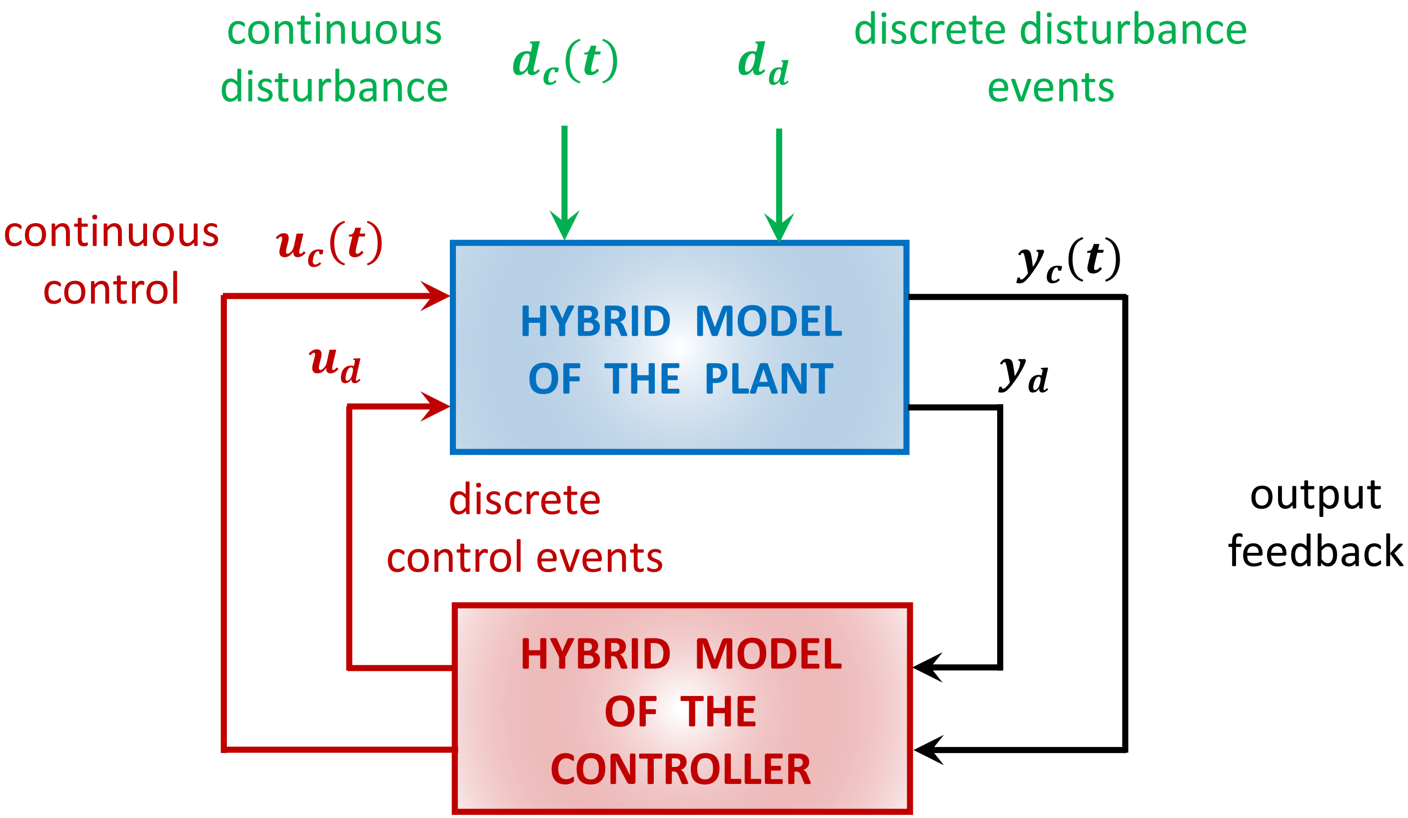
Synthesis of Maximal Controllers

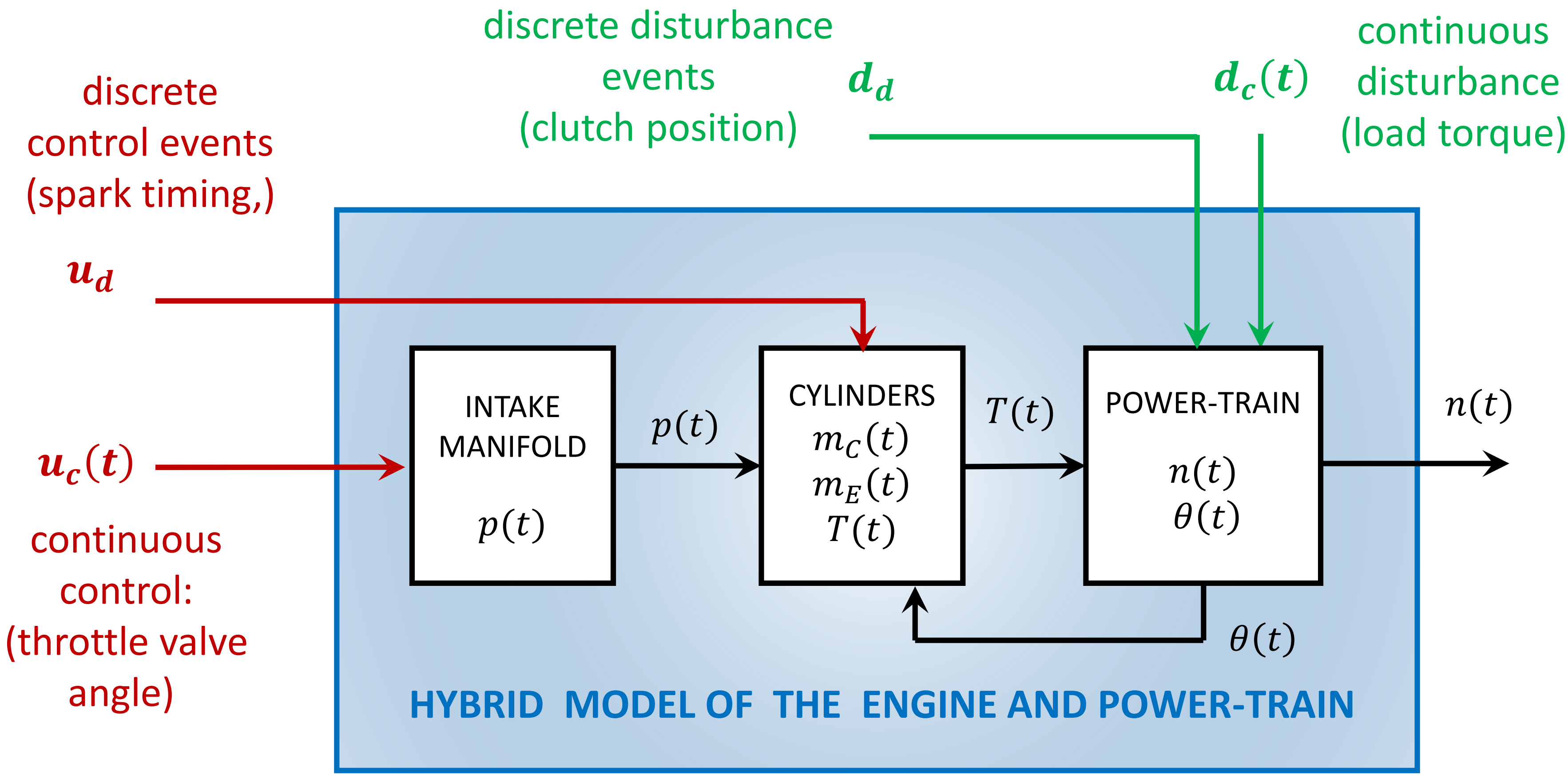
For Hybrid Systems

A. Balluchi, L. Benvenuti, T. Villa, H. Wong-Toi, and A. L. Sangiovanni-Vincentelli.
Controller synthesis for hybrid systems with a lower bound on event separation.
International Journal of Control, 76(12):1171–1200, August 2003.
https://doi.org/10.1080/0020717031000123616



continuous
disturbance
$d_c(t)$
d_d
discrete disturbance
events
continuous
control
$u_c(t)$
HYBRID MODEL
OF THE PLANT
$y_c(t)$
u_d
y_d
discrete
control events
output
feedback
HYBRID MODEL
OF THE
CONTROLLER

