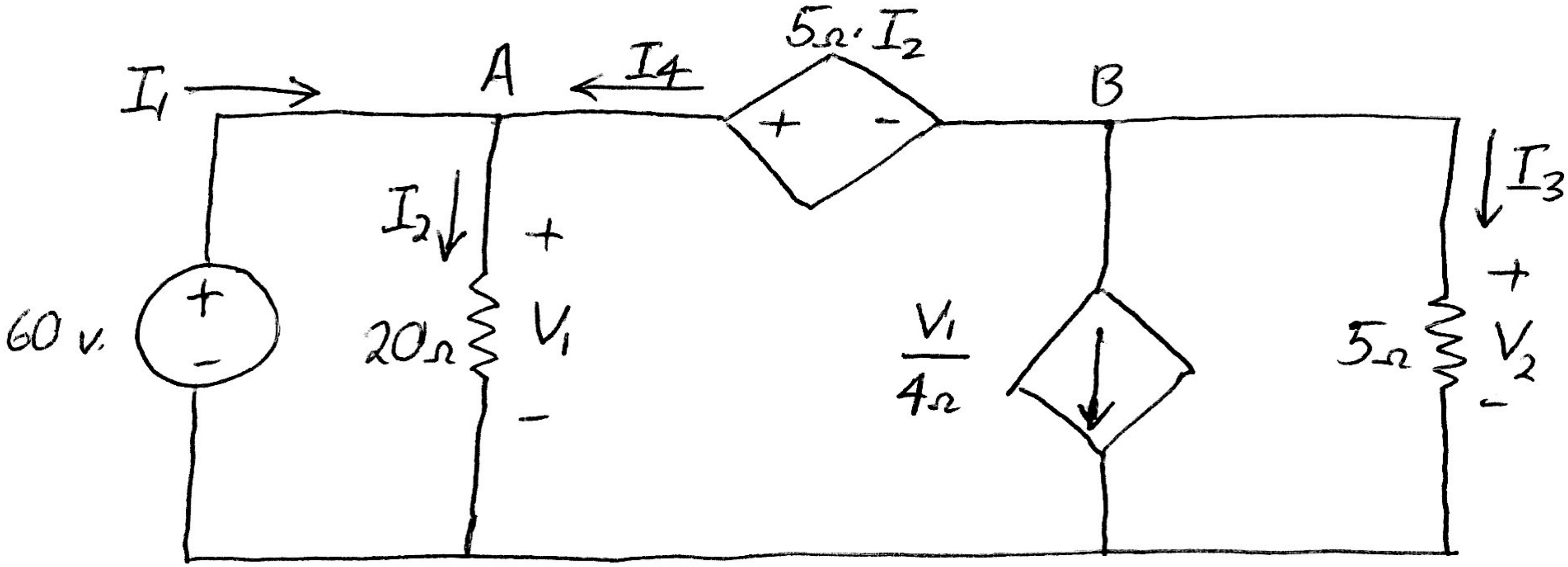
Circuits with Dependent Sources

Circuit with Dependent Sources



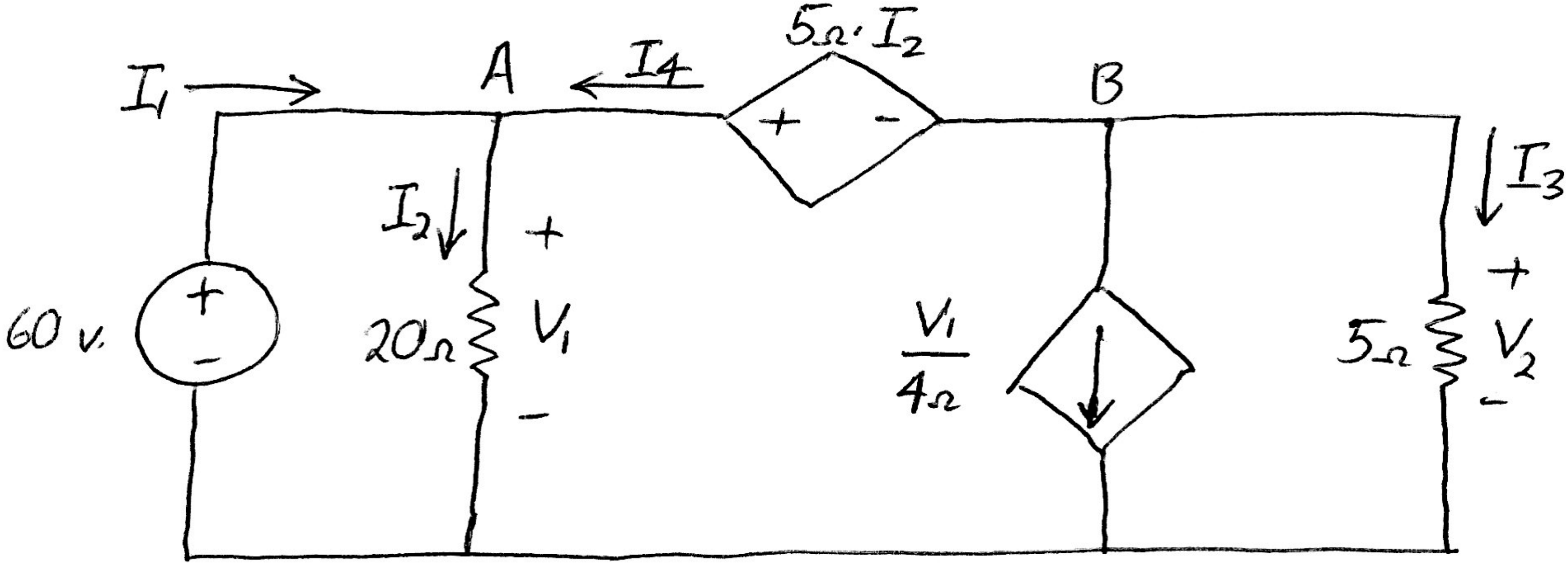
V_1 = 60 volts because the 20Ω resistor is in parallel; by Ohm's law, $V_1 = I_2 \cdot 20\Omega$;
so $I_2 = V_1/20\Omega = 60v/20\Omega = 3$ A

Circuit with Dependent Sources



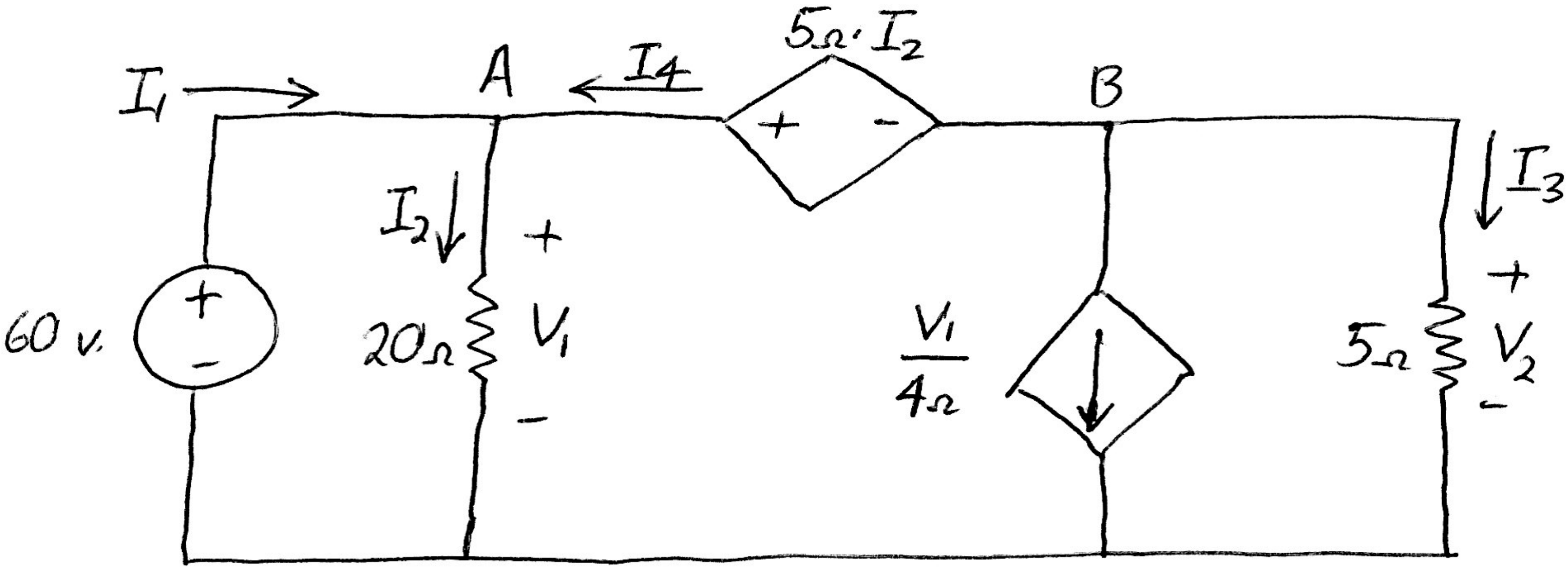
If $I_2 = 3$ A, then the $5\Omega \cdot I_2$ dependent source is 15 volts and if $V_1 = 60$ v., then the $V_1/4\Omega$ dependent source is 15 A

Circuit with Dependent Sources



Writing Kirchoff’s Voltage law around the outside loop, $-60\text{ v} + 5\Omega \cdot I_2 + 5\Omega \cdot I_3 = 0$ where $I_2=3$ A, so $I_3 = (60–15)\text{v} / 5\Omega = 9$ A

Circuit with Dependent Sources

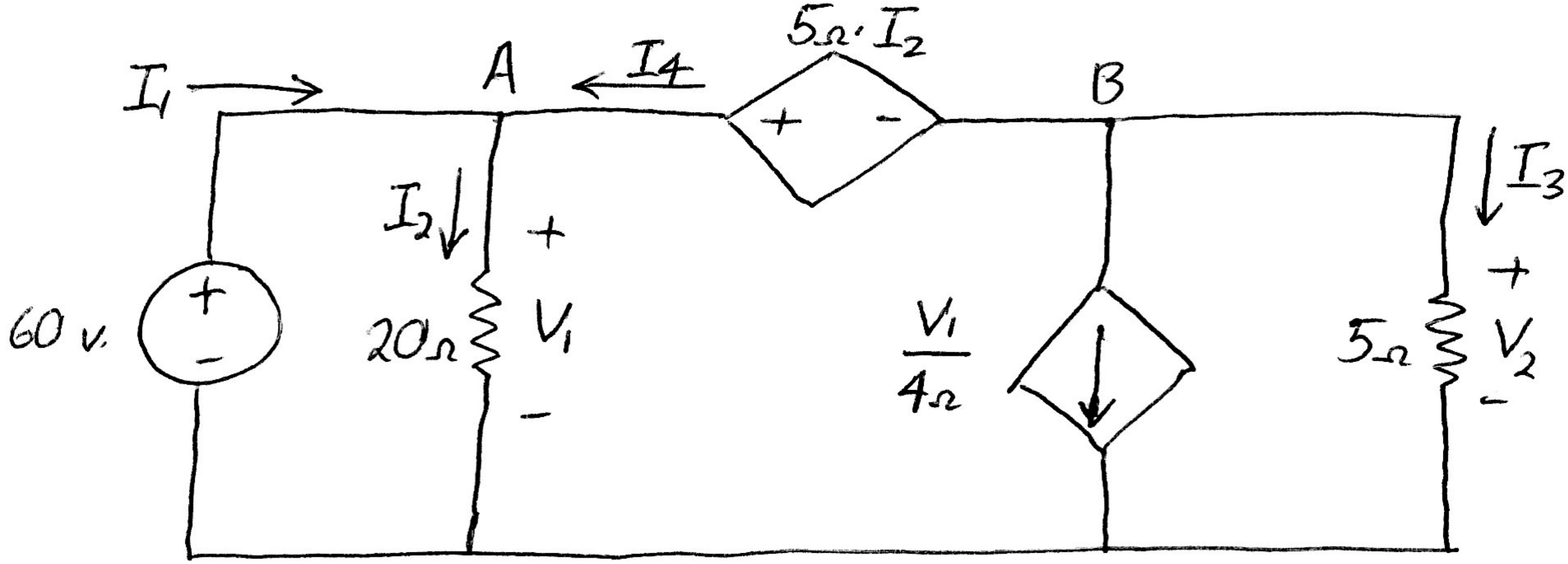


Writing Kirchoff’s Current law at B

$I_4 + I_3 + V_1/4 = 0$ (all leaving node B)

Since $V_1/4\Omega = 15$ A and $I_3 = 9$ A, $I_4 = -24$ A

Circuit with Dependent Sources

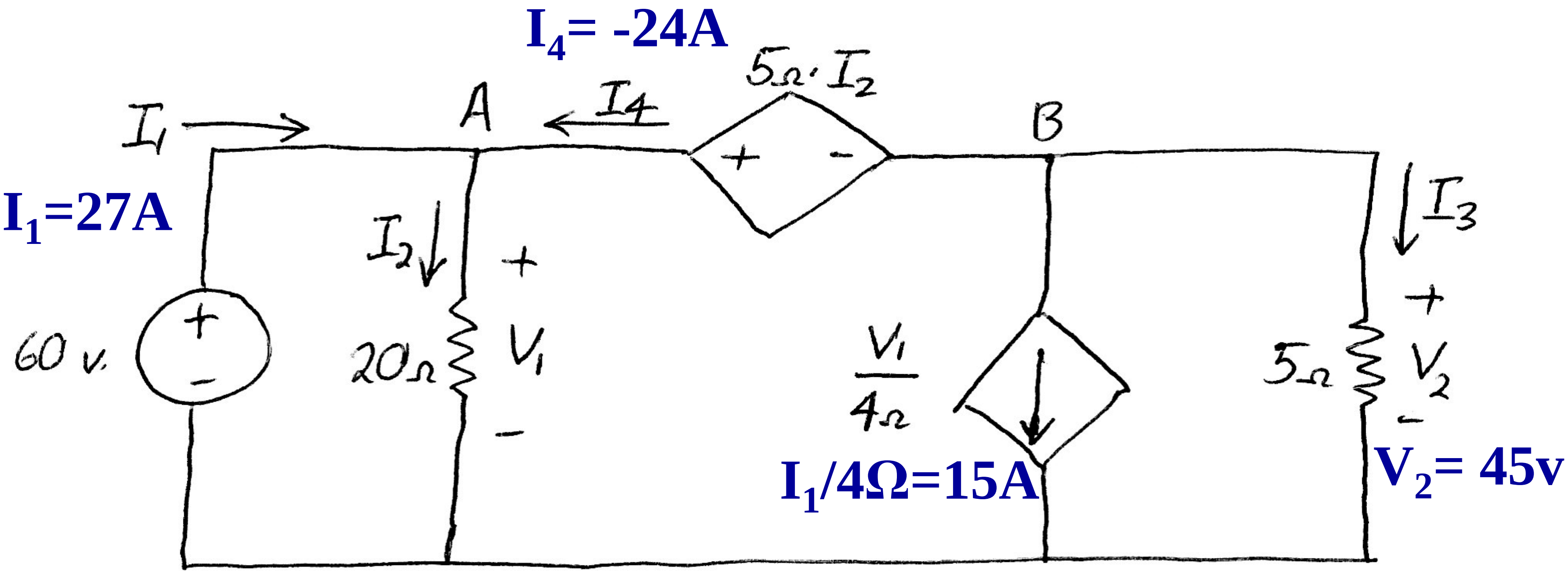


Writing Kirchoff's Current law at A

$I_4 + I_1 – I_2 = 0$

Since I_2 = 3 A and I_4 = -24 A, I_1 = 27 A

Circuit with Dependent Sources

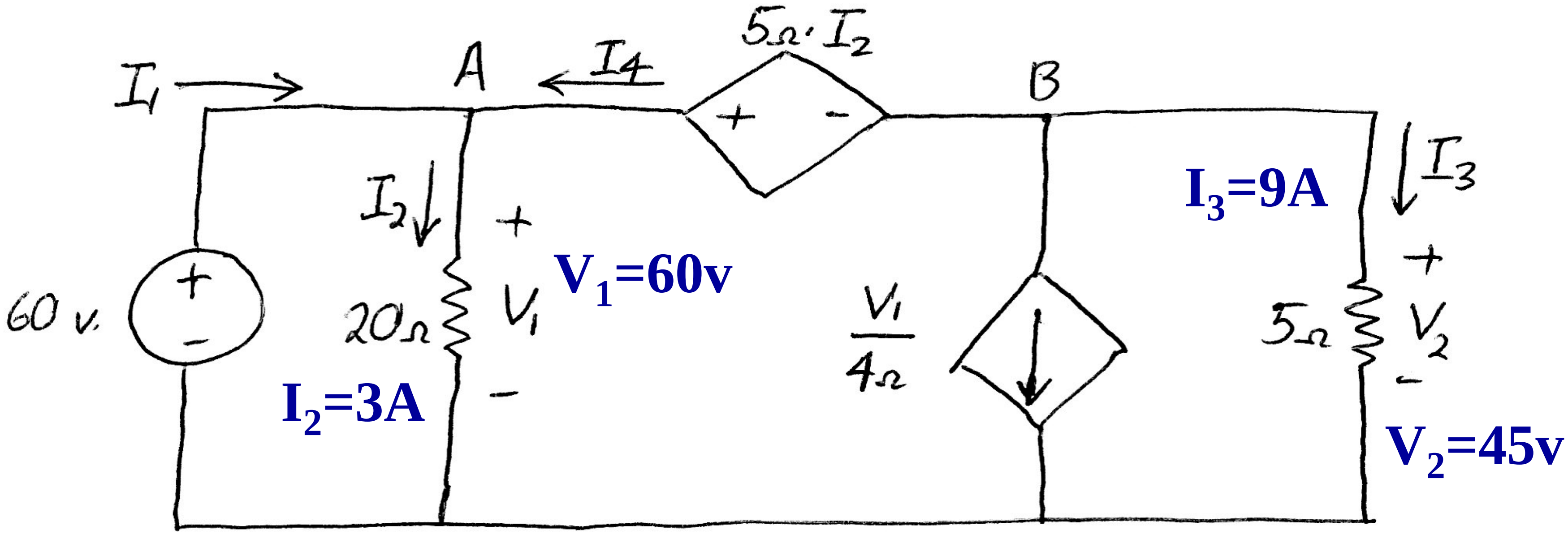


60 v source generating, P=-27A·60v=-1620 watts

$5 \cdot I_2$ source absorbing, P=24A·15v=360 watts

$V_1/4$ source absorbing, P=15A·45v= 675watts

Circuit with Dependent Sources

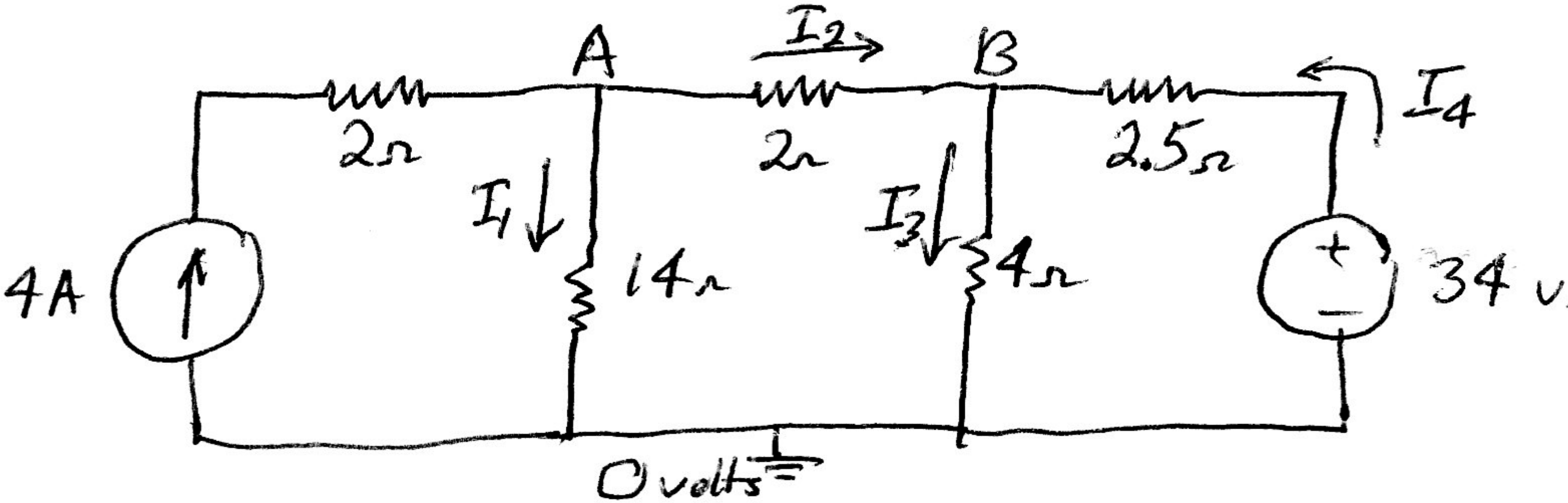


20 Ω resistor absorbing, P=3A·60v=180 watts

5 Ω resistor absorbing, P=9A·45v= 405 watts

-1620 w +360 w + 675 w + 180 w + 405 w = 0

VA = 28 volts



Find I_1 — **2 Amps by Ohm's law ($I_2 \cdot 14=28$)**

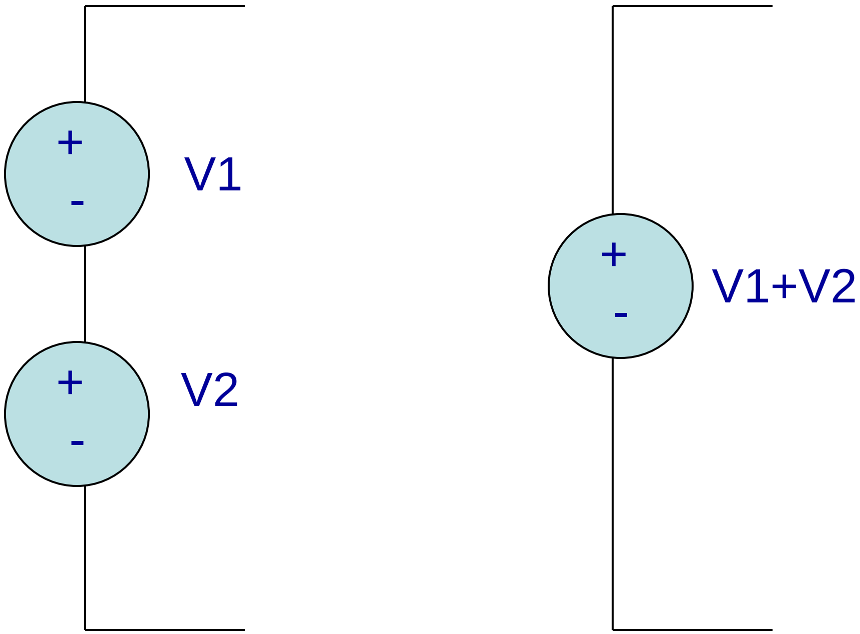
Find I_2 — **2 Amps by KCL ($4-I_1-I_2=0$)**

Find V_B — **24 volts by KVL ($-I_1 \cdot 14+I_2 \cdot 2+V_B=0$)**

Find I_3 — **6 Amps by Ohm's law ($I_3 \cdot 4=24$)**

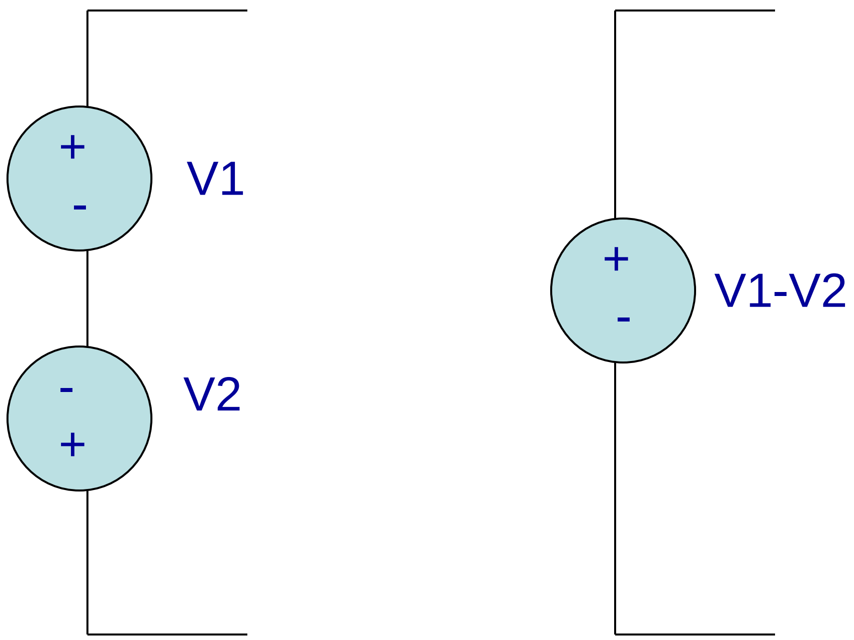
Find I_4 — **4 Amps by KCL ($I_2+I_4-I_3=0$)**

Sources in Series



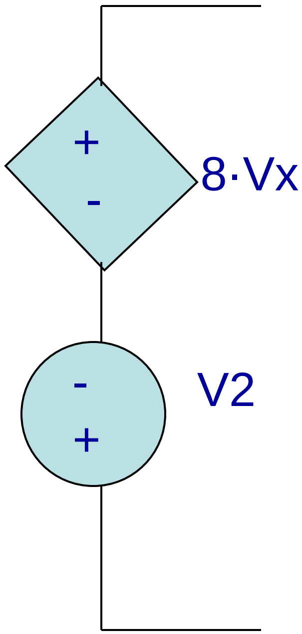
Voltage sources In series add algebraically

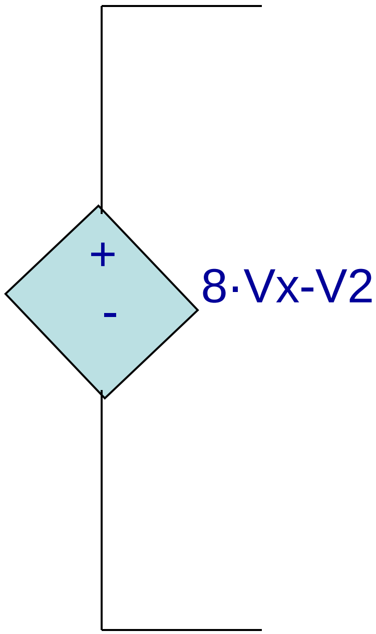
Sources in Series



Start at the top terminal and add.
If hit a + (+V1)
If you hit a – (-V2)

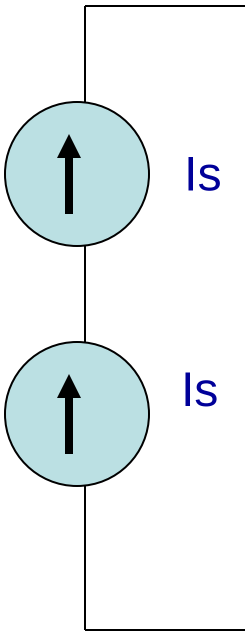
Sources in Series

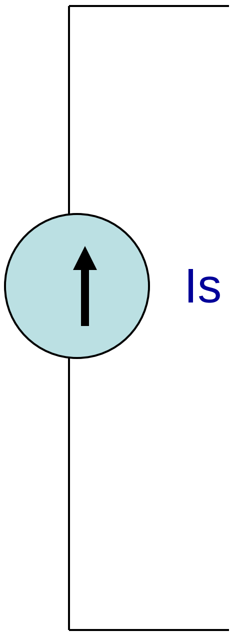




If one source is dependent, then so is the equivalent

Sources in Series

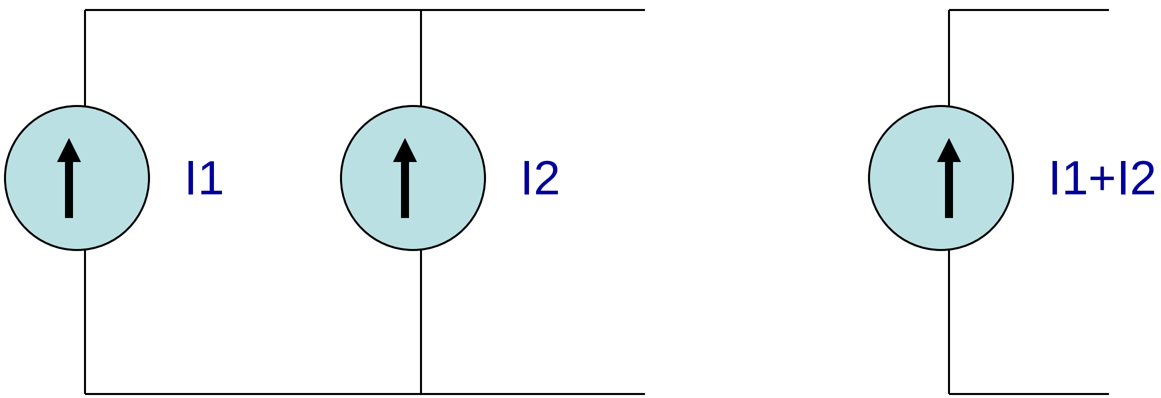




Current sources in series must be the same value and direction

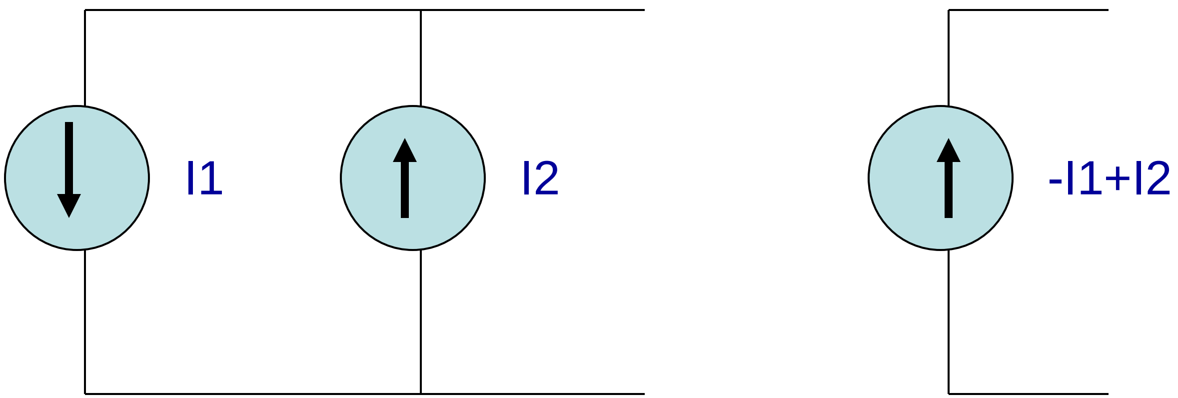
Sources in Parallel

Current sources in parallel add algebraically



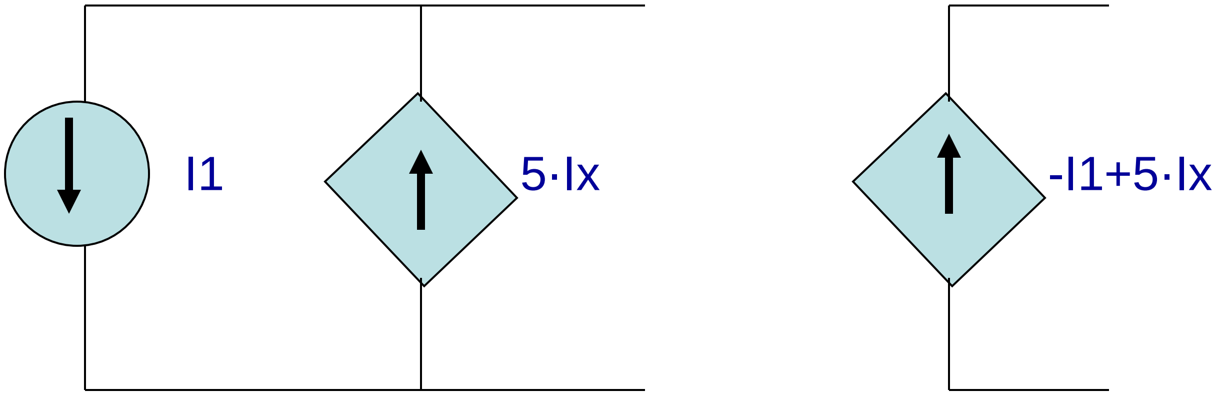
Sources in Parallel

Current sources in parallel add algebraically



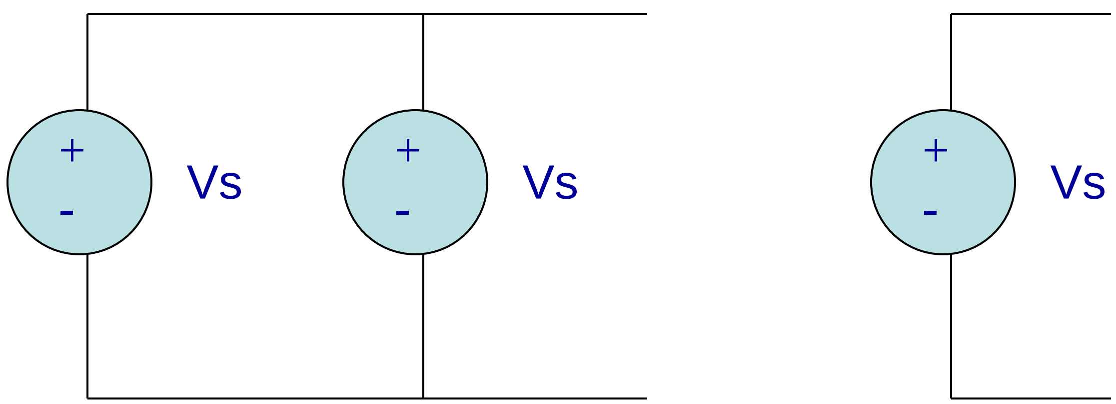
Sources in Parallel

If any source is dependent, then the combination is also dependent



Sources in Parallel

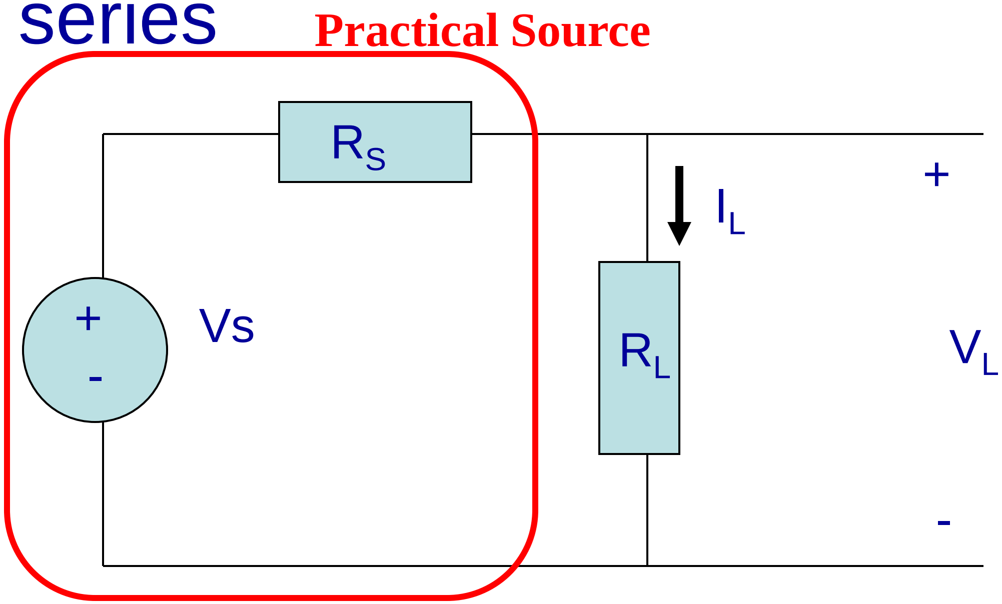
Voltage sources in parallel must be the same value and same direction



Source Transformation

Source Transformation

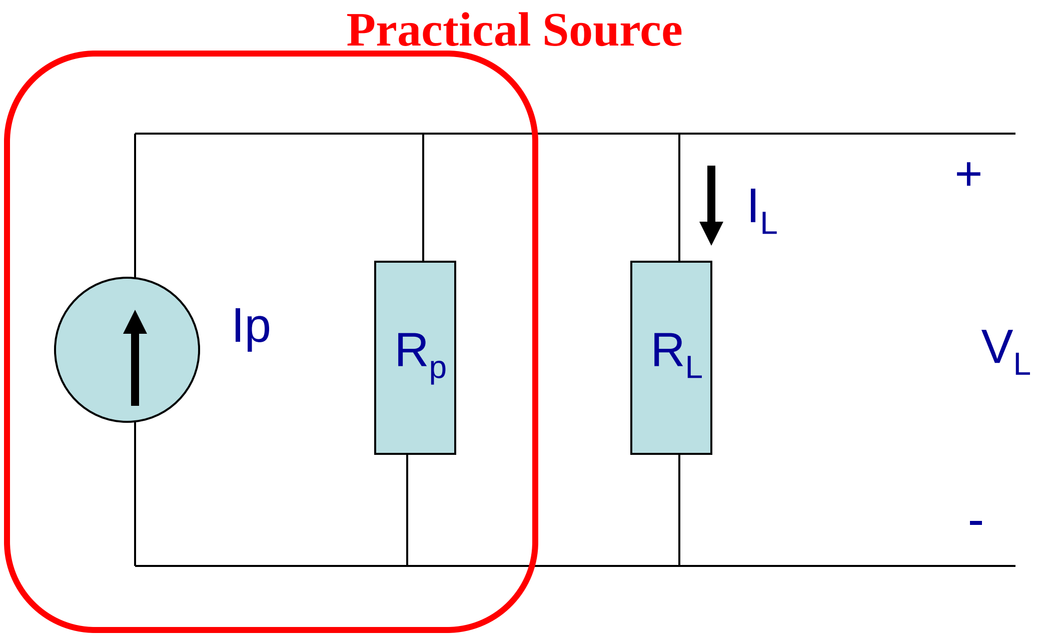
- Practical voltage sources are current limited and we can model them by adding a resistor in series



- We want to create an equivalent using a current source and parallel resistance for any R_L

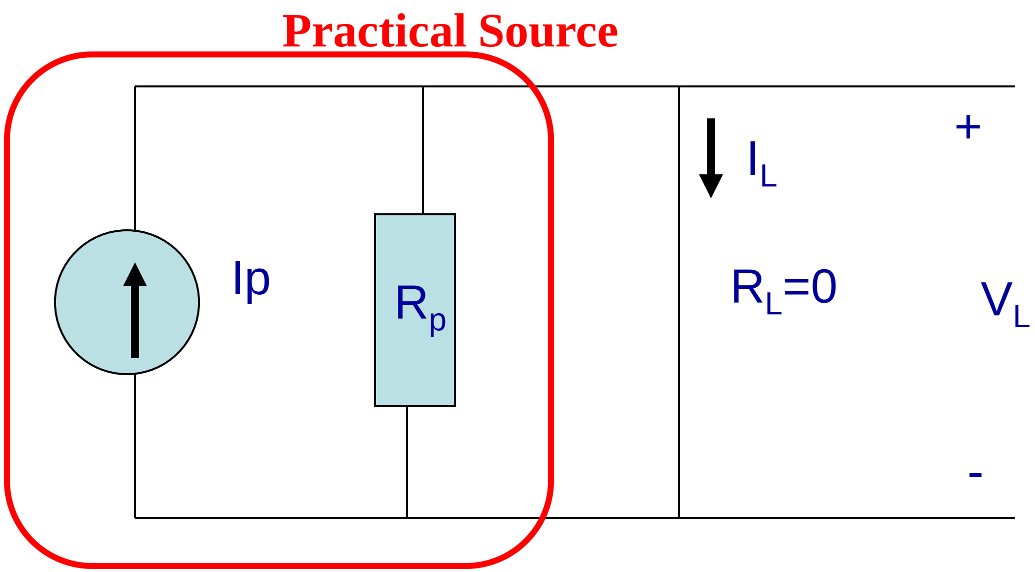
Source Transformation

- V_L and I_L must be the same in both circuits for any R_L



Source Transformation

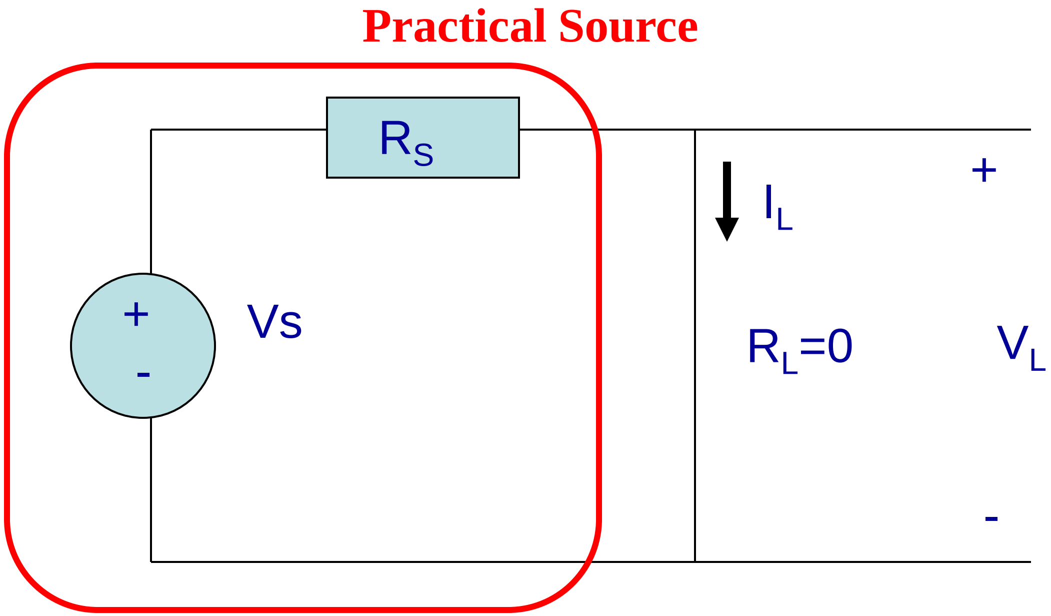
- V_L and I_L must be the same in both circuits for $R_L = 0$, or a short circuit



- $Ip = I_L$ and $V_L = 0$

Source Transformation

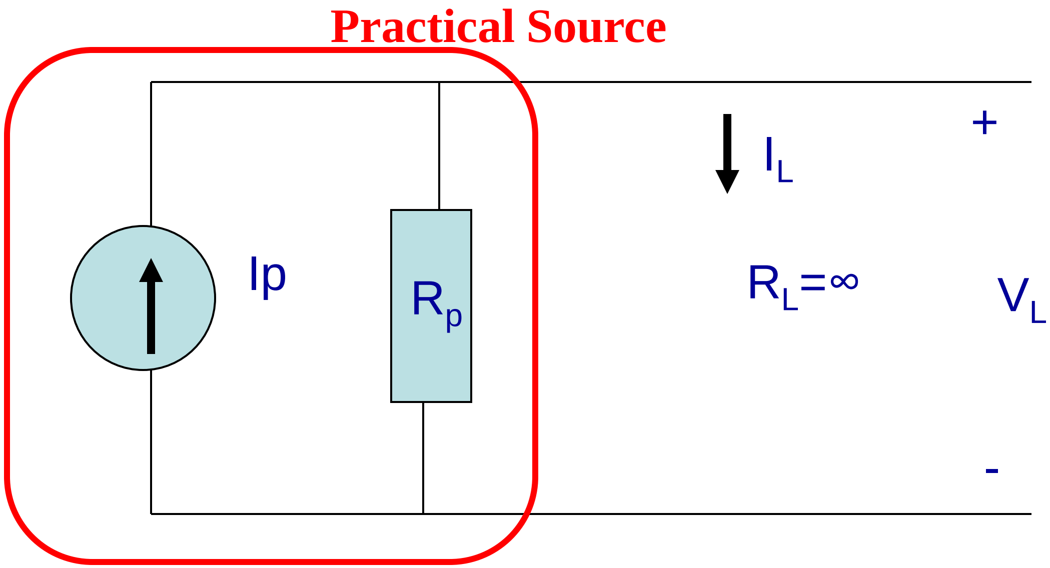
- Now look at the voltage source in series with the resistor with a short circuit



- I_L= Vs/Rs and V_L = 0
- So Ip = Vs/Rs

Source Transformation

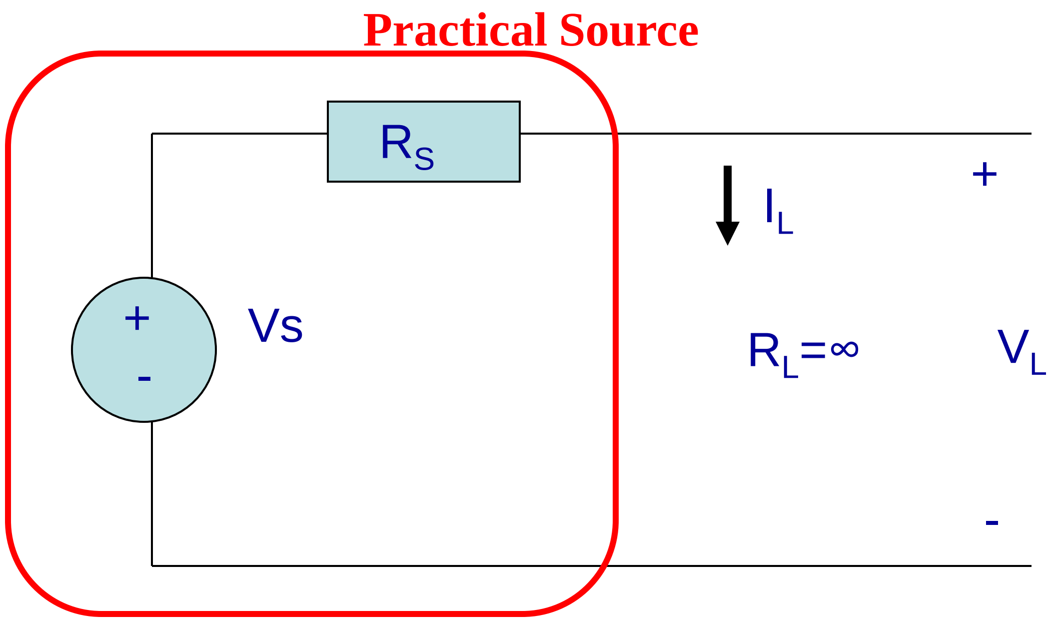
- V_L and I_L must also be the same in both circuits for $R_L = \infty$, or an open circuit