Example: Idle speed control of an automotive engine



$p(t)$: intake manifold pressure

$m_i(t)$: air loaded into cylinder in stroke i

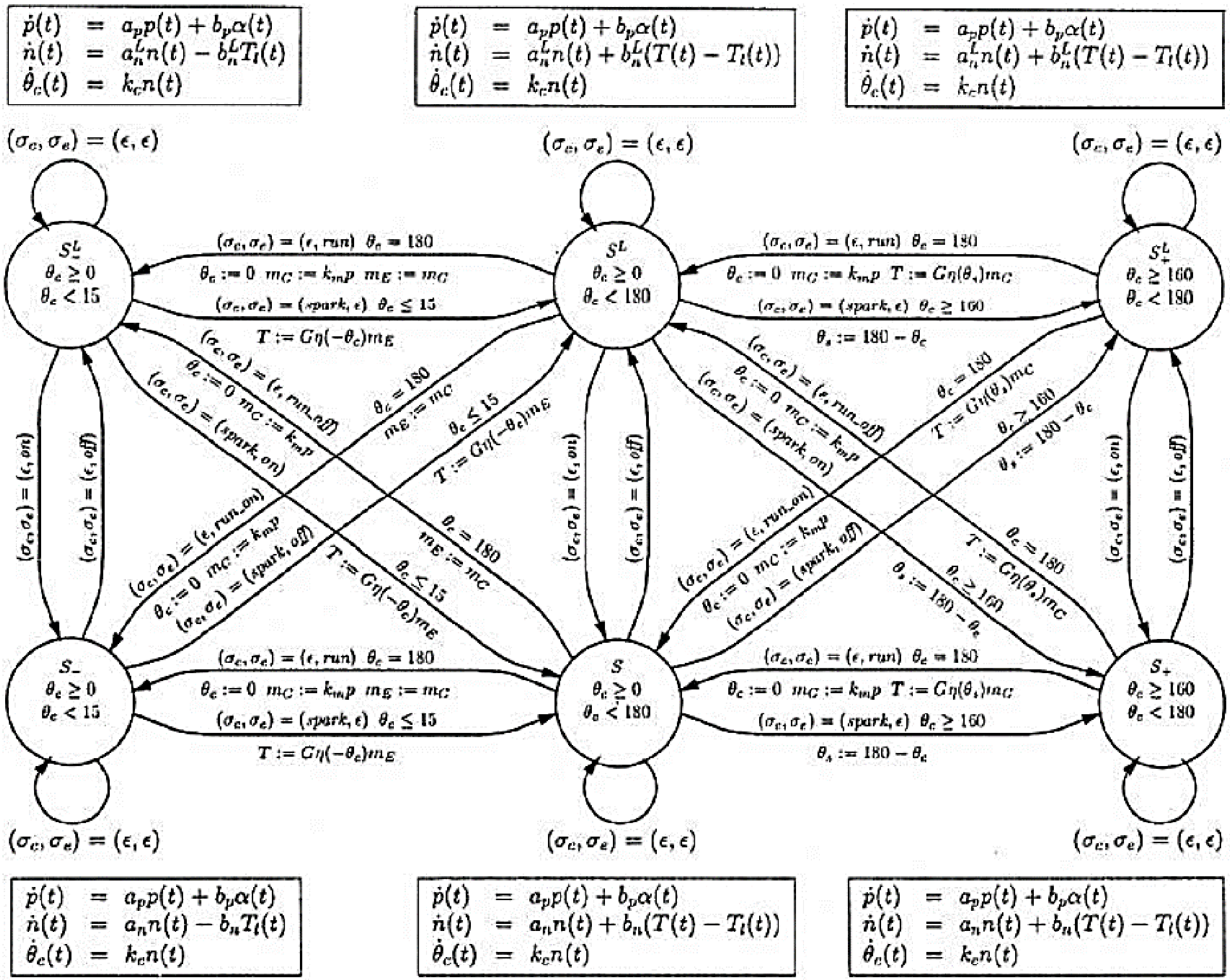
$\theta(t), n(t)$: crankshaft position and speed