- $I_L = 0$ and $V_L = Ip \cdot Rp$

Source Transformation

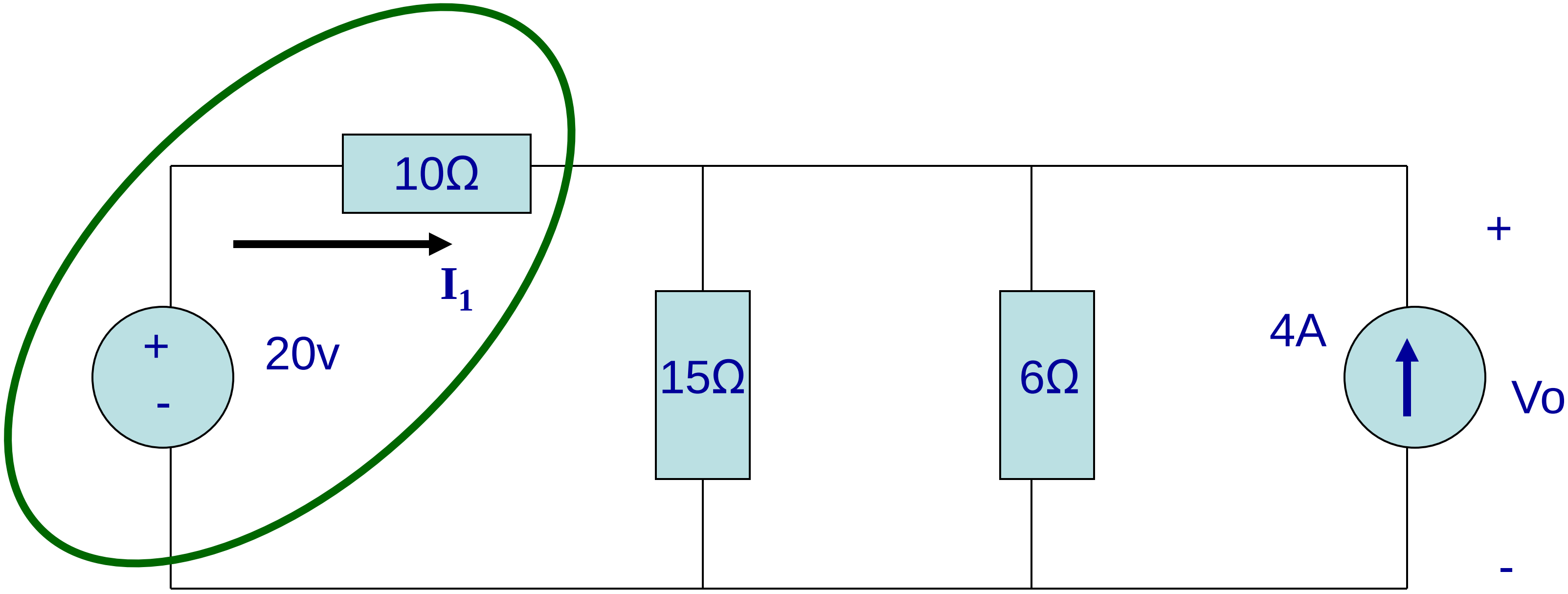
- Now look at the voltage source in series with the resistor with an open circuit



- $I_L = 0$ and V_L = Vs, so Vs = Ip·Rp
- If Ip = Vs/Rs, then Rp = Rs

Example

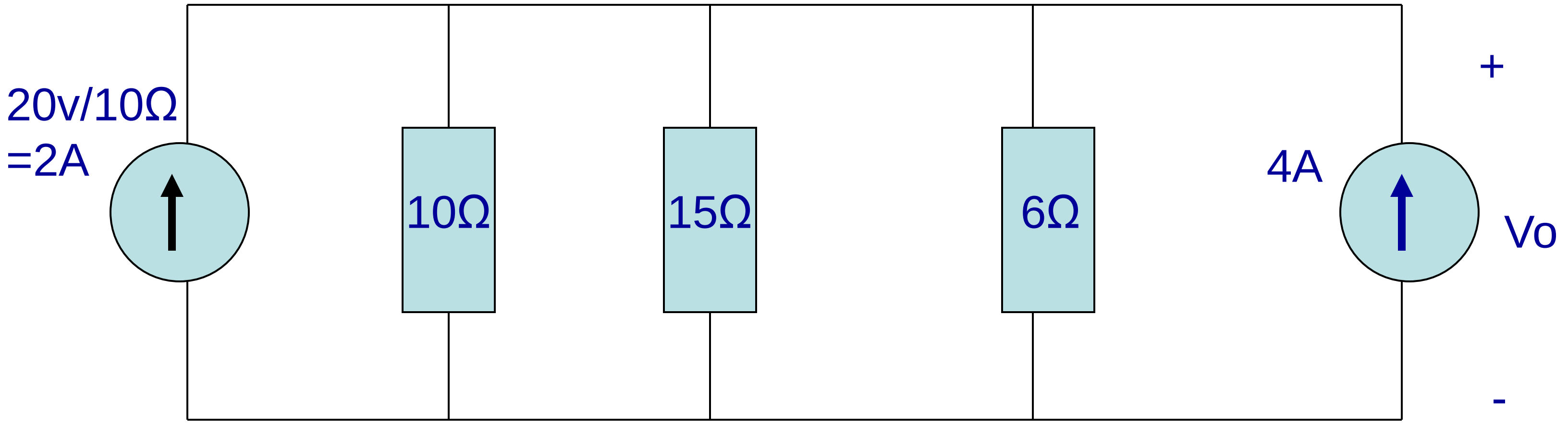
- We can transform the voltage source



- Why? Gets all components in parallel

Example

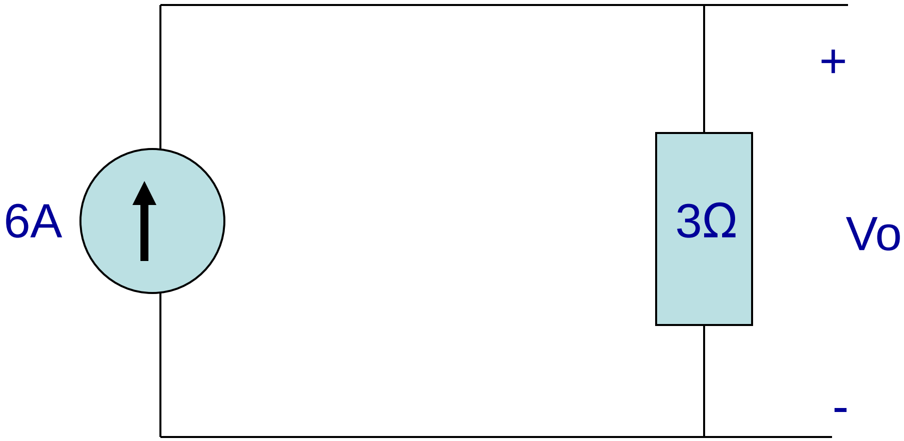
- We can combine sources and resistors



- Ieq = 2A+4A = 6A, Req = 3Ω

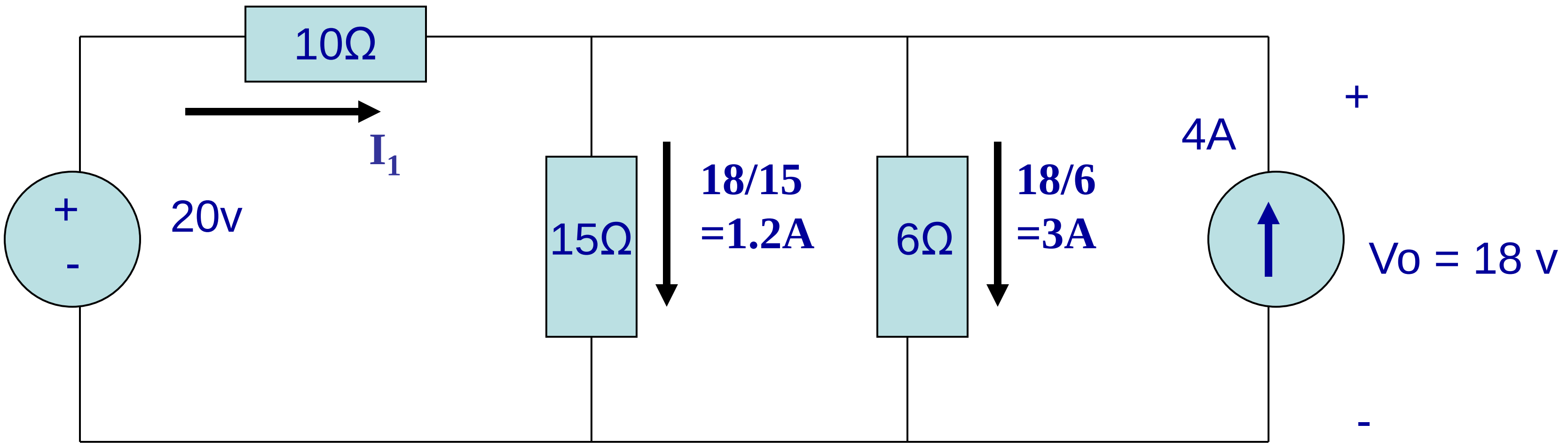
Example

- Vo = 6A· 3Ω = 18 v



Example

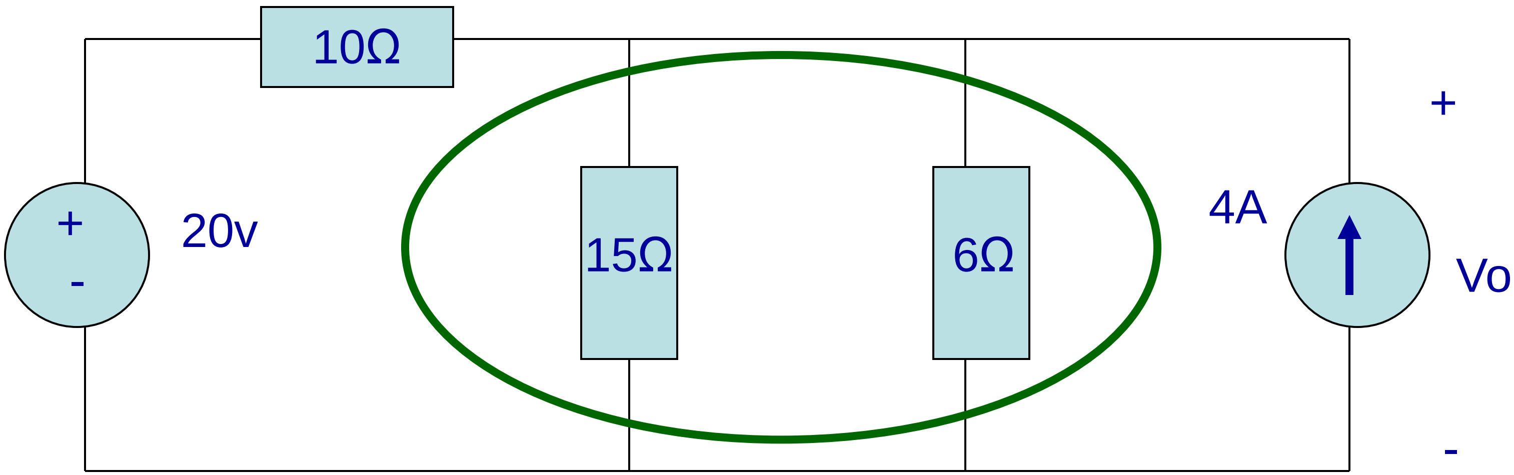
- Going back to the original circuit, Vo=18 v



- KCL: I_1+ 4A - 1.2A - 3A=0, so I_1=0.2A

Example

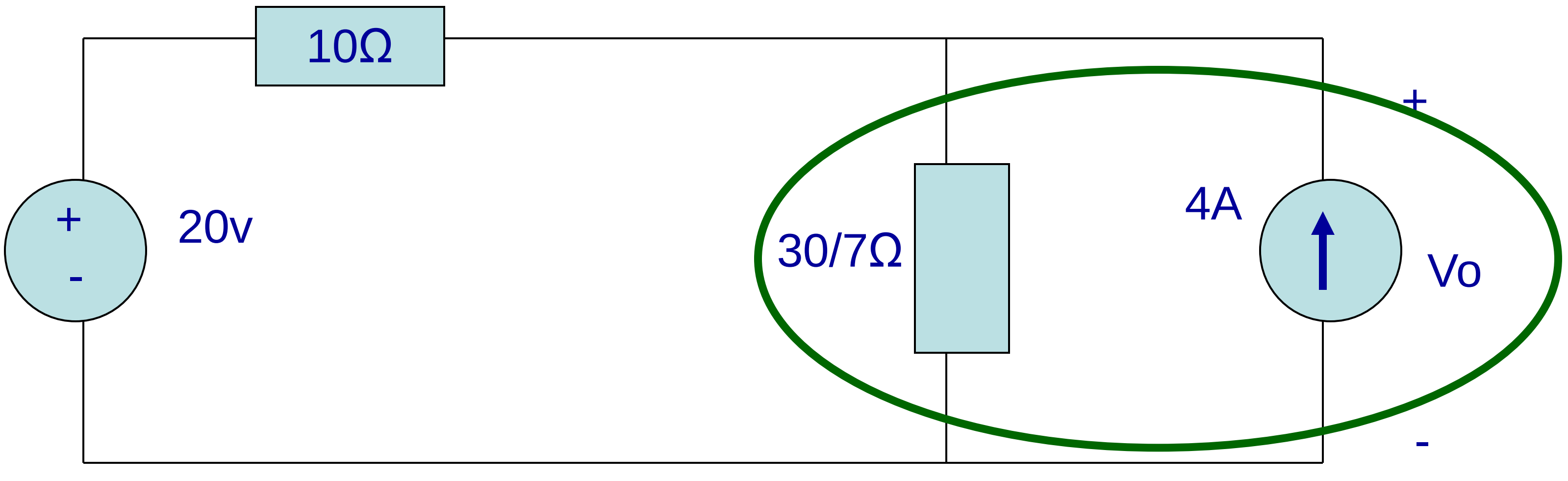
- We can transform the current source after first combining parallel resistances



- Why? Gets all components in series

Example

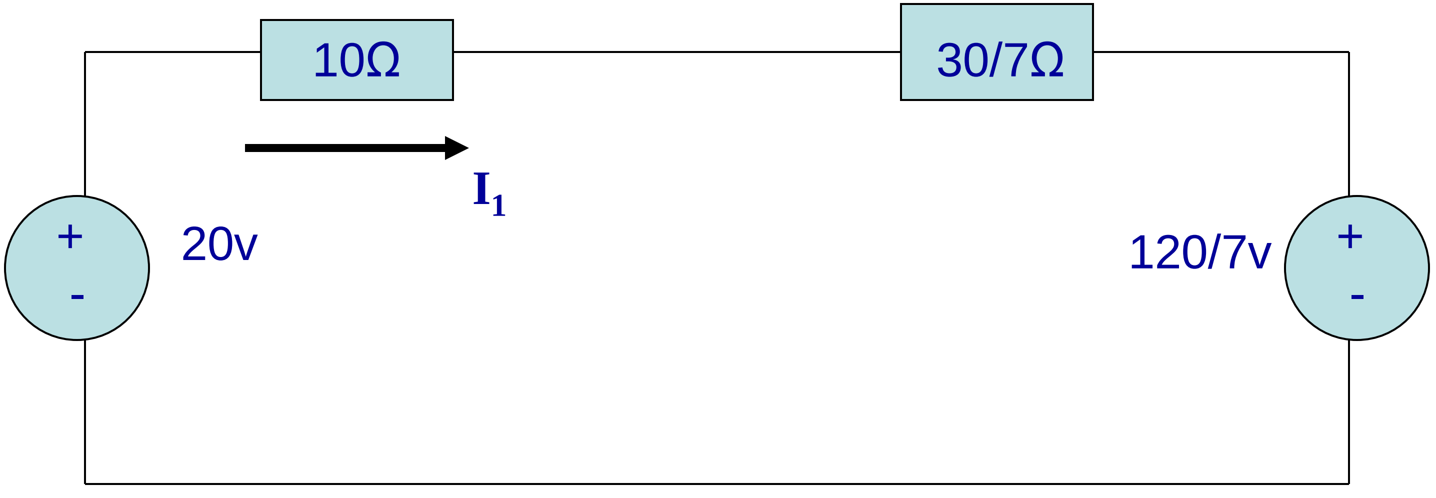
- We can transform the current source after first combining parallel resistances



- Req=6·15/(6+15)=30/7 Ω

Example

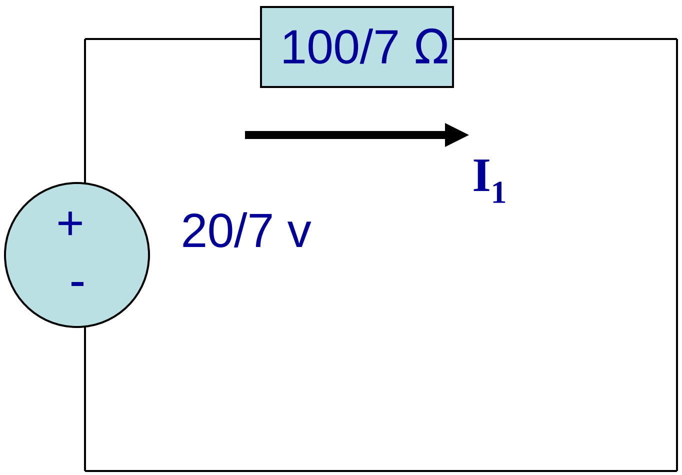
- We can now add the series voltage sources and resistances



- Rtotal=100/7 Ω and Vtotal=20/7 volts

Example

- We can easily solve using KVL for I_1

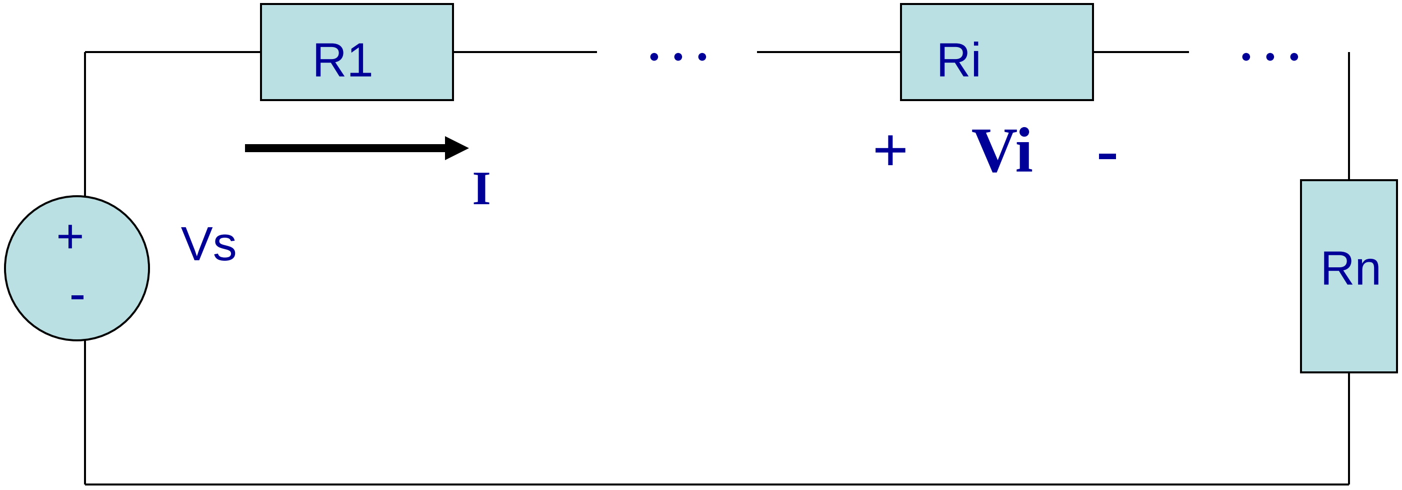


- $I_1 = 20/7 \div 100/7 = 0.2$ A

Voltage & Current Division

Voltage Division

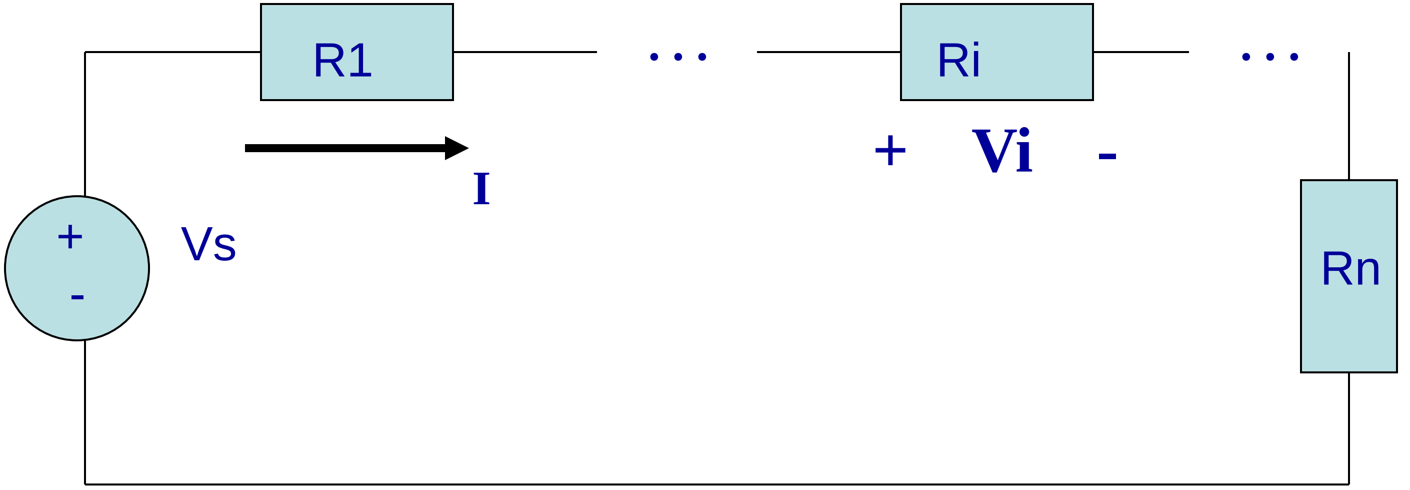
- We could have a circuit with multiple resistors in series where we want to be able to find the voltage across any resistor



- Clearly Req = Σ Ri, and I = Vs/Req
- So Vi = I·Ri = Vs ·(Ri/Req)

Voltage Division Application

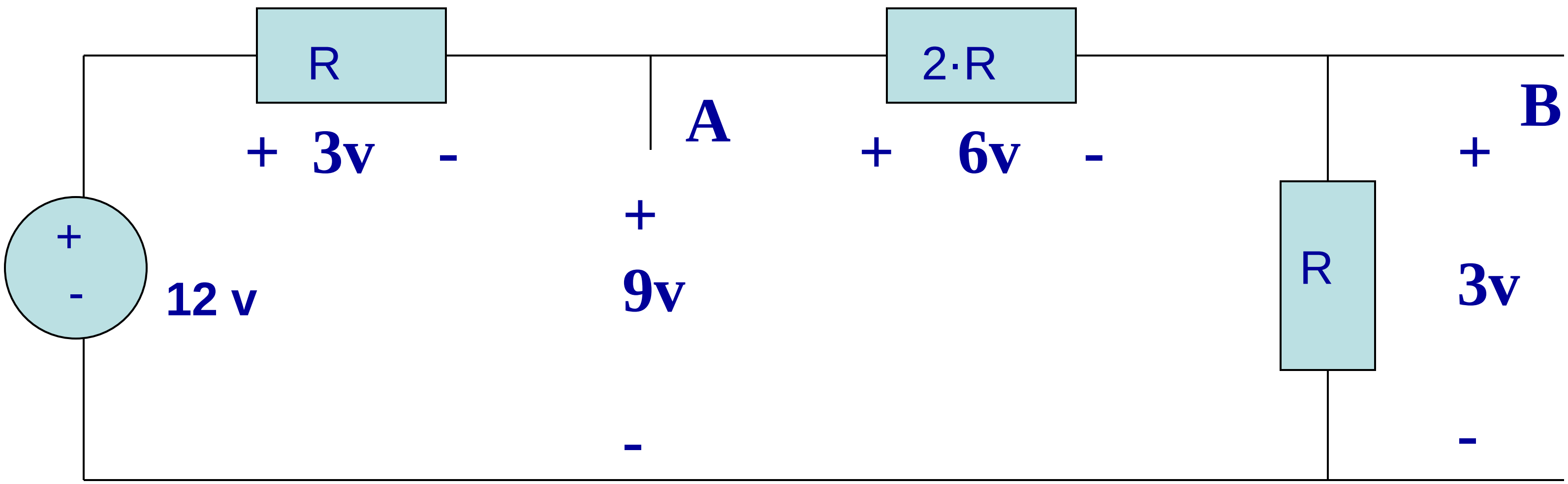
- You have a 12 volt source, but some devices in your circuit need voltages of 3 and 9 volts to run properly



- You can design a voltage divider circuit to produce the necessary voltages

Voltage Division Application

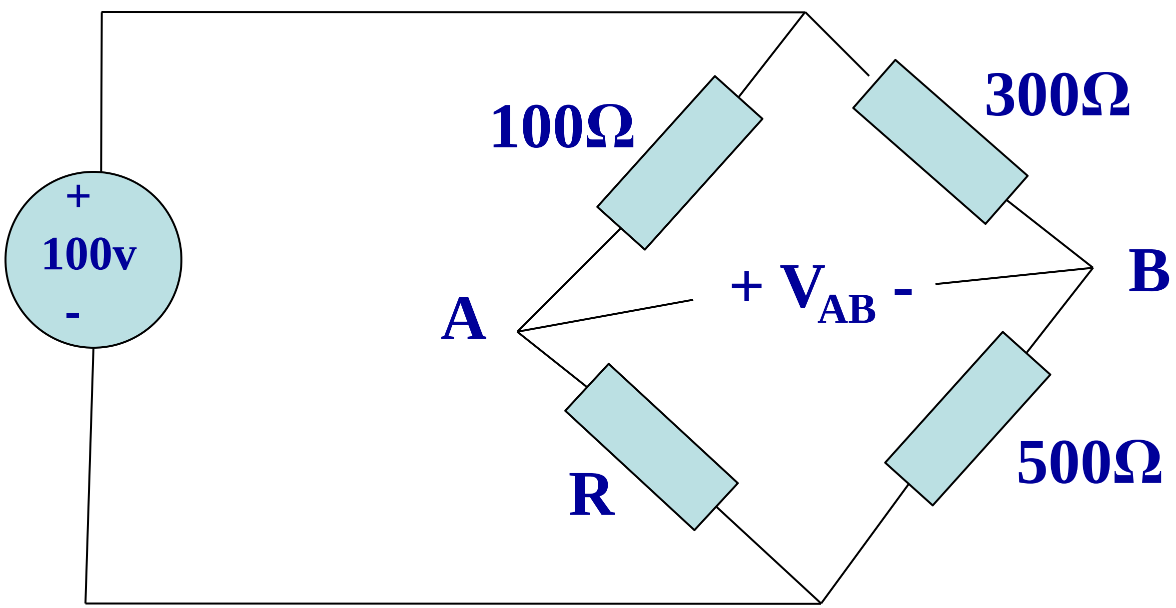
- To get 3, 6 and 3 across the three resistors, any R, 2·R and R could be used



- 9 volts is available at A, 3 volts at B

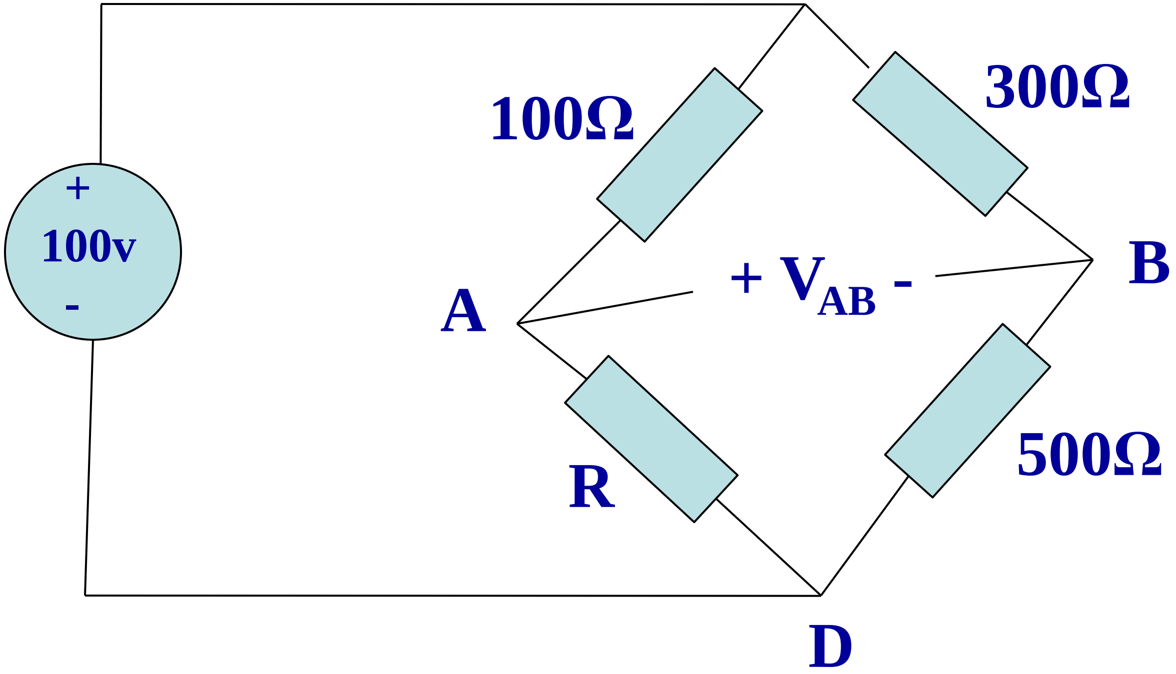
Wheatstone Bridge

- This circuit is often used to measure resistance or convert resistance into a voltage.



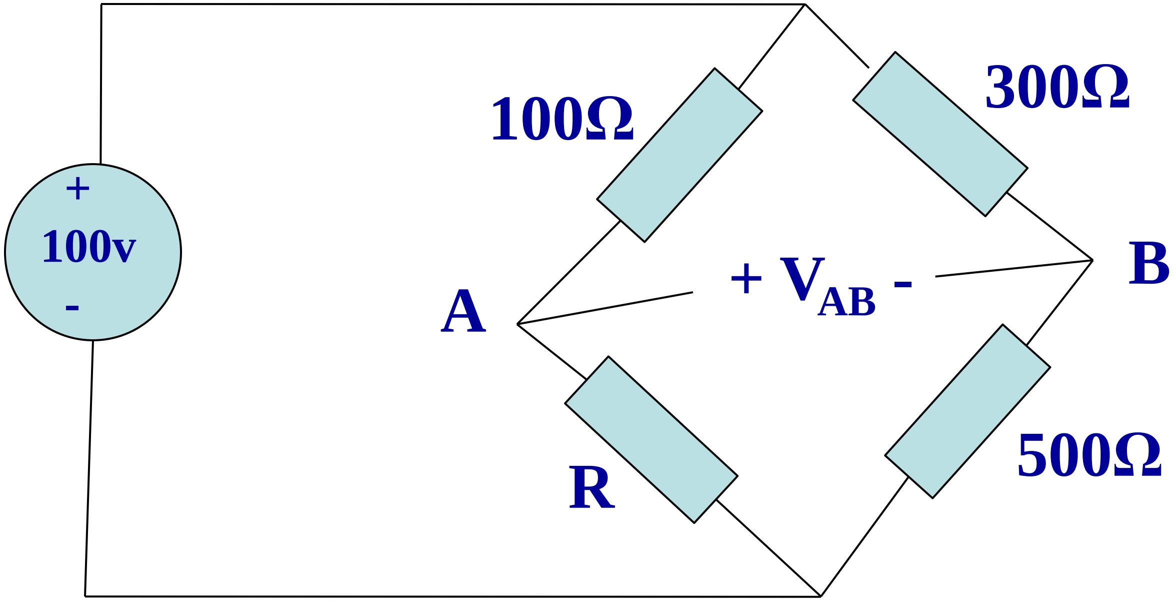
Wheatstone Bridge

- Using the voltage divider at A,
- $V_{AD} = 100 \text{ v} \cdot R/(100+R)\Omega$



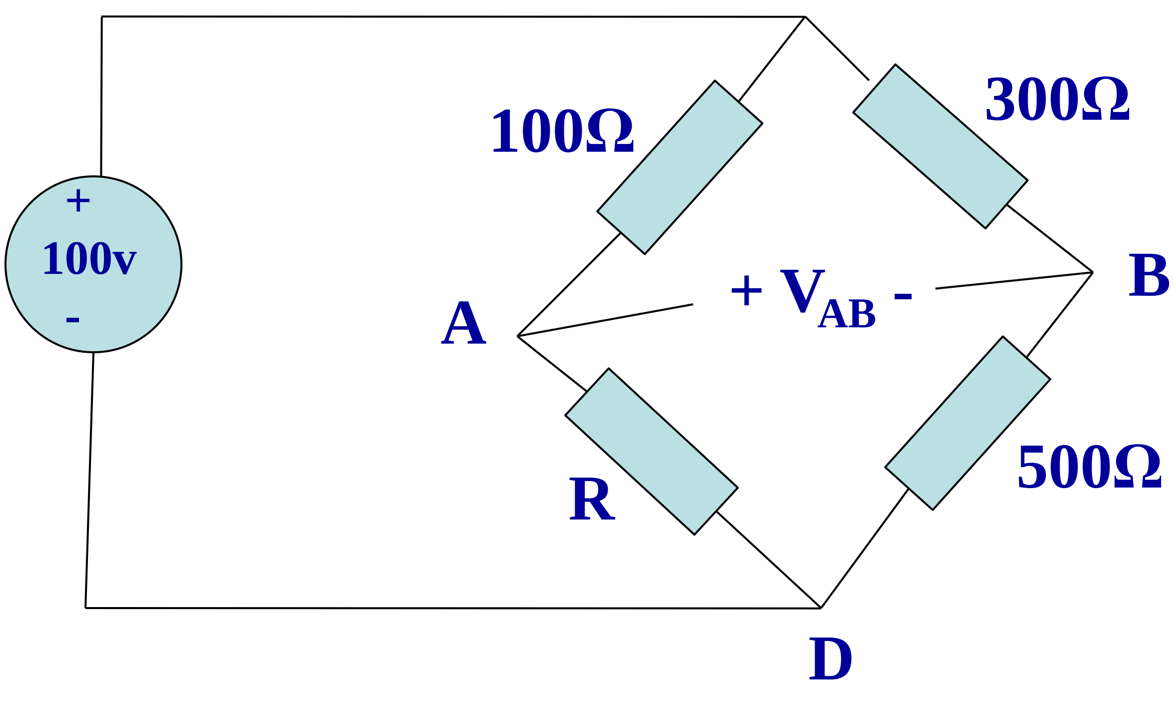
Wheatstone Bridge

- Find the Voltage at B, using the voltage divider theorem



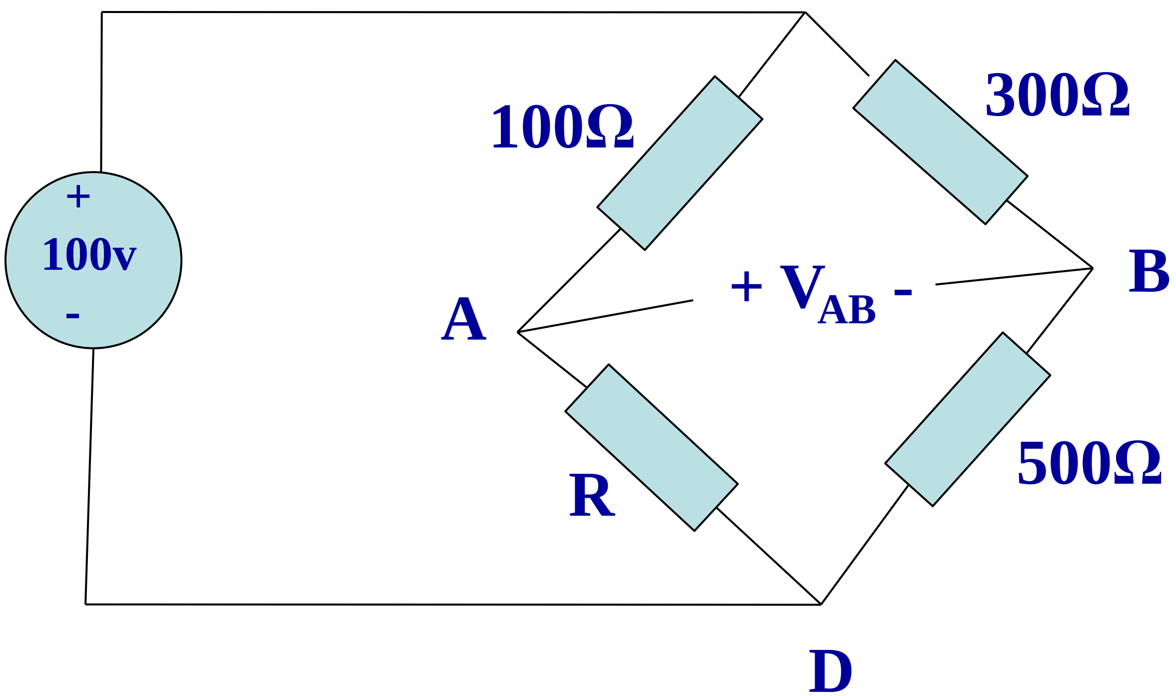
Wheatstone Bridge

- V_{BD} = 100 v · 500Ω/(300+500)Ω = 62.5 v



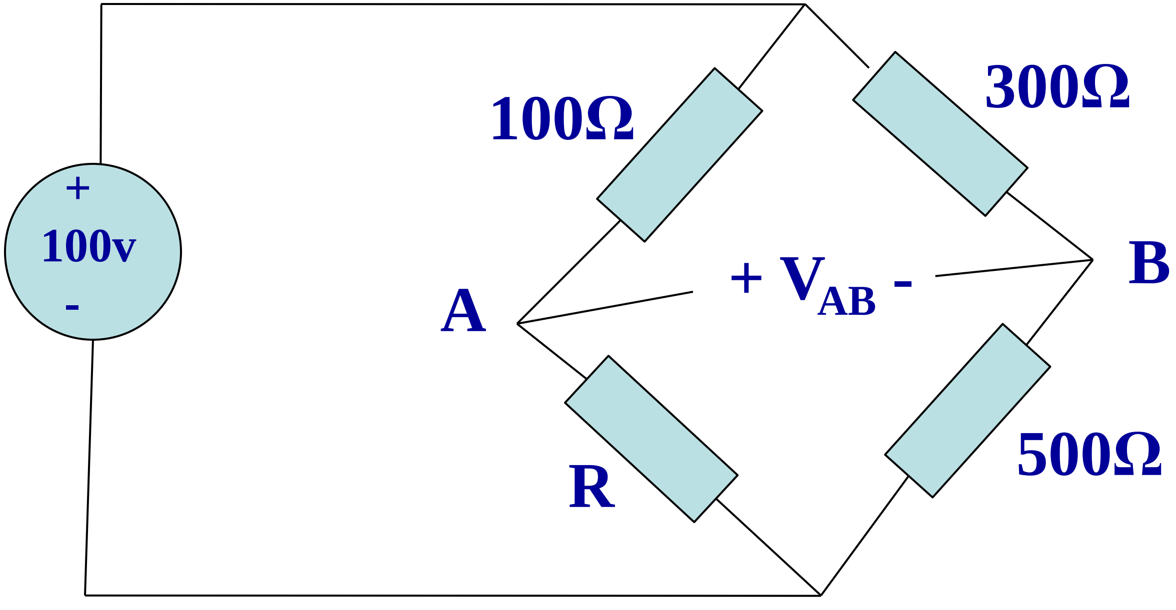
Wheatstone Bridge

- Let's find the relationship between V_{AB} & R
  - $V_{AB} = V_{AD} – V_{BD} = 100·R/(100+R) – 62.5$

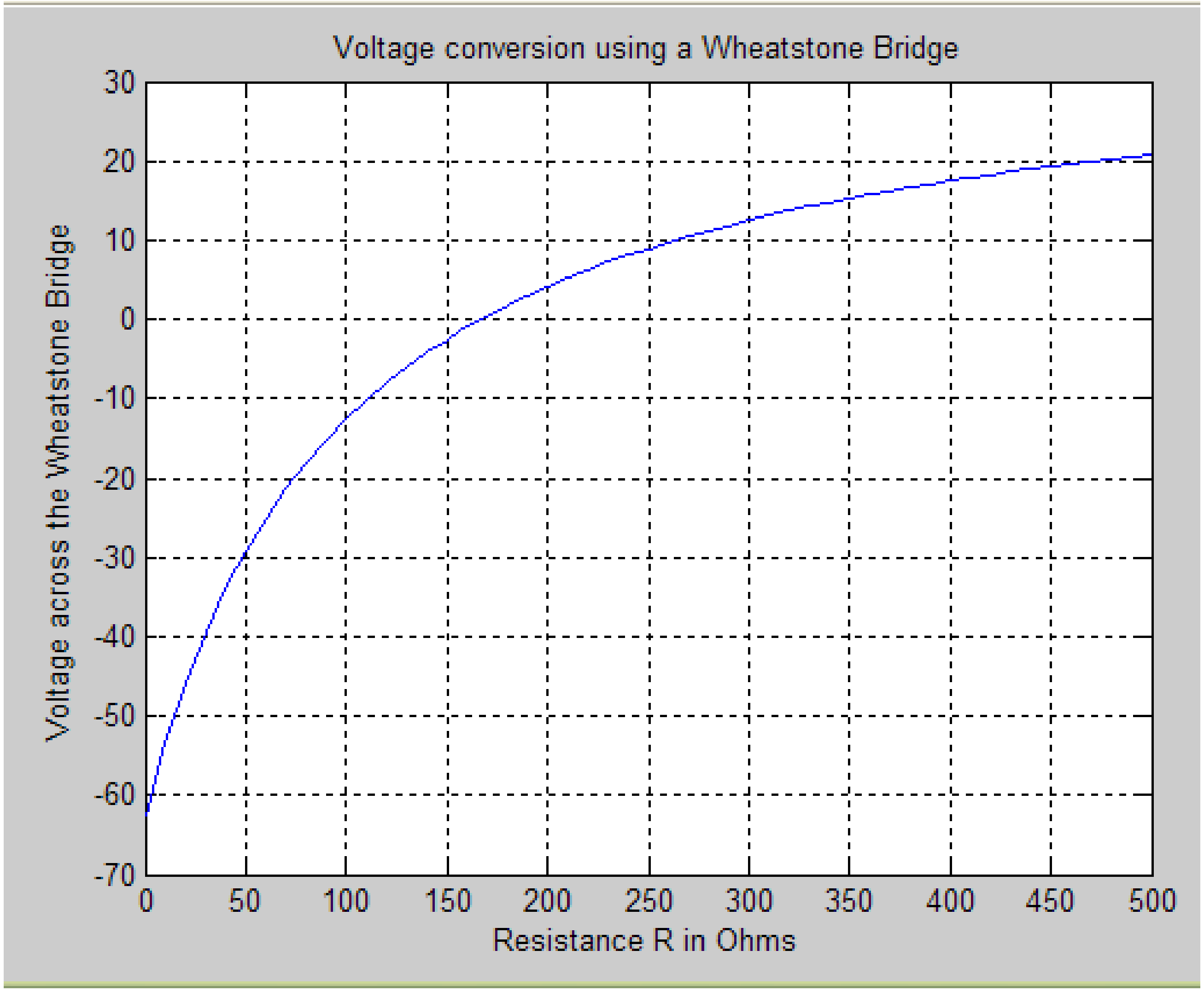


Wheatstone Bridge

- The Wheatstone bridge is considered balanced when V_{AB}=0 v.
- Find R
- R=167Ω

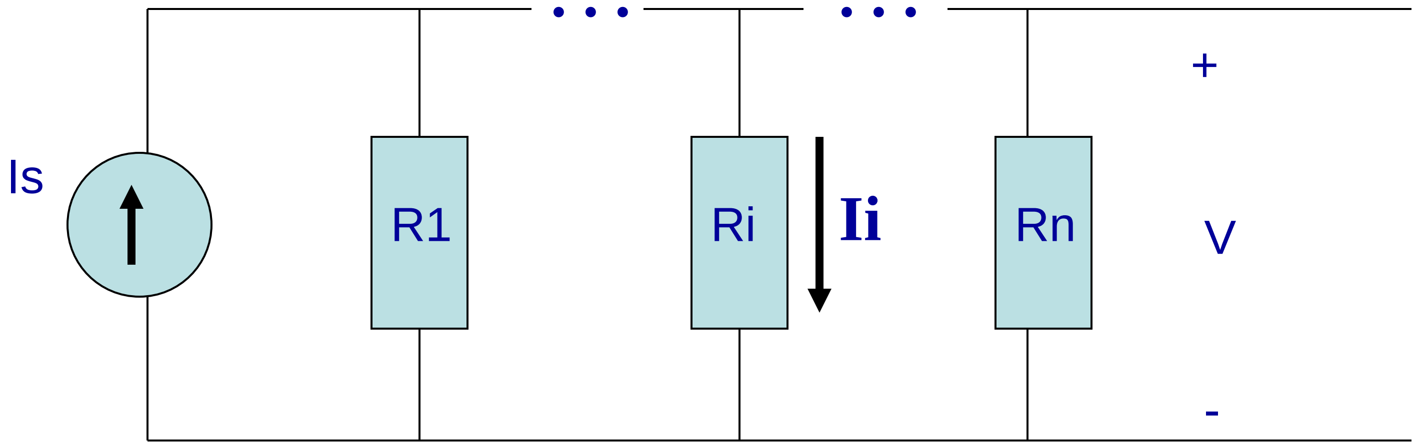


Wheatstone Bridge



Current Division

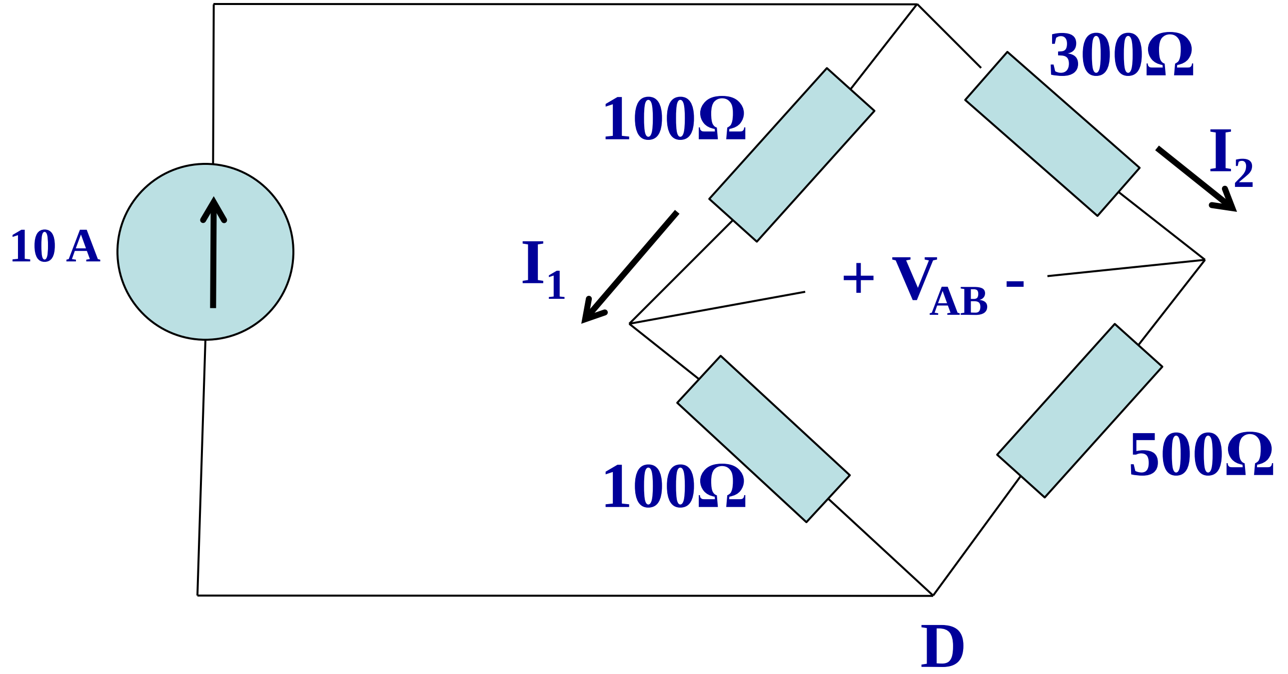
- What if we want to find the current through any parallel resistor?