$T(t)$: torque generated by the engine

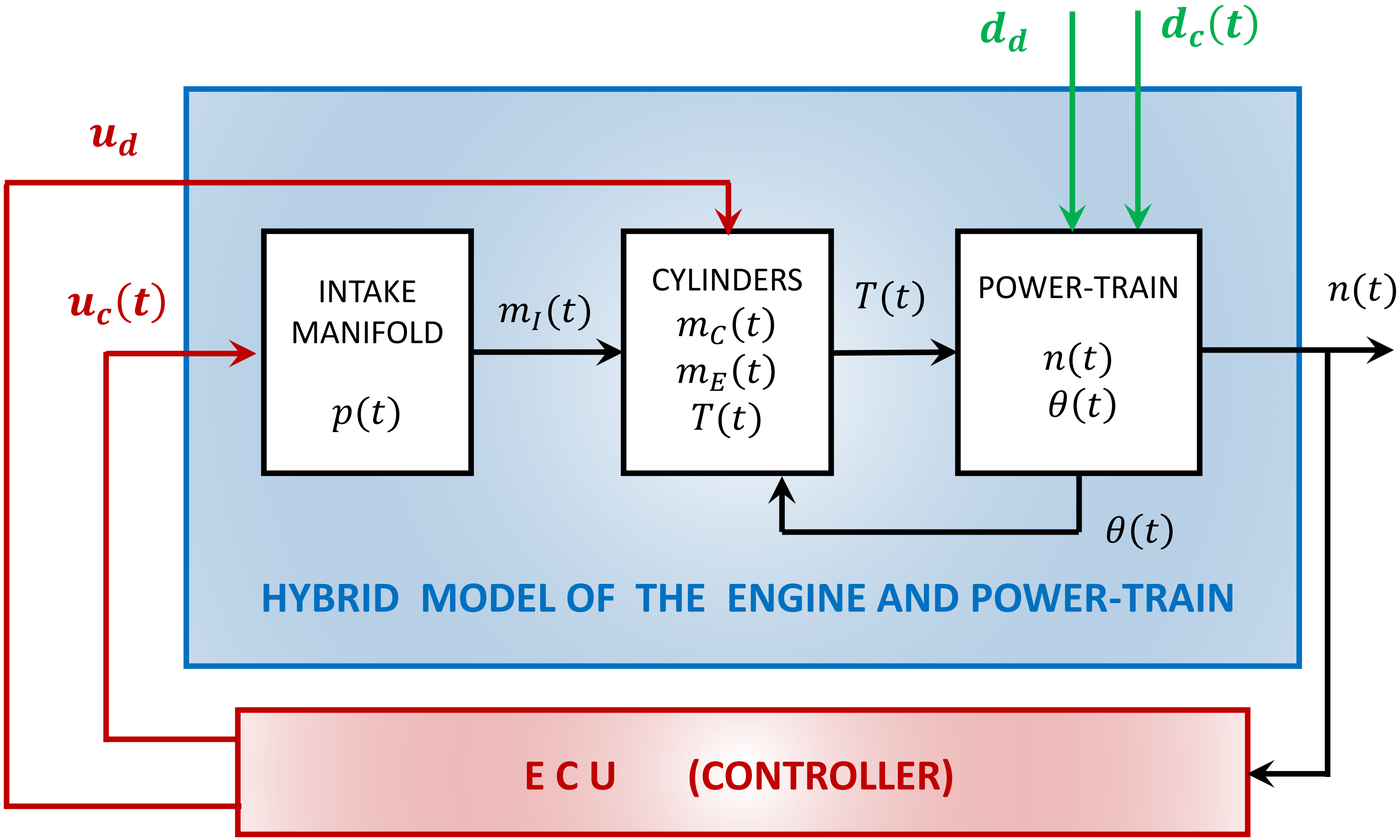
A. Balluchi, et al.

Maximal safe set computation for idle speed control of ana utomotive engine

Hybrid Systems: Computation and Control. Lecture Notes in Computer Science, vol 1790. Springer, Berlin, Heidelberg, 2000. https://doi.org/10.1007/3-540-46430-1_7