- Req = 1 / Σ(1/Ri) and V = Is·Req
- So Ii = V / Ri = Is·(Req/Ri)

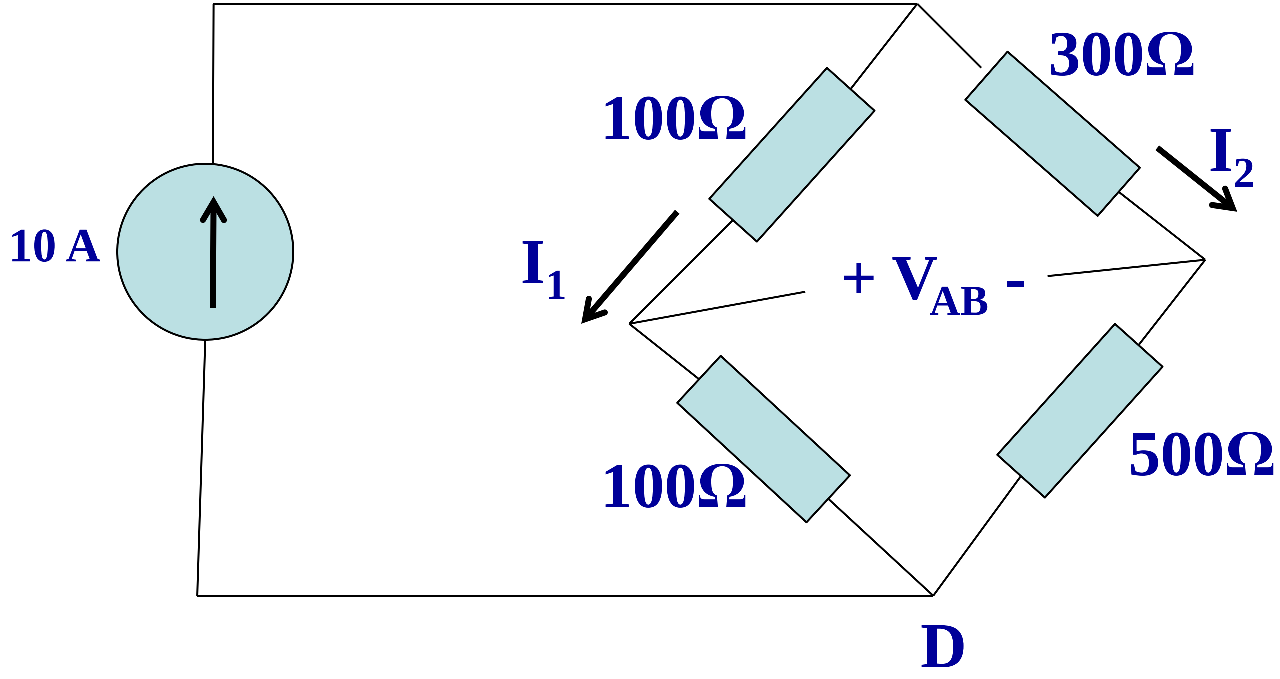
Wheatstone Bridge

- The 10 A current source divides between the two branches of the bridge circuit



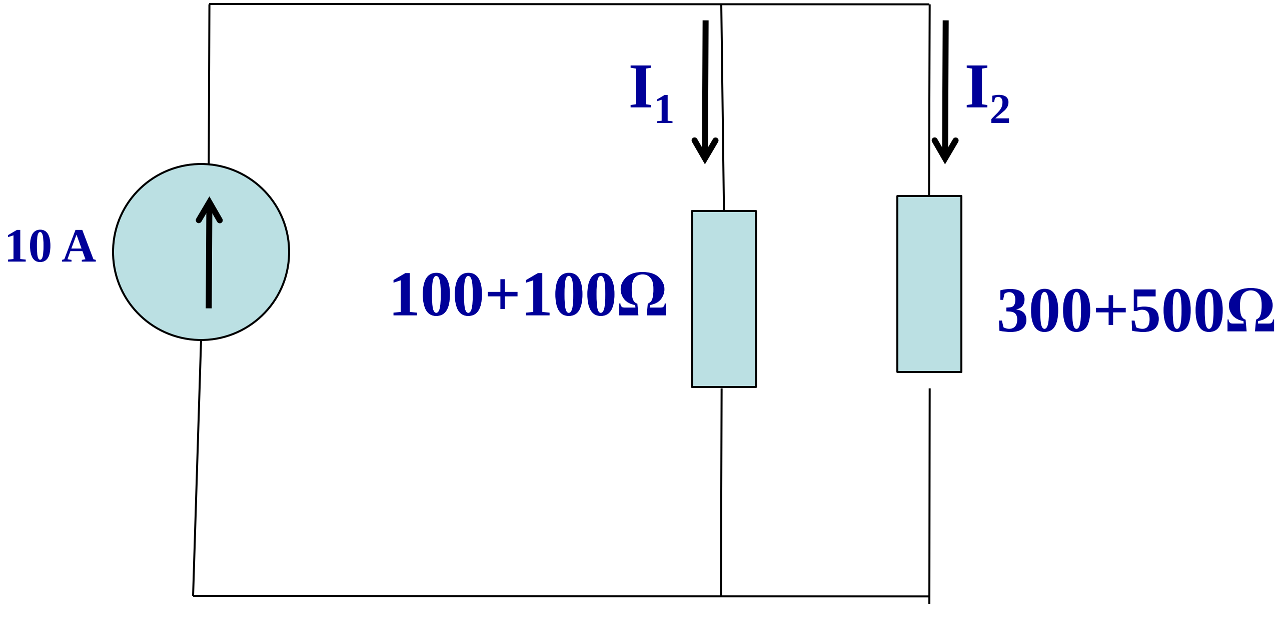
Wheatstone Bridge

- First, simplify by combining the series resistances in each branch of the bridge



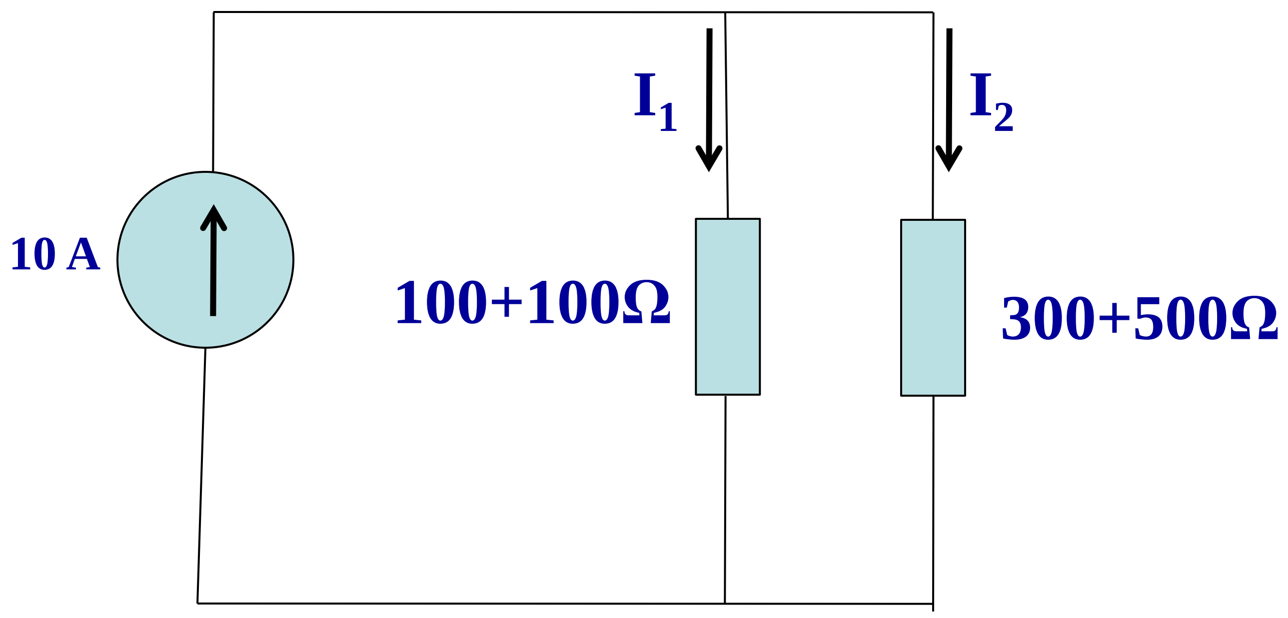
Wheatstone Bridge

- First, simplify by combining the series resistances in each branch of the bridge



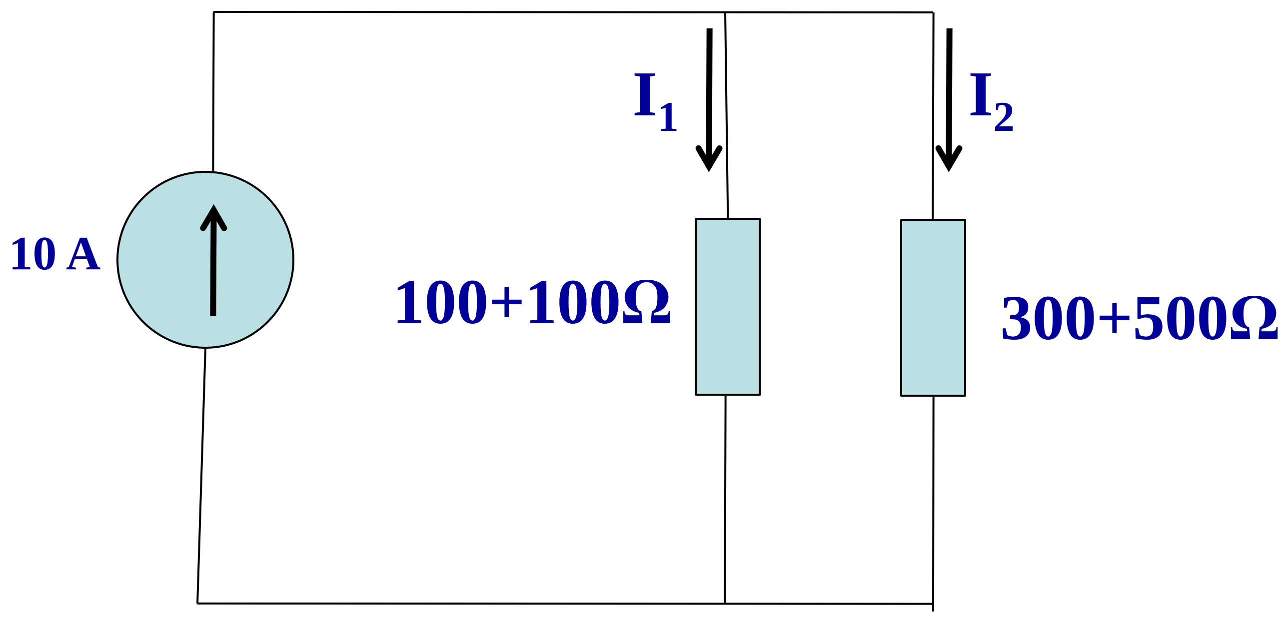
Wheatstone Bridge

- Find the parallel equivalent resistance
- Req = 200·800/(200+800) Ω = 160 Ω



Wheatstone Bridge

- $I_1 = 10\text{ A} \cdot (160/200) = 8\text{ A}$
- $I_2 = 10\text{ A} \cdot (160/800) = 2\text{ A}$



Voltage vs Current Division

- Voltage Division

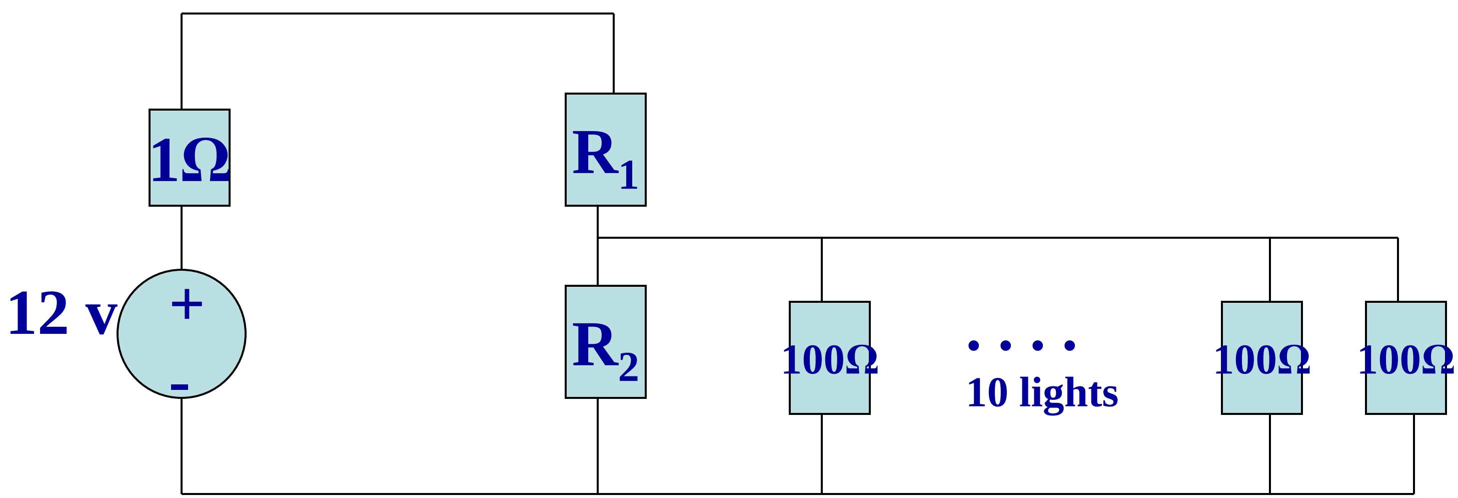
- $V_i = V_s \cdot R_i/R_{eq}$
- $R_{eq} = \Sigma R_i$
- $R_i/R_{eq} < 1$
- $V_i < V_s$
- Series resistors only

- Current Division

- $I_i = I_s \cdot R_{eq}/R_i$
- $R_{eq} = 1 \div \Sigma (1/R_i)$
- $R_{eq}/R_i < 1$
- $I_i < I_s$
- Parallel resistors only

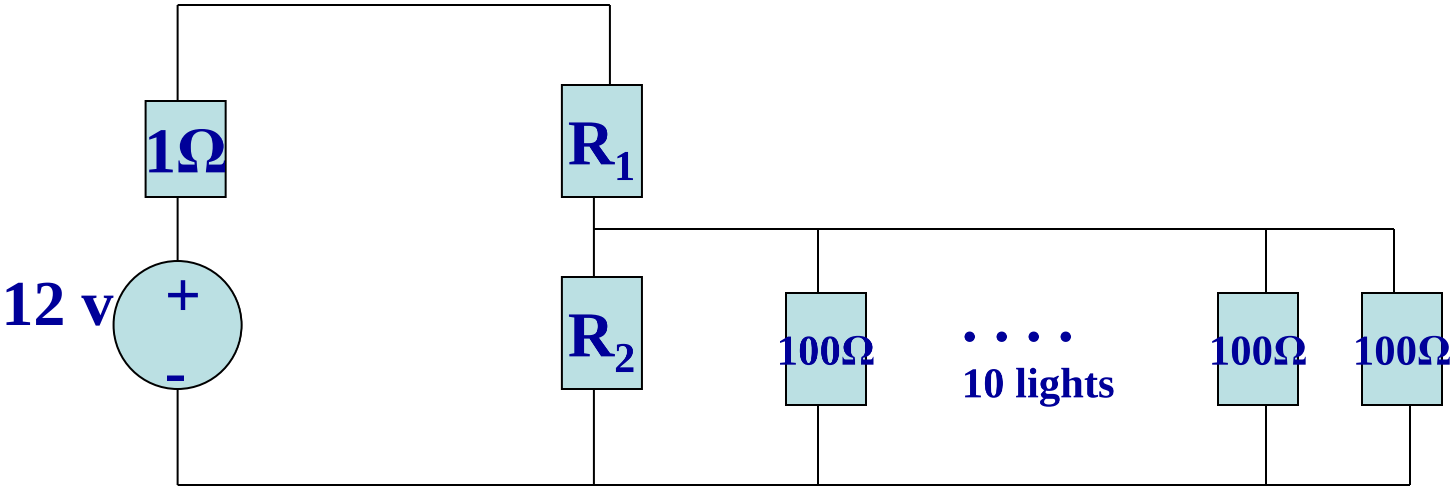
Application

- A practical source of 12 volts has a 1Ω internal resistance. Design a voltage divider that will be used to power up to ten 6 volt light bulbs in parallel.



Application

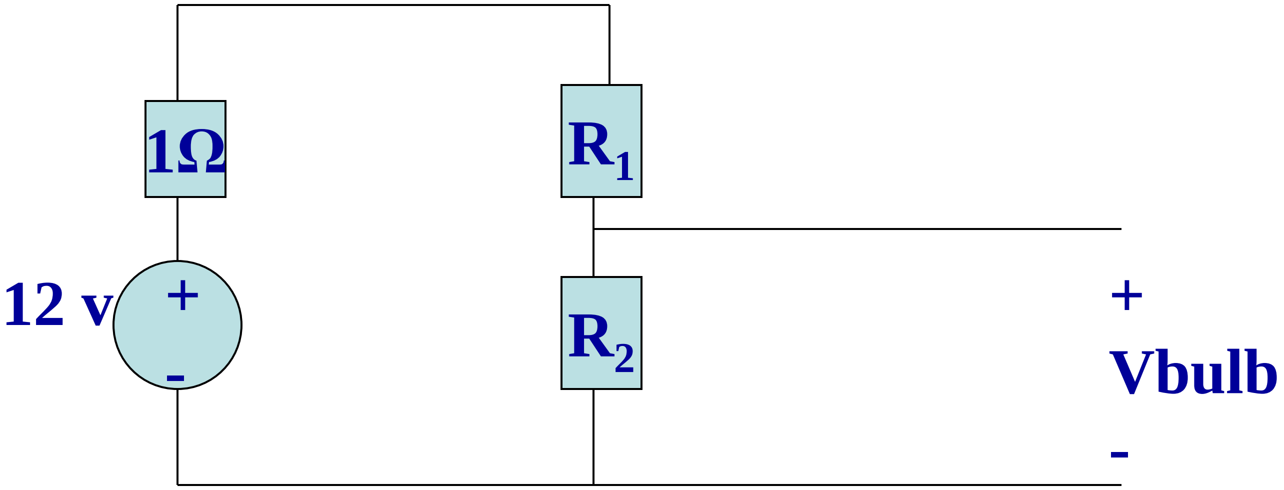
- The voltage across each bulb must be within the range of 5.5 to 6 volts in order for the bulbs to be bright enough. Find R_1 and R_2.



Application

- With no bulbs, the maximum voltage is obtained:

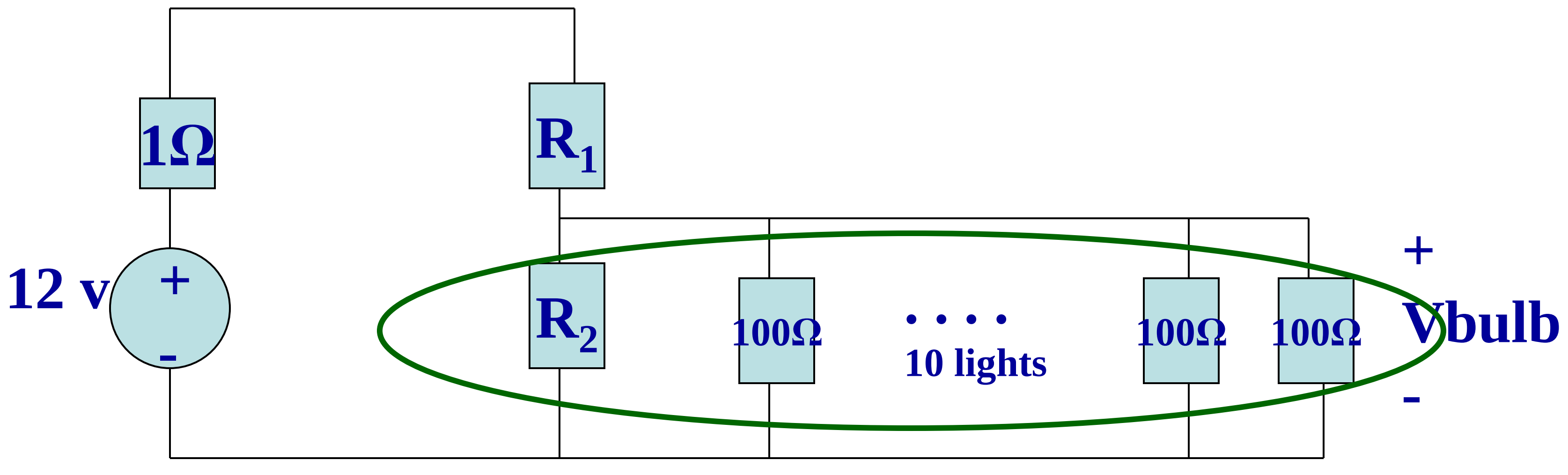
$$\mathbf{Vbulb = 6.0\ v =} \quad \frac{12\text{v}\cdot R_2}{1\Omega+R_1+R_2}$$



Application

- The parallel combination of ten 100Ω bulbs is 10Ω, the parallel combination with R2 is

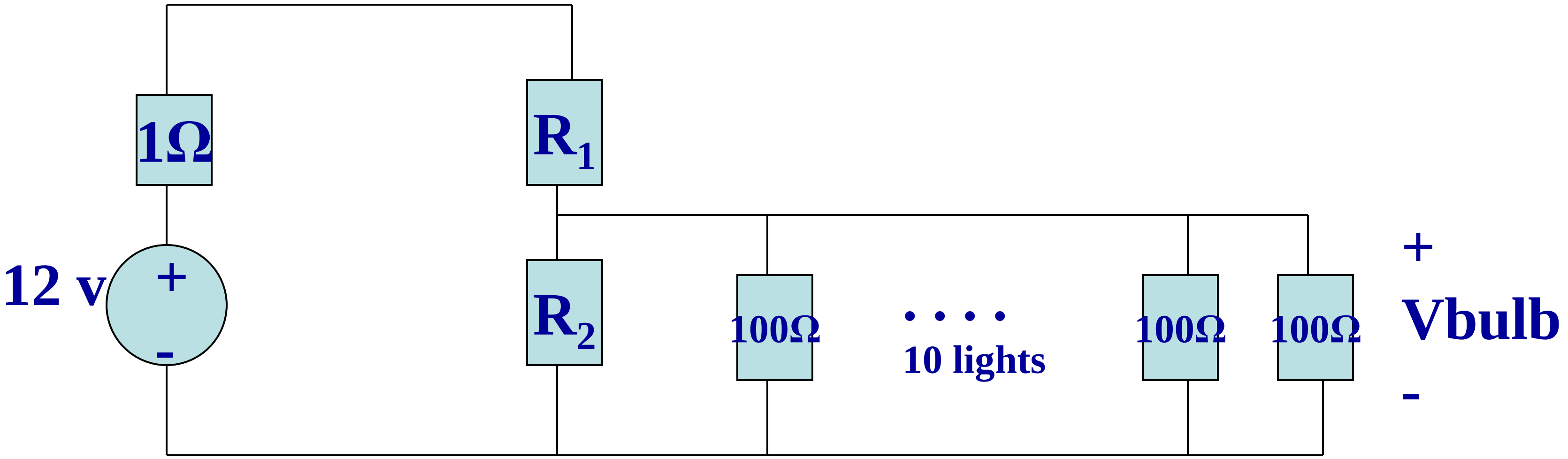
$$\frac{10\cdot R_2/(10+R_2)}{10\cdot R_2/(10+R_2)}\ \Omega$$



Application

- Using the voltage divider and the minimum voltage allowed:

$$\textbf{Vbulb} = \textbf{5.5 v} = \frac{12v \cdot 10 \cdot R_2/(10+R_2)}{1\Omega + R_1 + 10 \cdot R_2/(10+R_2)}$$



Solution

- Solving the two equations simultaneously, R_1 = .82Ω and R_2 = 1.82Ω
- With 10 bulbs, R_2 in parallel with ten 100Ω resistors would be 1.54Ω
- Using the voltage divider, with 10 bulbs

$$V_{bulb} = 12 \cdot 1.54\Omega \div (1 + .82\Omega + 1.54\Omega) = 5.5 \text{ v}$$

- Using the voltage divider, with no bulbs

$$V_{bulb} = 12 \cdot 1.82\Omega \div (1 + .82\Omega + 1.82\Omega) = 6.0 \text{ v}$$

Capacitors and RC circuits

Capacitors

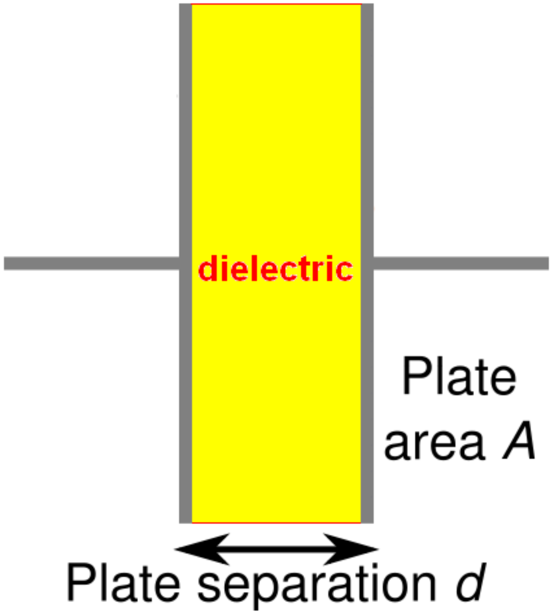
- Composed of two conductive plates separated by an insulator (or dielectric).
  - Commonly illustrated as two parallel metal plates separated by a distance, d.

$C = \varepsilon A/d$

where $\varepsilon = \varepsilon_r \varepsilon_o$

ε_r is the relative dielectric constant

ε_o is the vacuum permittivity



Effect of Dimensions

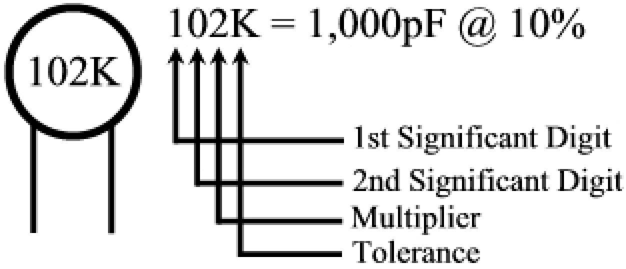
- Capacitance increases with
  - increasing surface area of the plates,
  - decreasing spacing between plates, and
  - increasing the relative dielectric constant of the insulator between the two plates.

Types of Capacitors

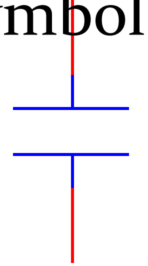
- Fixed Capacitors
  - Nonpolarized
    - May be connected into circuit with either terminal of capacitor connected to the high voltage side of the circuit.
      - Insulator: Paper, Mica, Ceramic, Polymer
  - Electrolytic
    - The negative terminal must always be at a lower voltage than the positive terminal
      - Plates or Electrodes: Aluminum, Tantalum

Nonpolarized

- Difficult to make nonpolarized capacitors that store a large amount of charge or operate at high voltages.
  - Tolerance on capacitance values is very large
    - +50%/-25% is not unusual

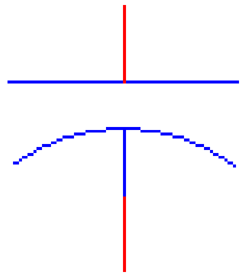


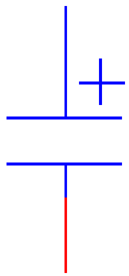
PSpice Symbol



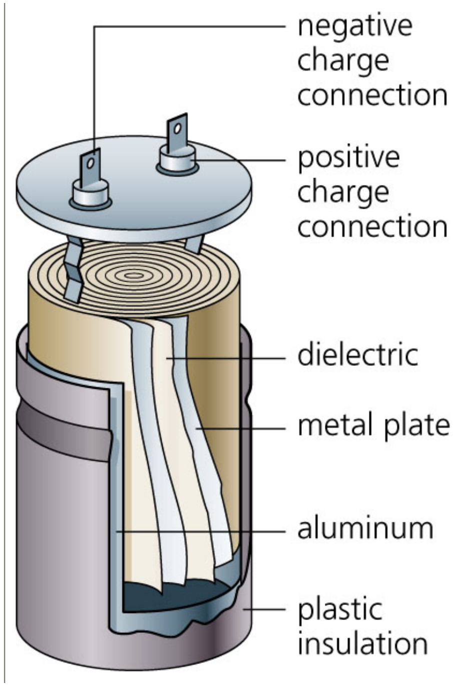
Electrolytic

Symbols



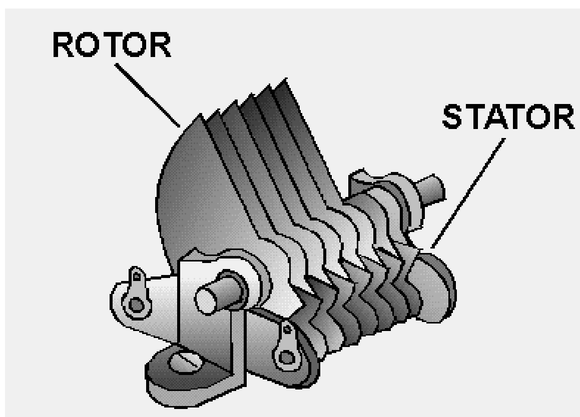


Fabrication



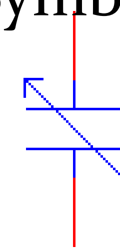
Variable Capacitors

- Cross-sectional area is changed as one set of plates are rotated with respect to the other.



http://www.tpub.com/neets/book2/3f.htm

PSpice Symbol

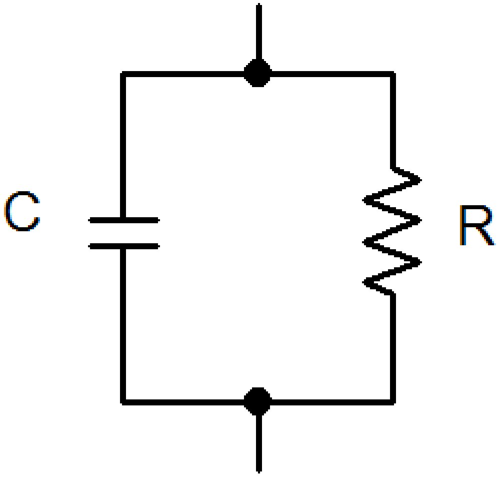


Electrical Properties of a Capacitor

- Acts like an open circuit at steady state when connected to a d.c. voltage or current source.
- Voltage on a capacitor must be continuous
  - There are no abrupt changes to the voltage, but there may be discontinuities in the current.
- An ideal capacitor does not dissipate energy, it takes power when storing energy and returns it when discharging.

Properties of a Real Capacitor

- A real capacitor does dissipate energy due leakage of charge through its insulator.
  - This is modeled by putting a resistor in parallel with an ideal capacitor.



Energy Storage

- Charge is stored on the plates of the capacitor.

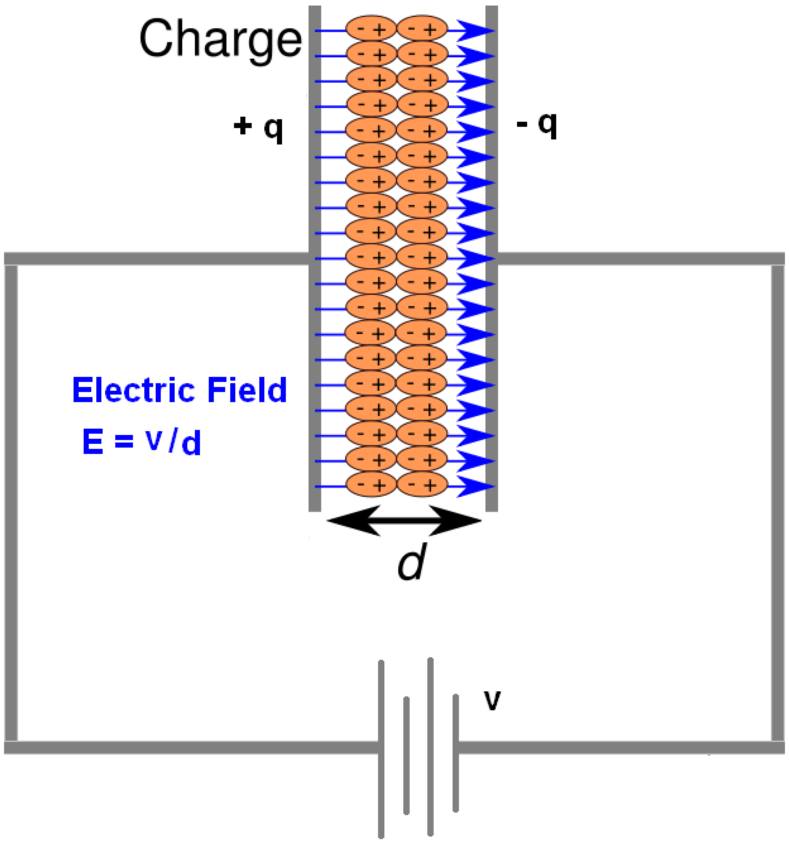
Equation:

$$Q = CV$$

Units:

Farad = Coulomb/Voltage

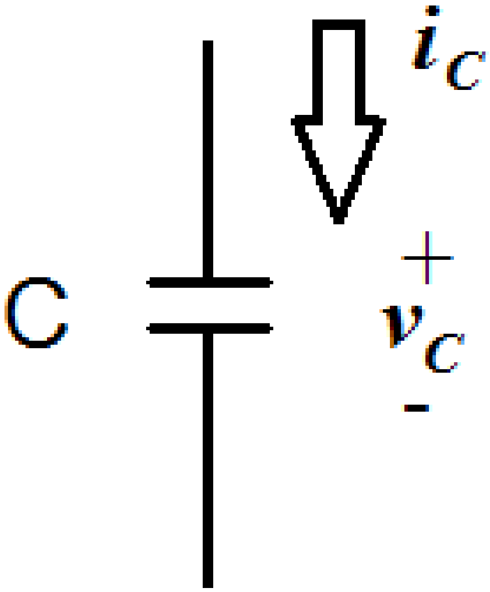
Farad is abbreviated as F



Sign Conventions

The sign convention used with a capacitor is the same as for a power dissipating device.

- When current flows into the positive side of the voltage across the capacitor, it is positive and the capacitor is dissipating power.
- When the capacitor releases energy back into the circuit, the sign of the current will be negative.



Charging a Capacitor

At first, it is easy to store charge in the capacitor.

As more charge is stored on the plates of the capacitor, it becomes increasingly difficult to place additional charge on the plates.

- Coulombic repulsion from the charge already on the plates creates an opposing force to limit the addition of more charge on the plates.
  - Voltage across a capacitor increases rapidly as charge is moved onto the plates when the initial amount of charge on the capacitor is small.
  - Voltage across the capacitor increases more slowly as it becomes difficult to add extra charge to the plates.

Adding Charge to Capacitor

- The ability to add charge to a capacitor depends on:
  - the amount of charge already on the plates of the capacitor

  and

  - the force (voltage) driving the charge towards the plates (i.e., current)

Discharging a Capacitor

At first, it is easy to remove charge in the capacitor.

- Coulombic repulsion from charge already on the plates creates a force that pushes some of the charge out of the capacitor once the force (voltage) that placed the charge in the capacitor is removed (or decreased).

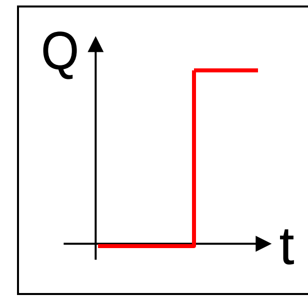
As more charge is removed from the plates of the capacitor, it becomes increasingly difficult to get rid of the small amount of charge remaining on the plates.

- Coulombic repulsion decreases as charge spreads out on the plates. As the amount of charge decreases, the force needed to drive the charge off of the plates decreases.
  - Voltage across a capacitor decreases rapidly as charge is removed from the plates when the initial amount of charge on the capacitor is small.
  - Voltage across the capacitor decreases more slowly as it becomes difficult to force the remaining charge out of the capacitor.

RC Circuits

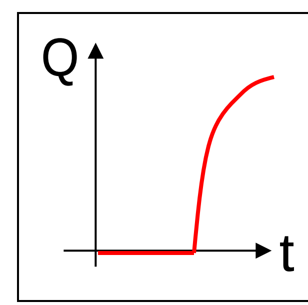
RC circuits contain both a resistor R and a capacitor C (duh).

Until now we have assumed that charge is instantly placed on a capacitor by an emf.



The approximation resulting from this assumption is reasonable, provided the resistance between the emf and the capacitor being charged/discharged is small.

If the resistance between the emf and the capacitor is finite, then the charge on the capacitor does not change instantaneously.

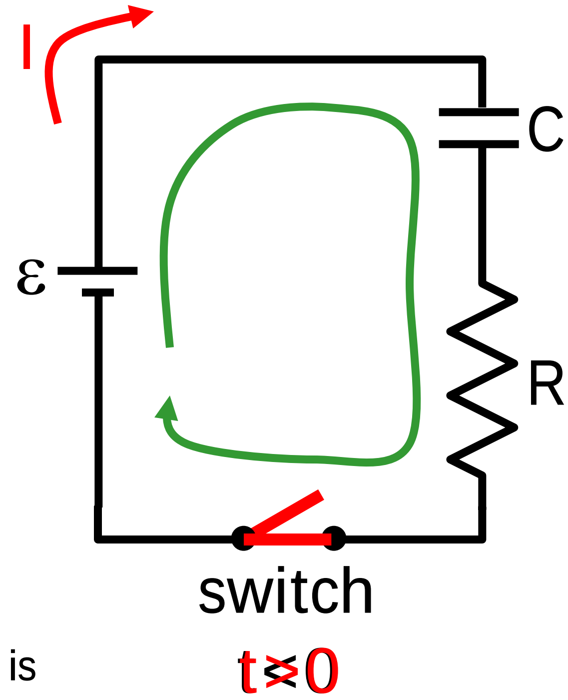


Charging a Capacitor

Switch open, no current flows.

Close switch, current flows.

Apply Kirchoff's loop rule* (green loop) at the instant charge on C is q.



$$\varepsilon - \frac{q}{C} - IR = 0$$

This equation is deceptively complex because I depends on q and both depend on time.

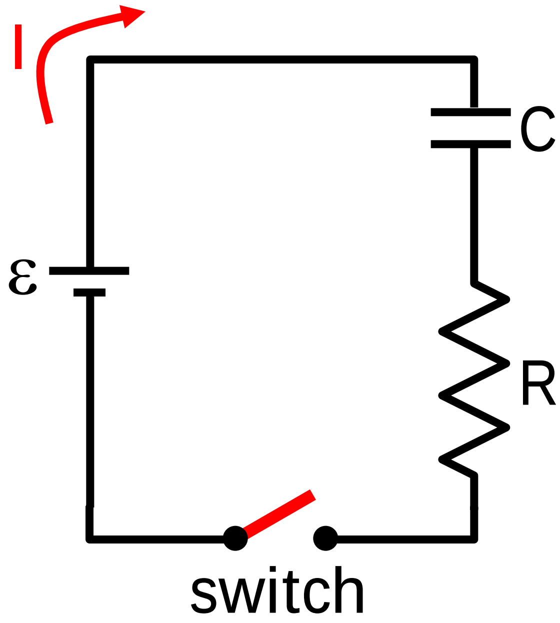
*Convention for capacitors is "like" batteries: negative if going across from + to -.

Limiting Cases

$$\varepsilon - \frac{q}{C} - IR = 0$$

When $t=0$, $q=0$ and $I_0=\varepsilon/R$.

When t is "large," the capacitor is fully charged, the current "shuts off," and $Q=C\varepsilon$.



Math:

$$\varepsilon - \frac{q}{C} - IR = 0$$

$$I = \frac{\varepsilon}{R} - \frac{q}{RC}$$

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} = \frac{C\varepsilon}{RC} - \frac{q}{RC} = \frac{C\varepsilon - q}{RC}$$

$$\frac{dq}{C\varepsilon - q} = \frac{dt}{RC}$$

$$\frac{dq}{q - C\varepsilon} = -\frac{dt}{RC}$$

More math:

$$\int_0^q \frac{dq}{q - C\varepsilon} = -\int_0^t \frac{dt}{RC}$$

$$\ln\left(q - C\varepsilon\right)\Big|_0^q = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

$$\frac{q - C\varepsilon}{-C\varepsilon} = e^{-\frac{t}{RC}}$$

$$q - C\varepsilon = -C\varepsilon\, e^{-\frac{t}{RC}}$$

Still more math:

$$q = C\varepsilon - C\varepsilon\, e^{-\frac{t}{RC}}$$

$$q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

$$q(t) = Q\left(1 - e^{-\frac{t}{RC}}\right)$$

Why not just solve this for q and I?

$$\varepsilon - \frac{q}{C} - IR = 0$$

$$I(t) = \frac{dq}{dt} = \frac{C\varepsilon}{RC} e^{-\frac{t}{RC}} = \frac{C\varepsilon}{RC} e^{-\frac{t}{RC}} = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

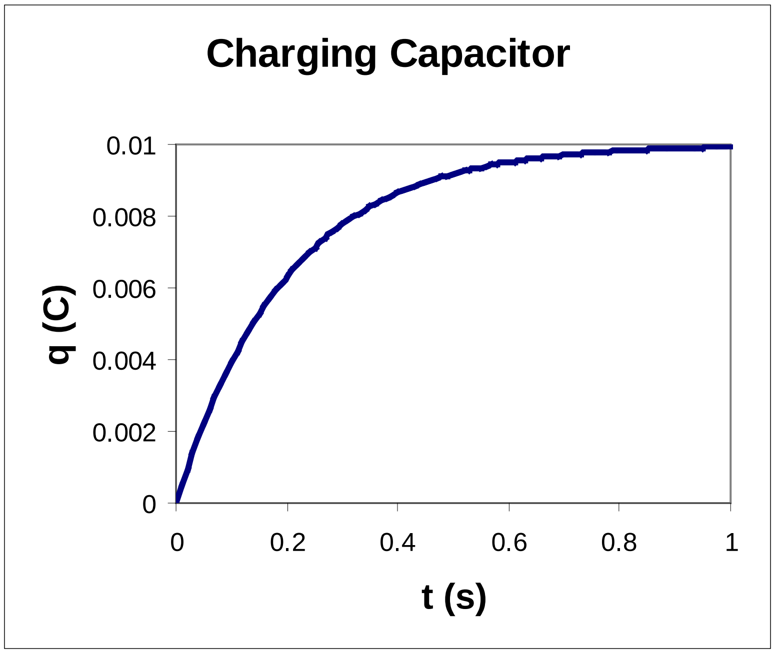
RC is the “time constant” of the circuit; it tells us “how fast” the capacitor charges and discharges.

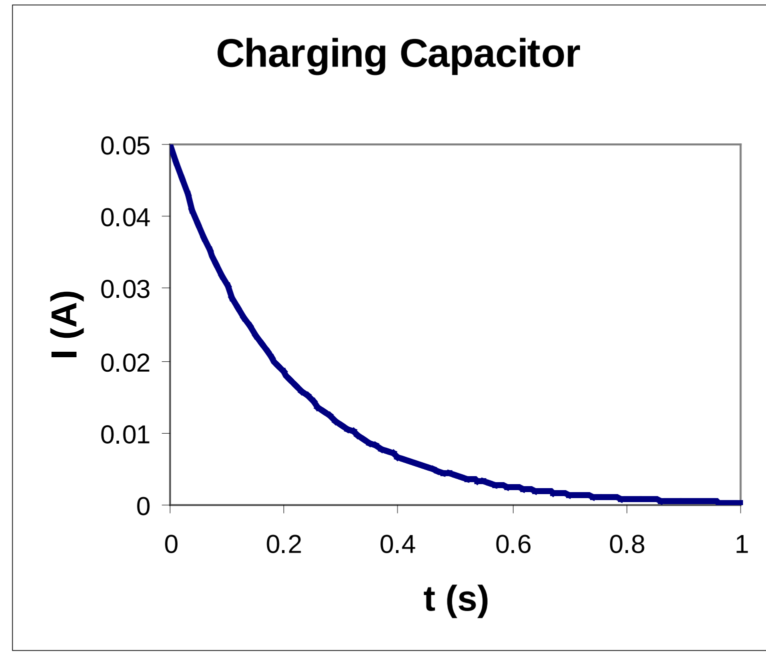
Charging a capacitor; summary:

$$\boxed{q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)}$$

recall that this is I_0, also called I_{max}

$$I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

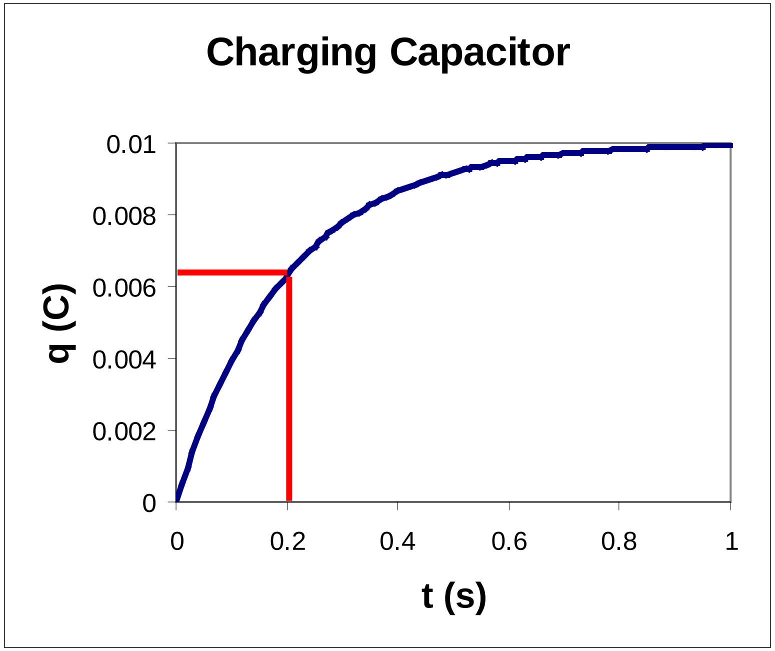


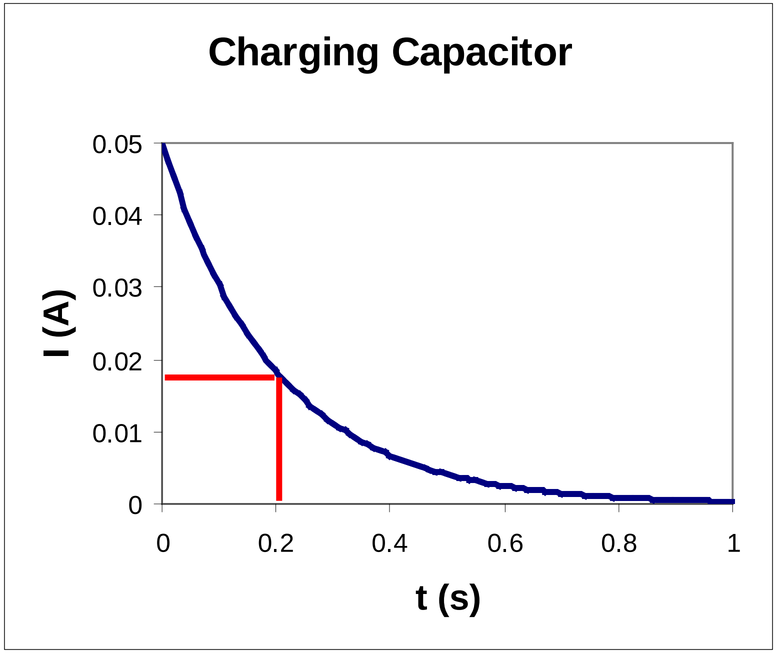


Sample plots with ε=10 V, R=200 Ω, and C=1000 μF.
RC=0.2 s

In a time t=RC, the capacitor charges to $Q(1-e^{-1})$ or 63% of its capacity...