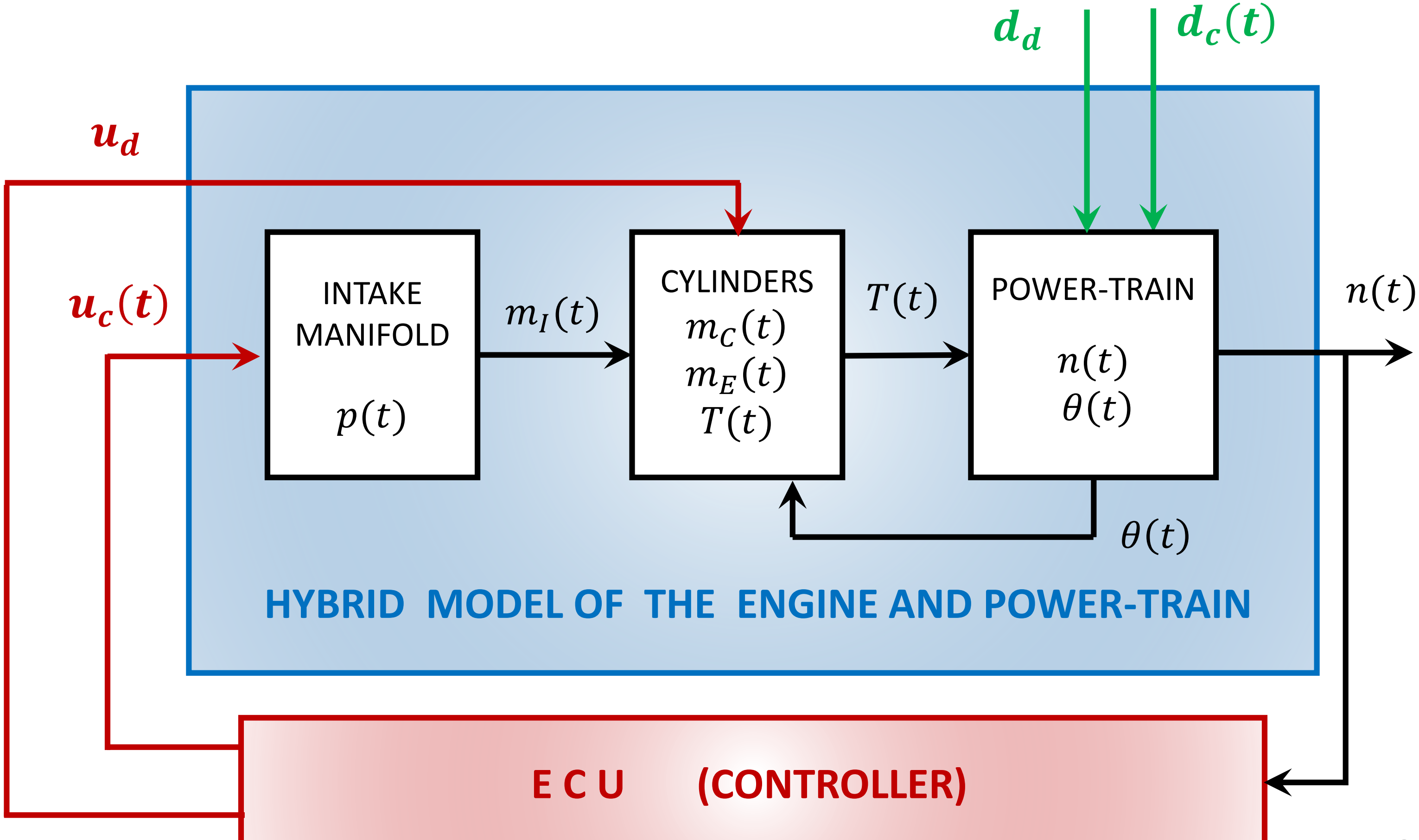


Example: Idle speed control of an automotive engine



Example: Idle speed control of an automotive engine

Find all the control strategies (if any) for the spark timing $\boldsymbol{u_d}$ and throttle valve position $\boldsymbol{u_c(t)} = \alpha(t) \in [0, \alpha^{max}]$, which keep the crankshaft speed $n(t)$ in a given range $[n_0 - \Delta n, n_0 + \Delta n]$, independently of the two disturbances given by the clutch $\boldsymbol{d_d}$ and the load torque $\boldsymbol{d_c(t)} = T_l(t) \in [0, T_l^{max}]$.



Control design for safety specifications

The control objective is to maintain the crankshaft speed $n(t)$ in a given range $[n_0 - \Delta n, n_0 + \Delta n]$, whatever the disturbances happen to be.

- A safety property for a hybrid system is specified by means of a set of $Good$ configurations that do not violate the property.

$$Good = \{Q \times [n_0 - \Delta n, n_0 + \Delta n]\}$$

- A configuration $(q, x) \in Good$ is said ***controllable safe*** $-$ with respect to the safety specification $Good$ $-$ if there exists a controller such that all the trajectories of the closed-loop system, starting from (q, x), remain forever within the set $Good$ for any admissible disturbances.