...and the current drops to $I_{max}(e^{-1})$ or 37% of its maximum.





RC=0.2 s

$\boxed{\tau=RC}$ is called the **time constant** of the RC circuit

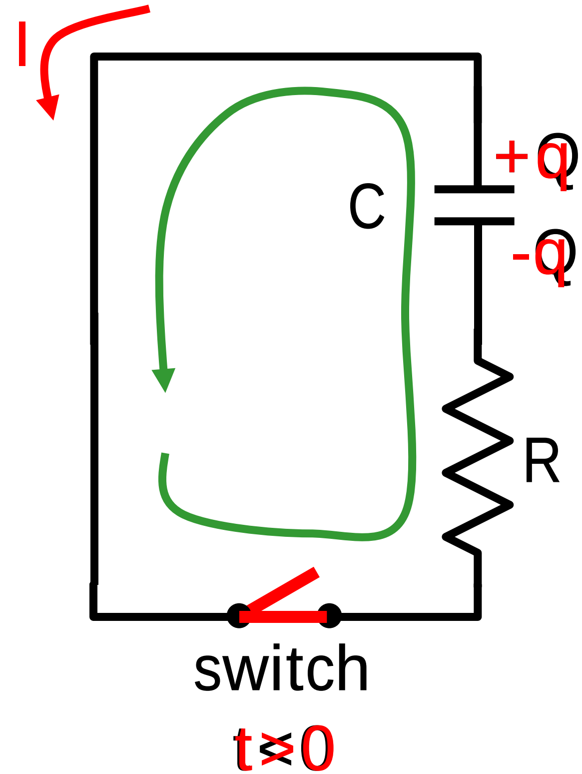
Discharging a Capacitor

Capacitor charged, switch open, no current flows.

Close switch, current flows.

Apply Kirchoff's loop rule* (green loop) at the instant charge on C is q.

$$\frac{q}{C} - IR = 0$$



*Convention for capacitors is "like" batteries: positive if going across from - to +.

Math:

$$\frac{q}{C} - IR = 0$$

$$IR = \frac{q}{C}$$

$$I = -\frac{dq}{dt}$$

negative because charge decreases

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

More math:

$$\int_Q^q \frac{dq}{q} = -\int_0^t \frac{dt}{RC} = -\frac{1}{RC}\int_0^t dt$$

$$\ln(q)\Big|_Q^q = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

$$q(t) = Q\,e^{-\frac{t}{RC}}$$

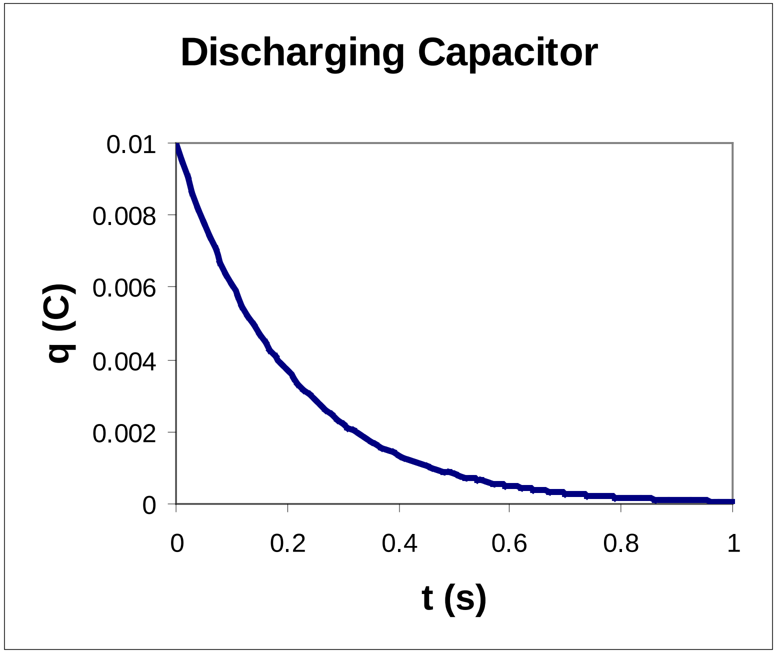
$$I(t) = -\frac{dq}{dt} = \frac{Q}{RC}\,e^{-\frac{t}{RC}} = I_0\,e^{-\frac{t}{RC}}$$

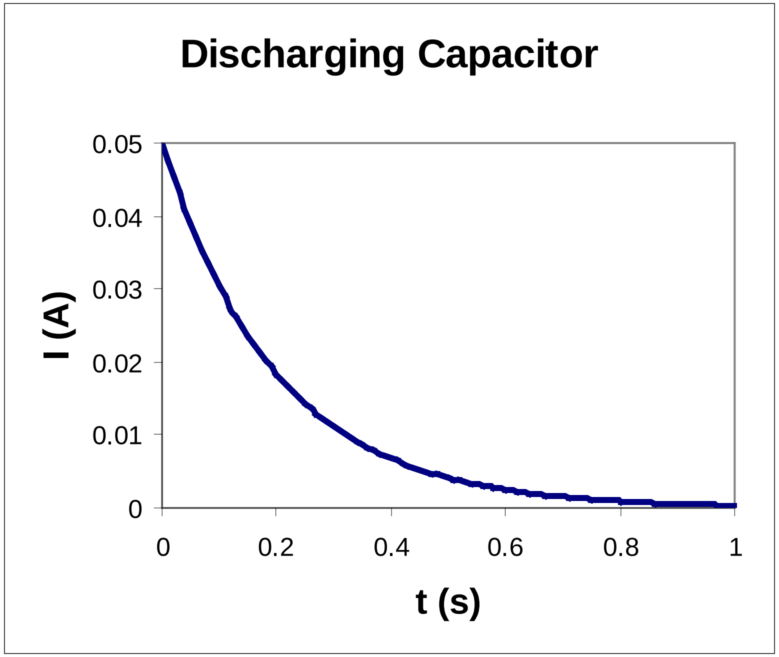
same equation
as for charging

Disharging a capacitor; summary:

$$\boxed{q(t) = Q_0\, e^{-\frac{t}{RC}}}$$

$$I(t) = I_0 e^{-\frac{t}{RC}}$$

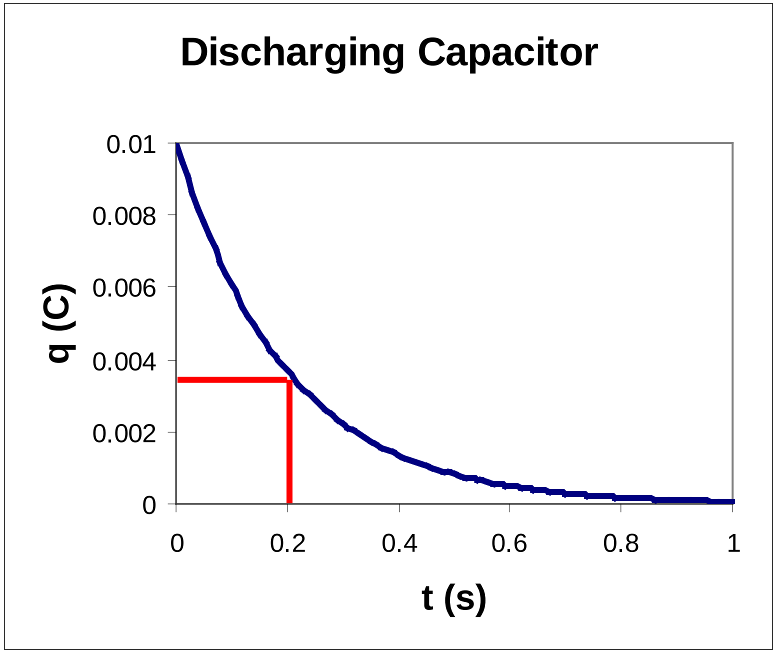


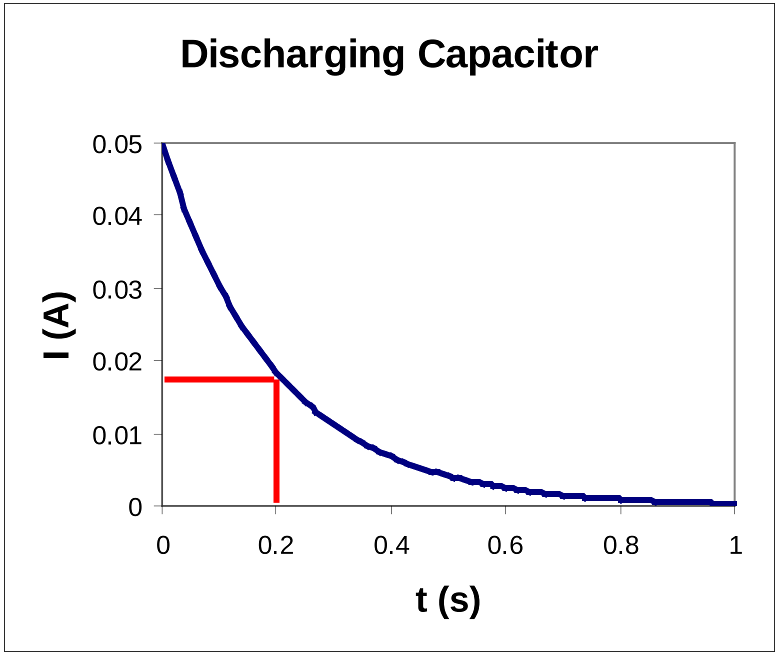


Sample plots with ε=10 V, R=200 Ω, and C=1000 μF.
RC=0.2 s

In a time t=RC, the capacitor discharges to Qe^{-1} or 37% of its capacity...

...and the current drops to $I_{max}(e^{-1})$ or 37% of its maximum.





RC=0.2 s

Notes

$$I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

This is for charging a capacitor. $\varepsilon/R = I_0 = I_{max}$ is the initial current, and depends on the charging emf and the resistor.

$$I(t) = I_0 e^{-\frac{t}{RC}}$$

This is for discharging a capacitor. $I_0 = Q/RC$, and depends on how much charge Q the capacitor started with.

I_0 for charging is equal to I_0 for discharging only if the discharging capacitor was fully charged.

SUMMARY

$$Q(t) = CV(t)$$

This is always true for a capacitor.

$$q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)$$

Q_{final} = CV, where V is the potential difference of the charging emf.

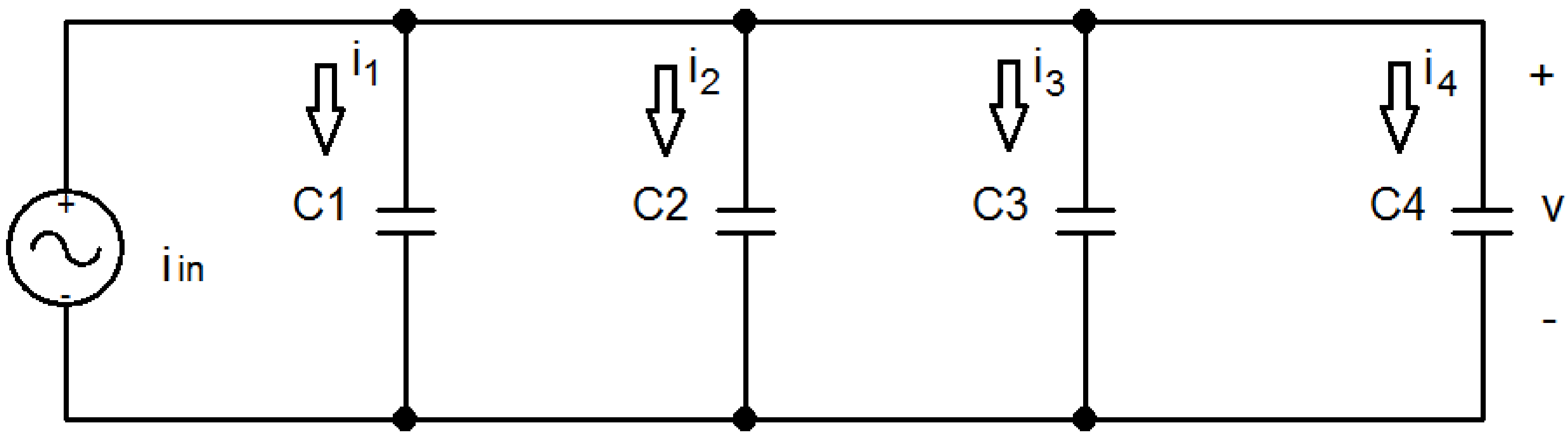
$$q(t) = Q_0\, e^{-\frac{t}{RC}}$$

Q_0 is the charge on the capacitor at the start of discharge. $Q_0 = C\varepsilon$ only if you let the capacitor charge for a "long time."

$$V = IR$$

Ohm's law applies to resistors, not capacitors. Sometimes you can get away with using this. Better to take dq/dt.

Capacitors in Parallel



C_{eq} for Capacitors in Parallel

$$i_{in} = i_1 + i_2 + i_3 + i_4$$

$$i_1 = C_1 \frac{dv}{dt} \qquad i_2 = C_2 \frac{dv}{dt}$$

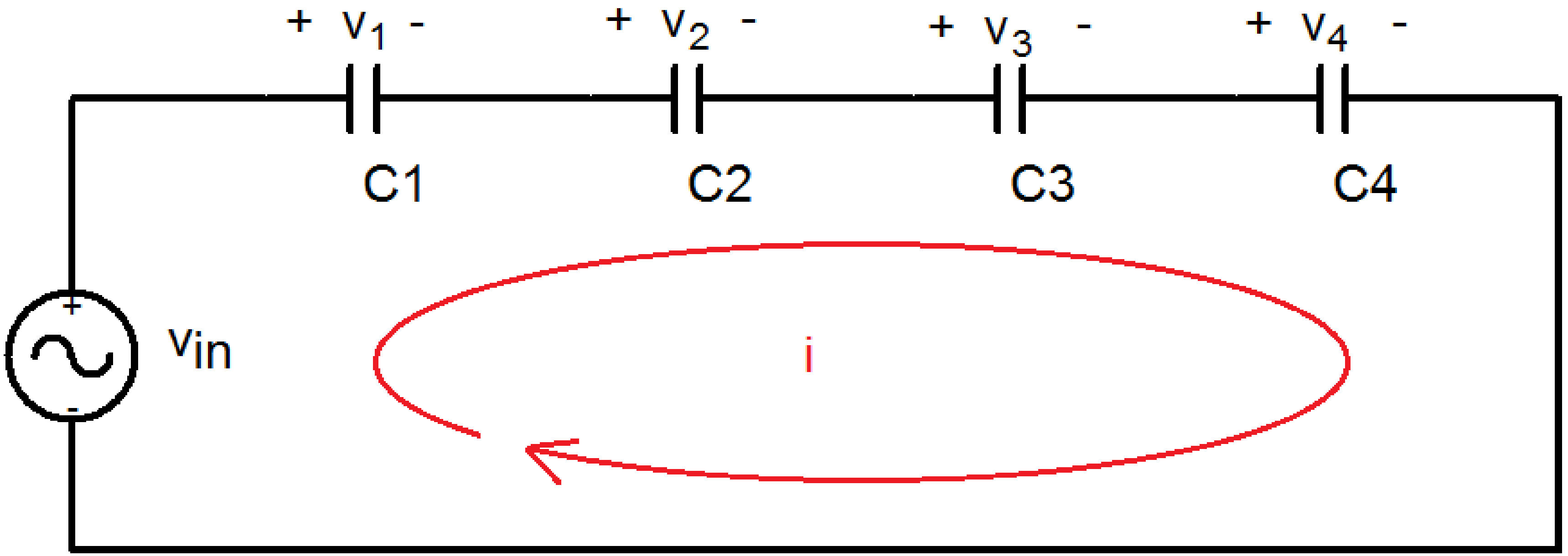
$$i_3 = C_3 \frac{dv}{dt} \qquad \overset{i}{i_4} = C_4 \frac{dv}{dt}$$

$$i_{in} = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + C_4 \frac{dv}{dt}$$

$$i_{in} = C_{eq} \frac{dv}{dt}$$

$$\mathrm{C}_{\mathrm{eq}} = C_1 + C_2 + C_3 + C_4$$

Capacitors in Series



C_{eq} for Capacitors in Series

$$v_{in} = v_1 + v_2 + v_3 + v_4$$

$$v_1 = \frac{1}{C_1}\int_{t_o}^{t_1} \mathrm{idt} \qquad v_2 = \frac{1}{C_2}\int_{t_o}^{t_1} \mathrm{idt}$$

i

$$v_3 = \frac{1}{C_3}\int_{t_o}^{t_1} \mathrm{idt} \qquad v_4 = \frac{1}{C_4}\int_{t_o}^{t_1} \mathrm{idt}$$

$$v_{in} = \frac{1}{C_1}\int_{t_o}^{t_1} \mathrm{idt} + \frac{1}{C_2}\int_{t_o}^{t_1} \mathrm{idt} + \frac{1}{C_3}\int_{t_o}^{t_1} \mathrm{idt} + \frac{1}{C_4}\int_{t_o}^{t_1} \mathrm{idt}$$

$$v_{in} = \frac{1}{C_{eq}}\int_{t_o}^{t_1} \mathrm{idt}$$

$$\mathrm{C_{eq}} = \left[(1/C_1) + (1/C_2) + (1/C_3) + (1/C_4)\right]^{-1}$$

General Equations for C_{eq}

Parallel Combination

- If P capacitors are in parallel, then

$$C_{eq} = \sum_{p=1}^{P} C_P$$

Series Combination

- If S capacitors are in series, then:

$$C_{eq} = \left[\sum_{s=1}^{S} \frac{1}{C_s} \right]^{-1}$$

Summary

- Capacitors are energy storage devices.
- An ideal capacitor act like an open circuit at steady state when a DC voltage or current has been applied.
- The voltage across a capacitor must be a continuous function; the current flowing through a capacitor can be discontinuous.

$$i_C = C\frac{dv_C}{dt} \qquad v_C = \frac{1}{C}\int_{t_o}^{t_1} i_C\,dt$$

- The equations for equivalent capacitance for

capacitors in parallel capacitors in series

$$C_{eq} = \sum_{p=1}^{P} C_P$$

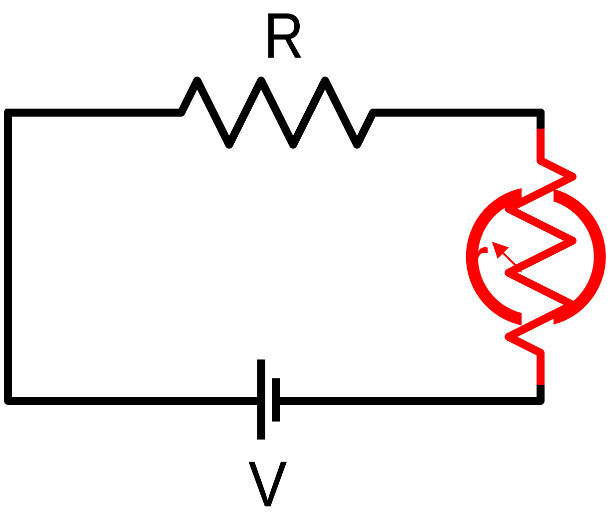
$$C_{eq} = \left[\sum_{s=1}^{S}\frac{1}{C_s}\right]^{-1}$$

Measuring Instruments

Measuring Instruments: Ammeter

You know how to calculate the current in this circuit:

$$I = \frac{V}{R}.$$



If you don't know V or R, you can measure I with an ammeter.

Any ammeter has a resistance r. The current you measure is

$$I = \frac{V}{R + r}.$$

To minimize error the ammeter resistance r should very small.

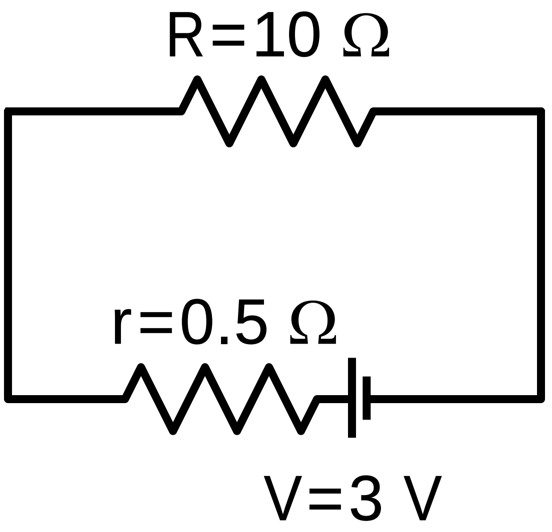
Example: an ammeter of resistance 10 mΩ is used to measure the current through a 10 Ω resistor in series with a 3 V battery that has an internal resistance of 0.5 Ω. What is the percent error caused by the nonzero resistance of the ammeter?

Actual current:

$$I = \frac{V}{R + r}$$

$$I = \frac{3}{10 + 0.5}$$

$$I = 0.286\ \text{A} = 286\ \text{mA}$$



You might see the symbol ε used instead of V.