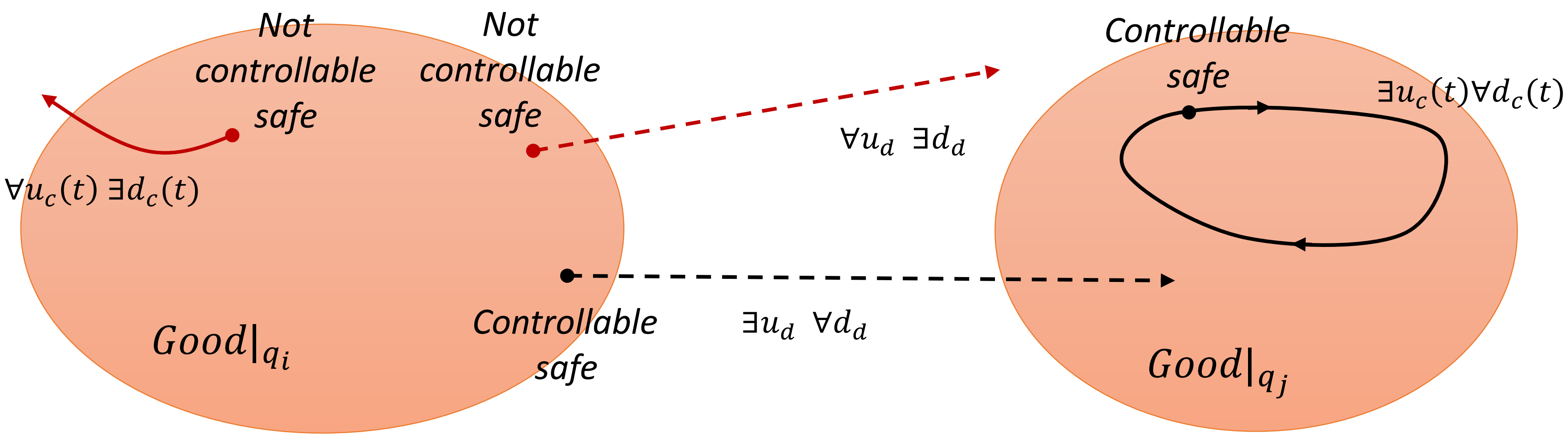
- The **maximal safe set** for a hybrid system and a safety specification $Good$, is the largest set of controllable safe configurations.

A game between control and disturbance

The control objective is to maintain the state (q, x) **inside** the set $Good$, whatever the disturbances happen to be.

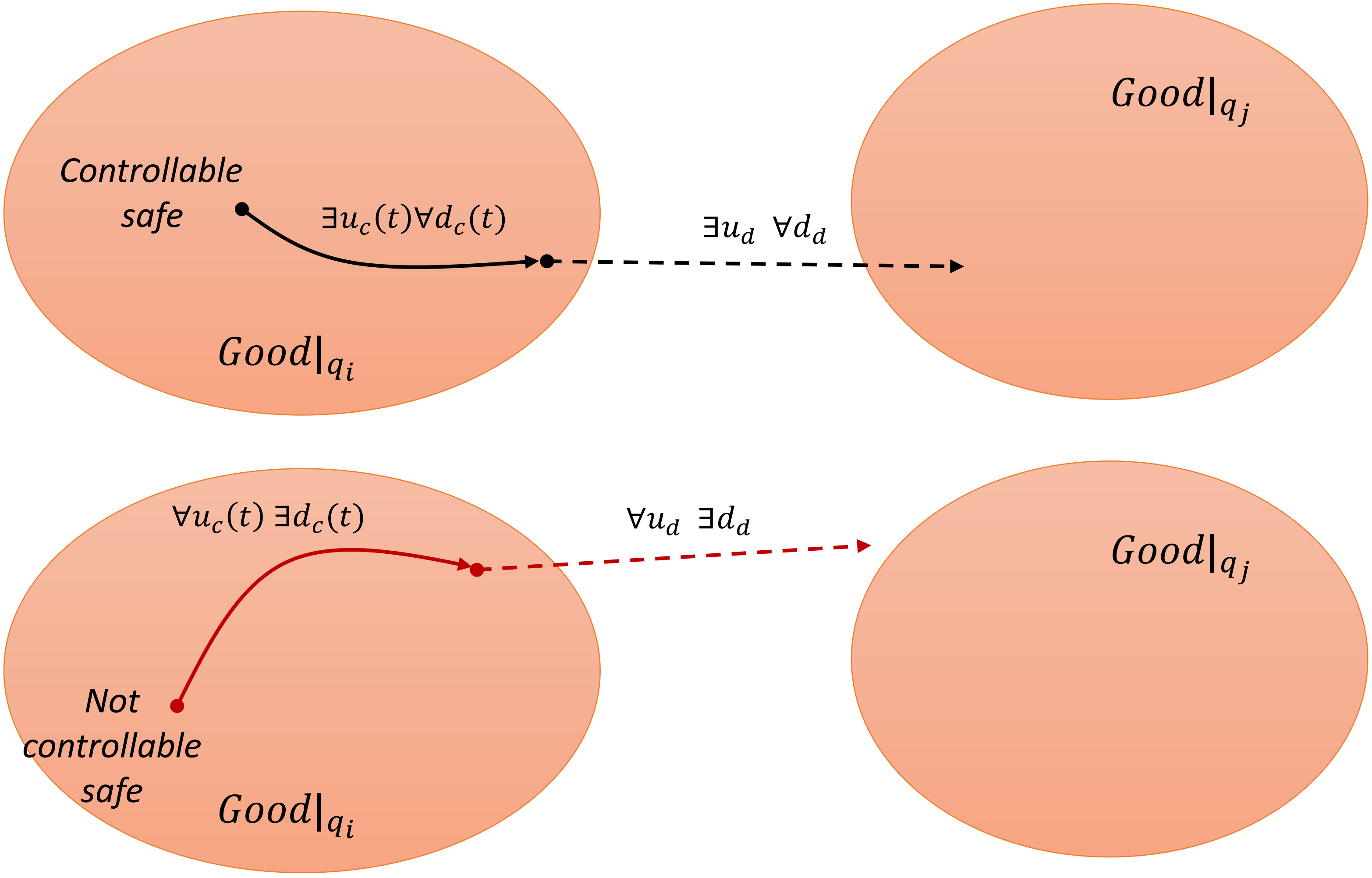
The disturbance objective is to drive the the state (q, x) **outside** the set $Good$.

The two players (control and disturbance) affect both the continuous and the discrete evolution of the system.



A game between control and disturbance

One may think of the interaction between the players as a continuous game with occasional discrete interruptions.



Hybrid system with control and disturbance

A hybrid system H is a collection

$$H = ((Q, X), (U, \Sigma_c), (D, \Sigma_d), Init, (f, \delta))$$

- $Q = \{q_1, q_2, \dots\}$ is the set of **discrete states**
- $X = \mathbb{R}^n$ is the set of **continuous states**
- $U \subseteq \mathbb{R}^m$ is the domain of **continuous control variables**
- Σ_c is the finite set of **discrete control events**
- $D \subseteq \mathbb{R}^p$ is the domain of **continuous disturbance variables**
- Σ_d is the finite set of **discrete disturbance events**
- $Init \subseteq Q \times X$ is the set of **initial states**

- $f: Q \times X \times U \times D \rightarrow \mathbb{R}^n$

  is the **vector field** defining the continuous dynamics

- $\delta: Q \times X \times (\Sigma_c \cup \epsilon) \times (\Sigma_d \cup \epsilon) \rightarrow 2^{Q \times X} / \{\}$

  is the **transition function** defining the discrete dynamics

ϵ is the **null event**, i.e., no discrete event is given.

When no discrete input and disturbance control is given, that is

$$u_d = \epsilon \text{ and } d_d = \epsilon$$

no transition takes place, i.e.,

$$\delta(q, x, \epsilon, \epsilon) = \{(q, x)\}$$

In this case, the location q remains fixed, and the continuous variables $x(t)$ evolve according to the continuous control $u_c(t) \in U$, the continuous disturbance $d_c(t) \in D$, and the continuous dynamics specified by the function f.