Current with ammeter:

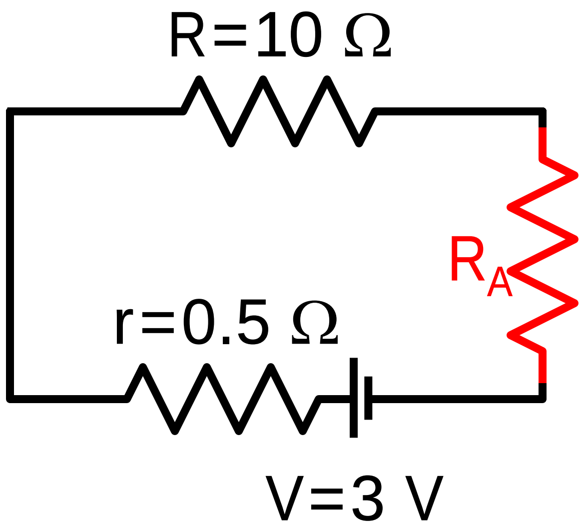
$$I = \frac{V}{R + r + R_A}$$

$$I = \frac{3}{10 + 0.5 + 0.01}$$

$$I = 0.285\,\text{A} = 285\,\text{mA}$$

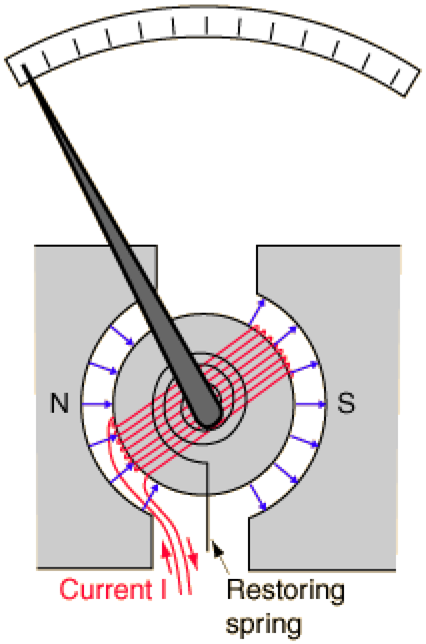
$$\%\,\text{Error} = \frac{0.286 - 0.285}{0.286} \times 100$$

$$\%\,\text{Error} = 0.3\,\%$$



A Galvanometer

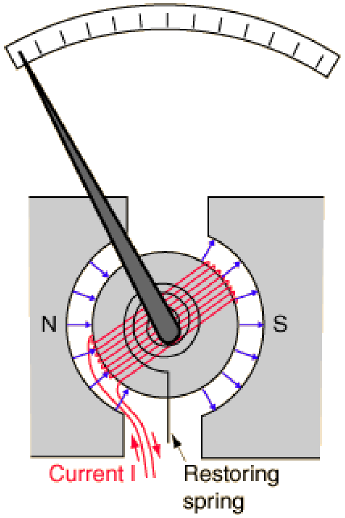
When a current is passed through a coil connected to a needle, the coil experiences a torque and deflects.



http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/galvan.html#c1

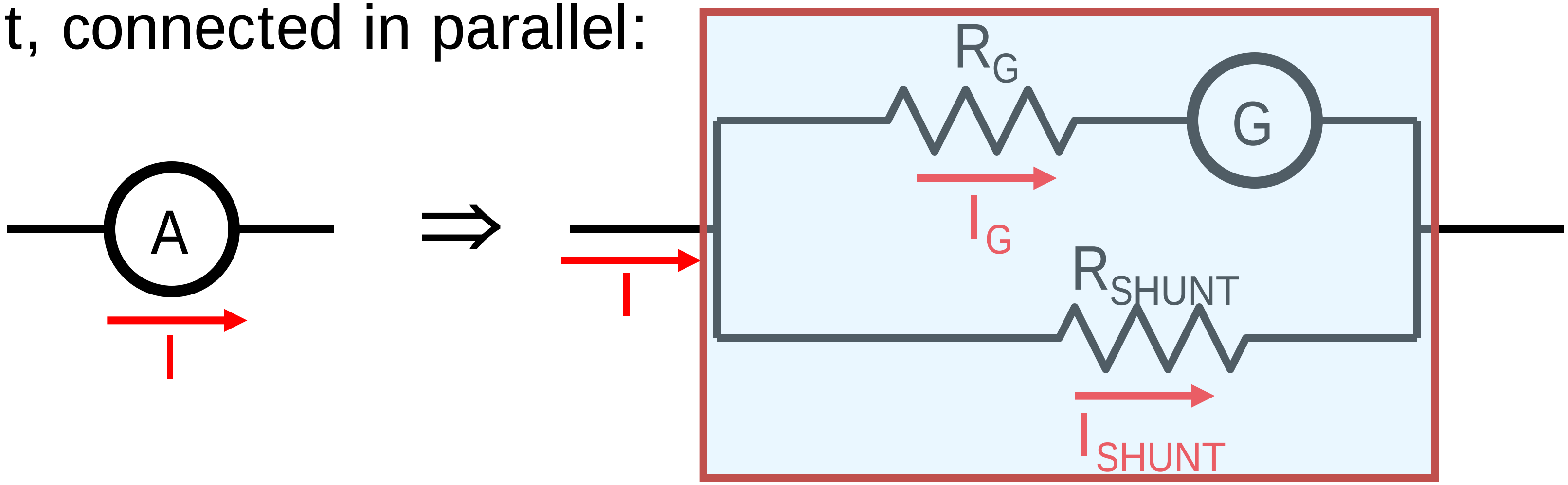
An ammeter (and a voltmeter) is based on a galvanometer.

For now, all you need to know is that the deflection of the galvanometer needle is proportional to the current in the coil (red).



A typical galvanometer has a resistance of a few tens of ohms.

A galvanometer-based ammeter uses a galvanometer and a shunt, connected in parallel:

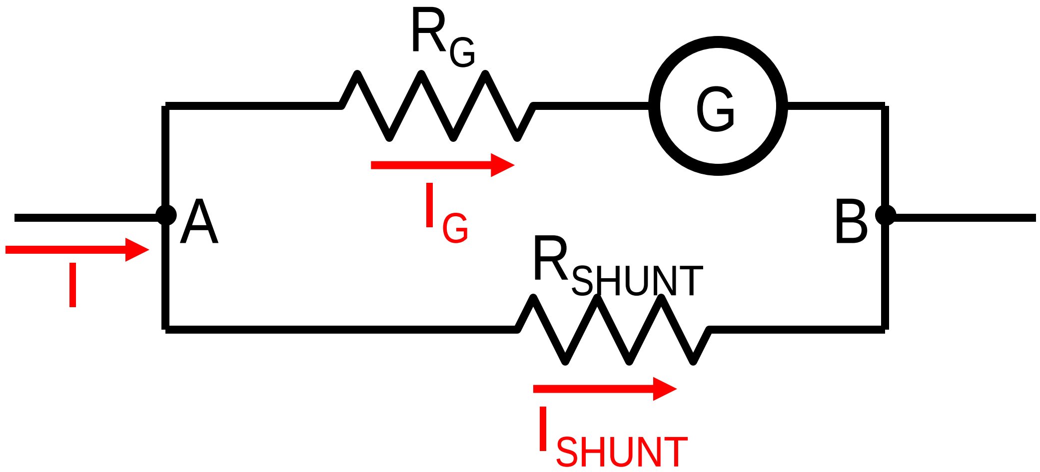


Everything inside the blue box is the ammeter.

The resistance of the ammeter is

$$\frac{1}{R_A} = \frac{1}{R_G} + \frac{1}{R_{SHUNT}}$$

$$R_A = \frac{R_G \, R_{SHUNT}}{R_G + R_{SHUNT}}$$



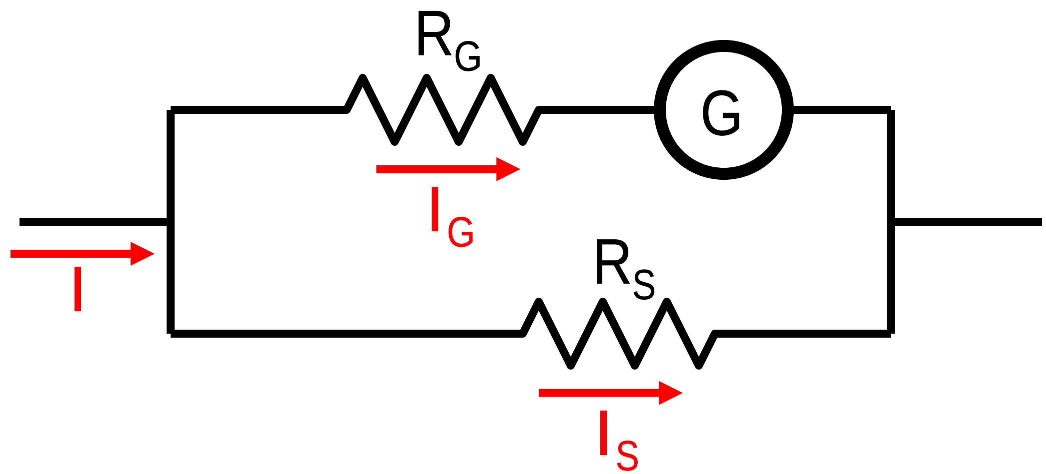
Homework hint: “the galvanometer reads 1A full scale” means a current of I_G=1A produces a full-scale deflection of the galvanometer needle. The needle deflection is proportional to the current I_G.

If you want the ammeter shown to read 5A full scale, then the selected R_{SHUNT} must result in I_G=1A when I=5A. In that case, what are I_{SHUNT} and V_{AB} (=V_{SHUNT})?

Example: what shunt resistance is required for an ammeter to have a resistance of 10 mΩ, if the galvanometer resistance is 60 Ω?

$$\frac{1}{R_A} = \frac{1}{R_G} + \frac{1}{R_S}$$

$$\frac{1}{R_S} = \frac{1}{R_A} - \frac{1}{R_G}$$



$$R_S = \frac{R_G\, R_A}{R_G - R_A} = \frac{(60)(.01)}{60 - .01} = 0.010\ \Omega$$

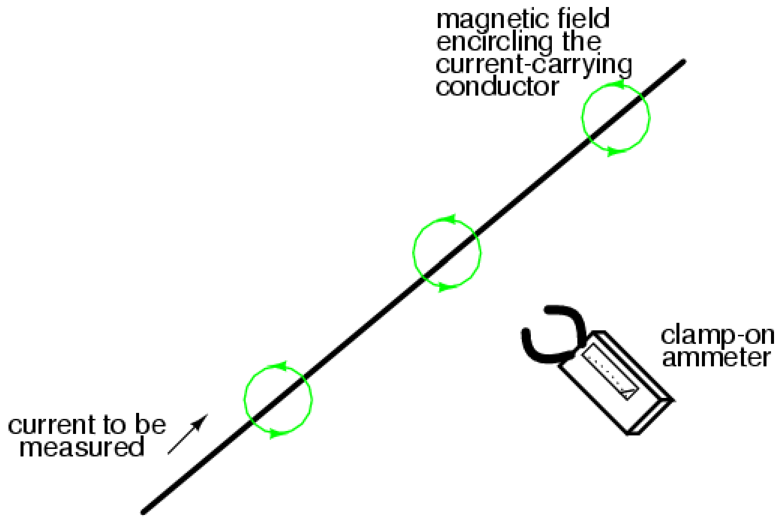
(actually 0.010002 Ω)

The shunt resistance is chosen so that I_G does not exceed the maximum current for the galvanometer and so that the effective resistance of the ammeter is very small.

$$R_S = \frac{R_G\, R_A}{R_G - R_A} = \frac{(60)(.01)}{60 - .01} = 0.010\,\Omega$$

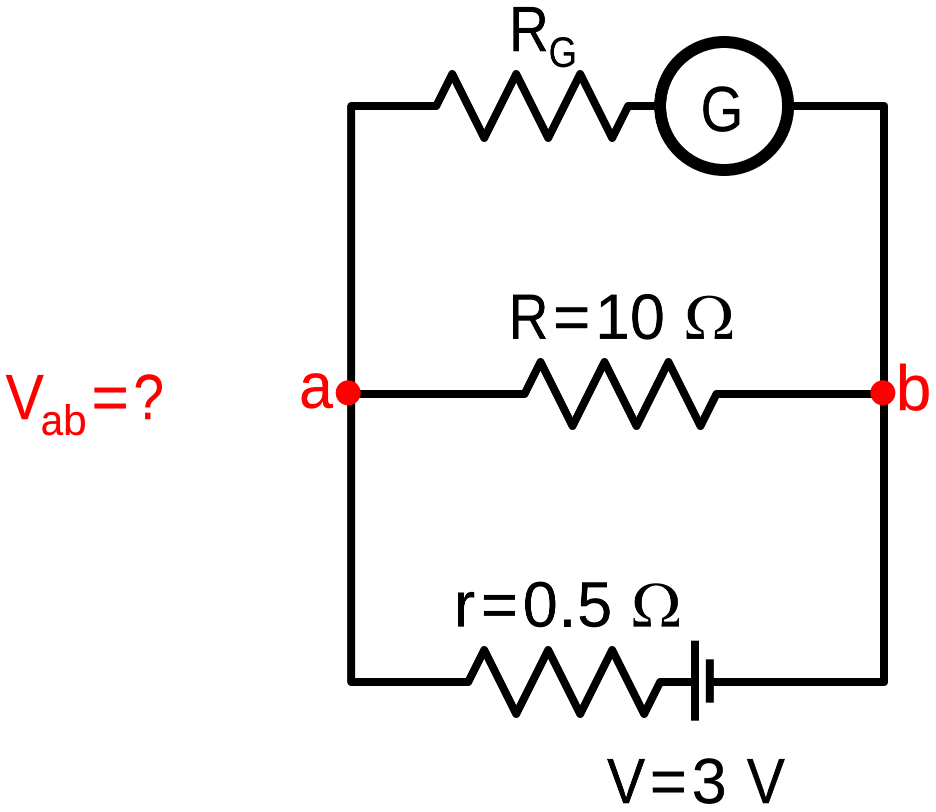
To achieve such a small resistance, the shunt is probably a large-diameter wire or solid piece of metal.

Web links: ammeter design, ammeter impact on circuit, clamp-on ammeter (based on principles we will soon be studying).



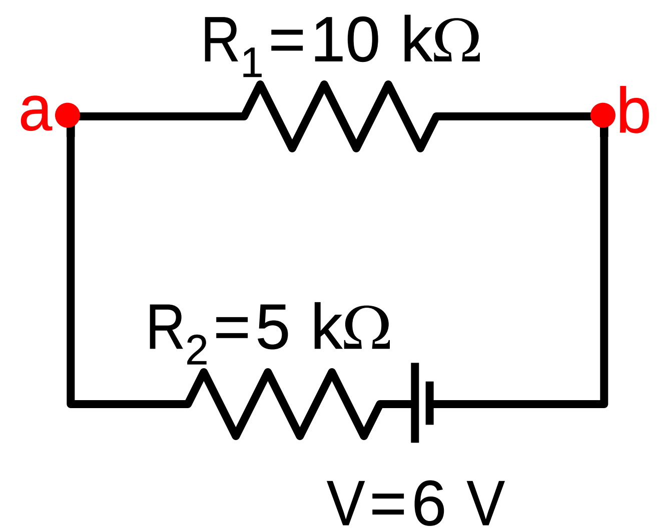
Measuring Instruments: Voltmeter

You can measure a voltage by placing a galvanometer in parallel with the circuit component across which you wish to measure the potential difference.



Example: a galvanometer of resistance 60 Ω is used to measure the voltage drop across a 10 kΩ resistor in series with a 6 V battery and a 5 kΩ resistor (neglect the internal resistance of the battery). What is the percent error caused by the nonzero resistance of the galvanometer?

First calculate the actual voltage drop.



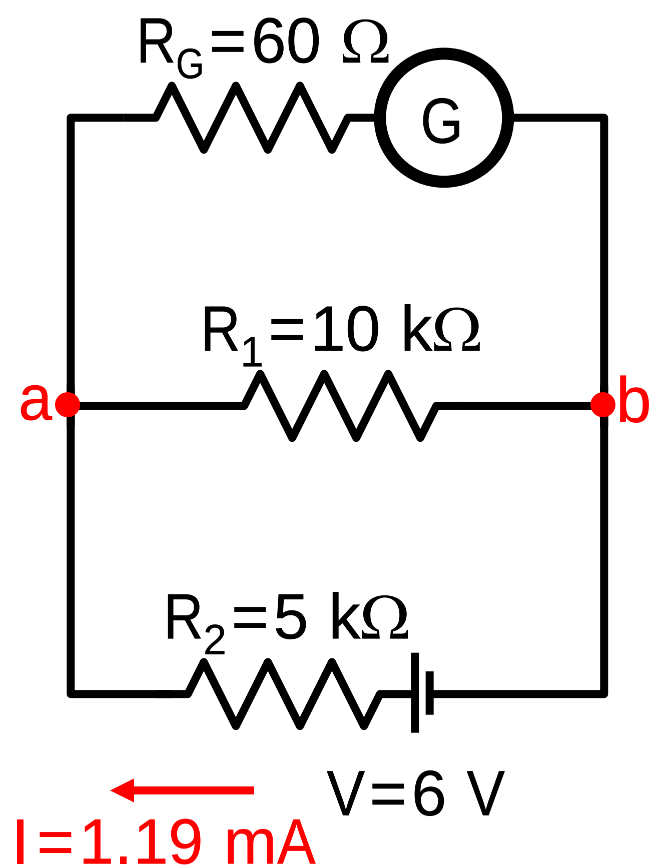
$$R_{eq} = R_1 + R_2 = 15\times10^3\ \Omega$$

$$I = \frac{V}{R_{eq}} = \frac{6\ V}{15\times10^3\ \Omega} = 0.4\times10^{-3}\ A$$

$$V_{ab} = IR = \left(0.4\times10^{-3}\right)\left(10\times10^3\ \Omega\right) = 4\ V$$

The measurement is made with the galvanometer.

60 Ω and 10 kΩ resistors in parallel are equivalent to an 59.6 Ω resistor. The total equivalent resistance is 5059.6 Ω, so 1.19×10^{-3} A of current flows from the battery.

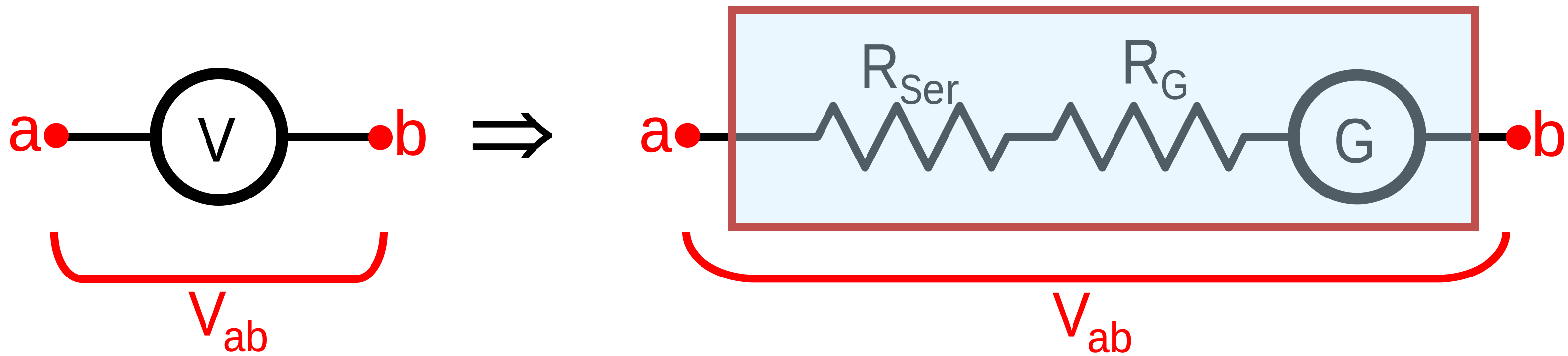


The voltage drop from a to b is then measured to be $6-(1.19\times10^{-3})(5000)=0.07$ V.

The percent error is.

$$\%\ \text{Error} = \frac{4-.07}{4}\times 100 = 98\%$$

To reduce the percent error, the device being used as a voltmeter must have a very large resistance, so a voltmeter can be made from galvanometer in series with a large resistance.

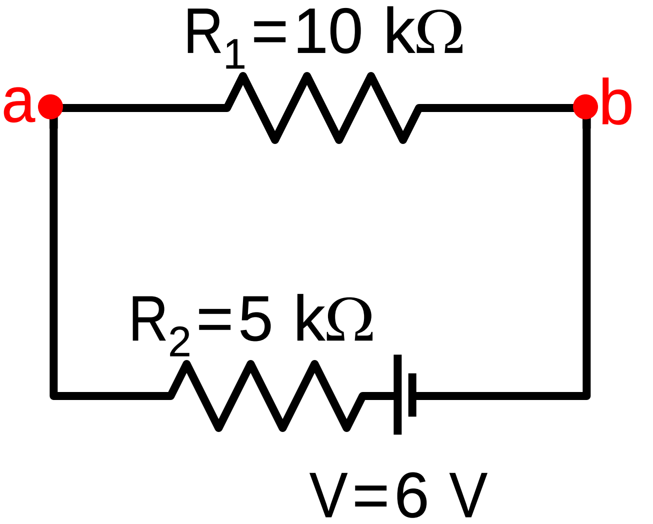


Everything inside the blue box is the voltmeter.

Homework hints: "the galvanometer reads 1A full scale" would mean a current of I_G=1A would produce a full-scale deflection of the galvanometer needle.

If you want the voltmeter shown to read 10V full scale, then the selected R_{Ser} must result in I_G=1A when V_{ab}=10V.

Example: a voltmeter of resistance 100 kΩ is used to measure the voltage drop across a 10 kΩ resistor in series with a 6 V battery and a 5 kΩ resistor (neglect the internal resistance of the battery). What is the percent error caused by the nonzero resistance of the voltmeter?



We already calculated the actual voltage drop (3 slides back).

$$V_{ab} = IR = \left(0.4\times10^{-3}\right)\left(10\times10^{3}\ \Omega\right) = 4\ \text{V}$$

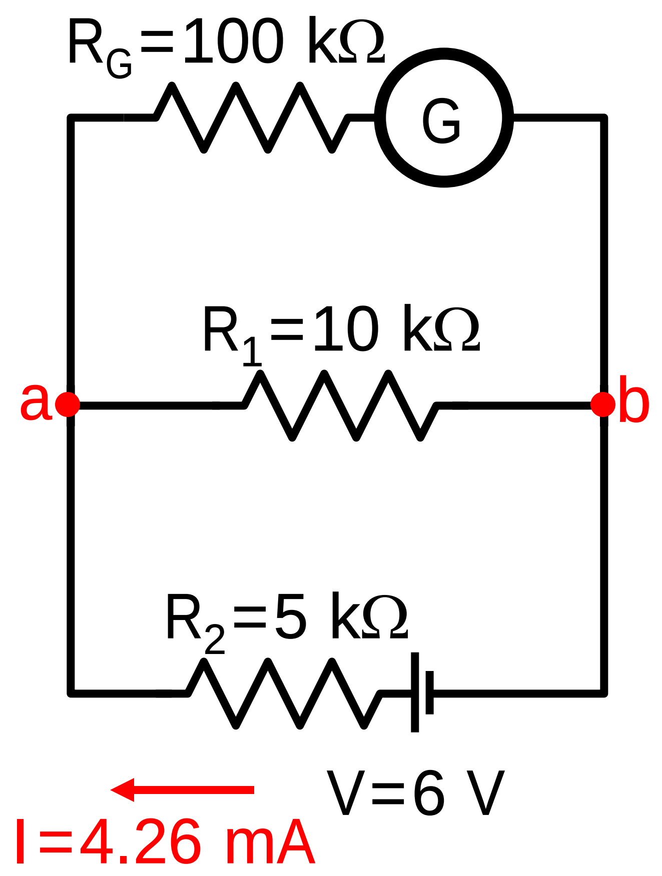
The measurement is now made with the voltmeter.

100 kΩ and 10 kΩ resistors in parallel are equivalent to an 9090 Ω resistor. The total equivalent resistance is 14090 Ω, so 4.26×10^{-3} A of current flows from the battery.

The voltage drop from a to b is then measured to be $6-(4.26\times10^{-3})(5000)=3.9$ V.

The percent error is.

$$\%\ \text{Error} = \frac{4-3.9}{4}\times 100 = 2.5\%$$



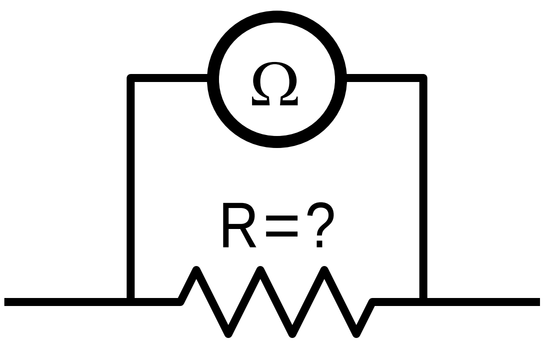
Not great, but much better. Larger R_{ser} is needed for high accuracy.

Measuring Instruments: Ohmmeter

An ohmmeter measures resistance. An ohmmeter is made from a galvanometer, a series resistance, and a battery.

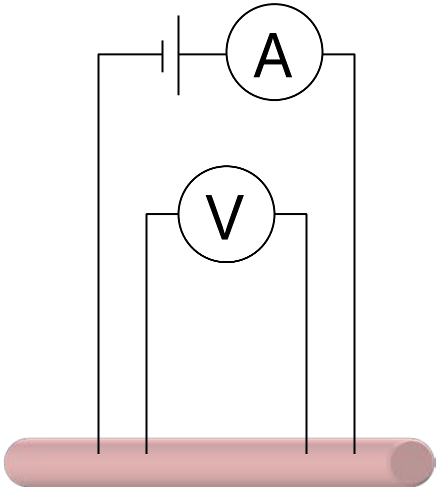
Everything inside the blue box is the ohmmeter.

The ohmmeter is connected in parallel with the unknown resistance **with external power off**. The ohmmeter battery causes current to flow, and Ohm's law is used to determine the unknown resistance.



To measure a really small resistance, an ohmmeter won't work.

Solution: four-point probe.



Measure current and voltage separately, apply Ohm's law.

reference: http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/movcoil.html#c4