HEAT PUMP
Whirlpool
DOOR
APPLIANCES
SAMSUNG
ELECTRIC
STOVE


CONTINUOUS
HEATING CONTROL
$u_c(t) \in [0, U]$


DISCRETE
(on/off)
HEATING
CONTROL
$u_d = on$
$w_h(on) = W$
Whirlpool
SAMSUNG

DISCRETE (on/off) HEATING CONTROL

$$u_d = off$$

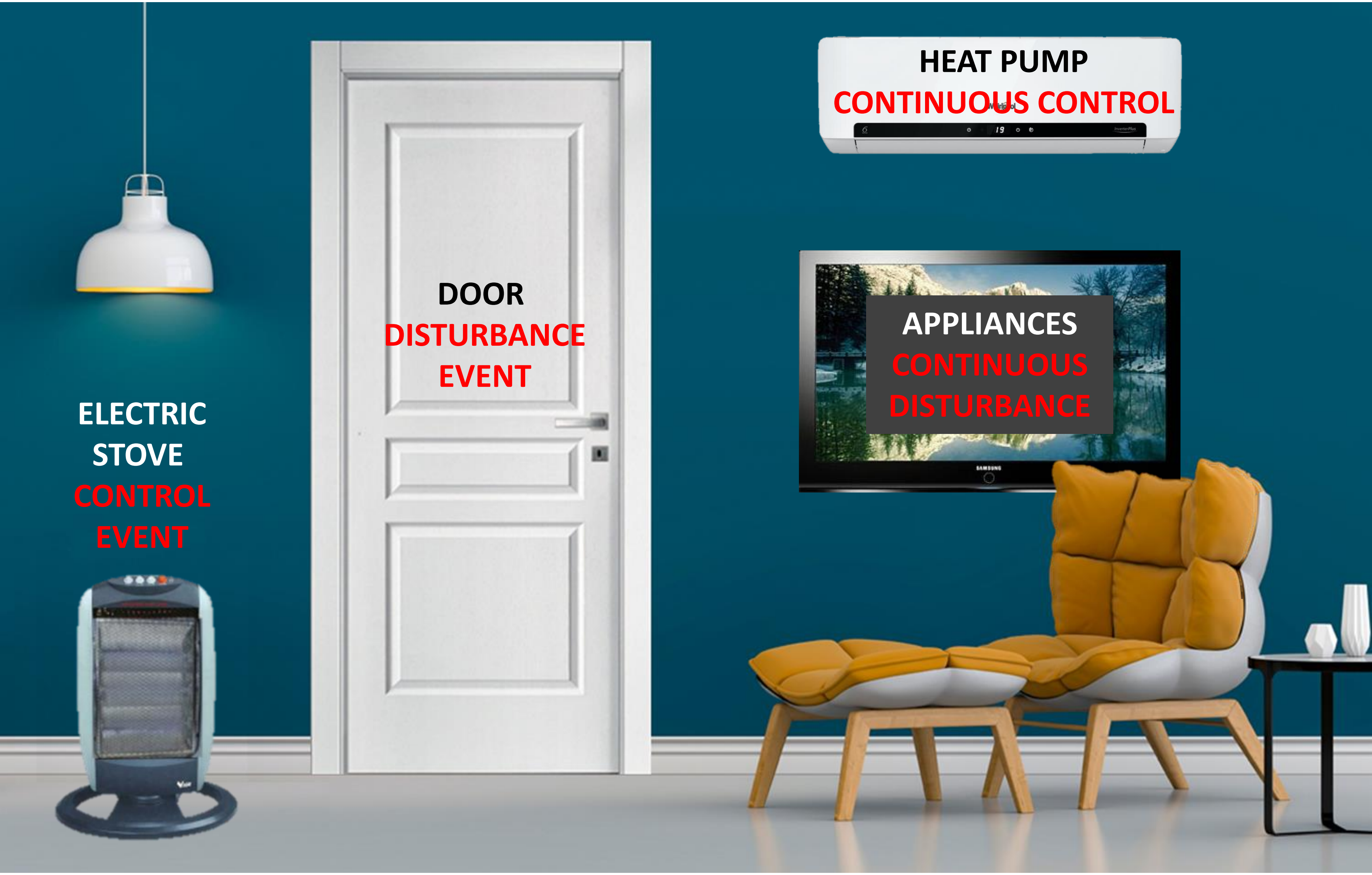
$$w_h(off) = 0$$


APPLIANCES
CONTINUOUS
HEATING DISTURBANCE
$d_c(t) \in [0, D]$

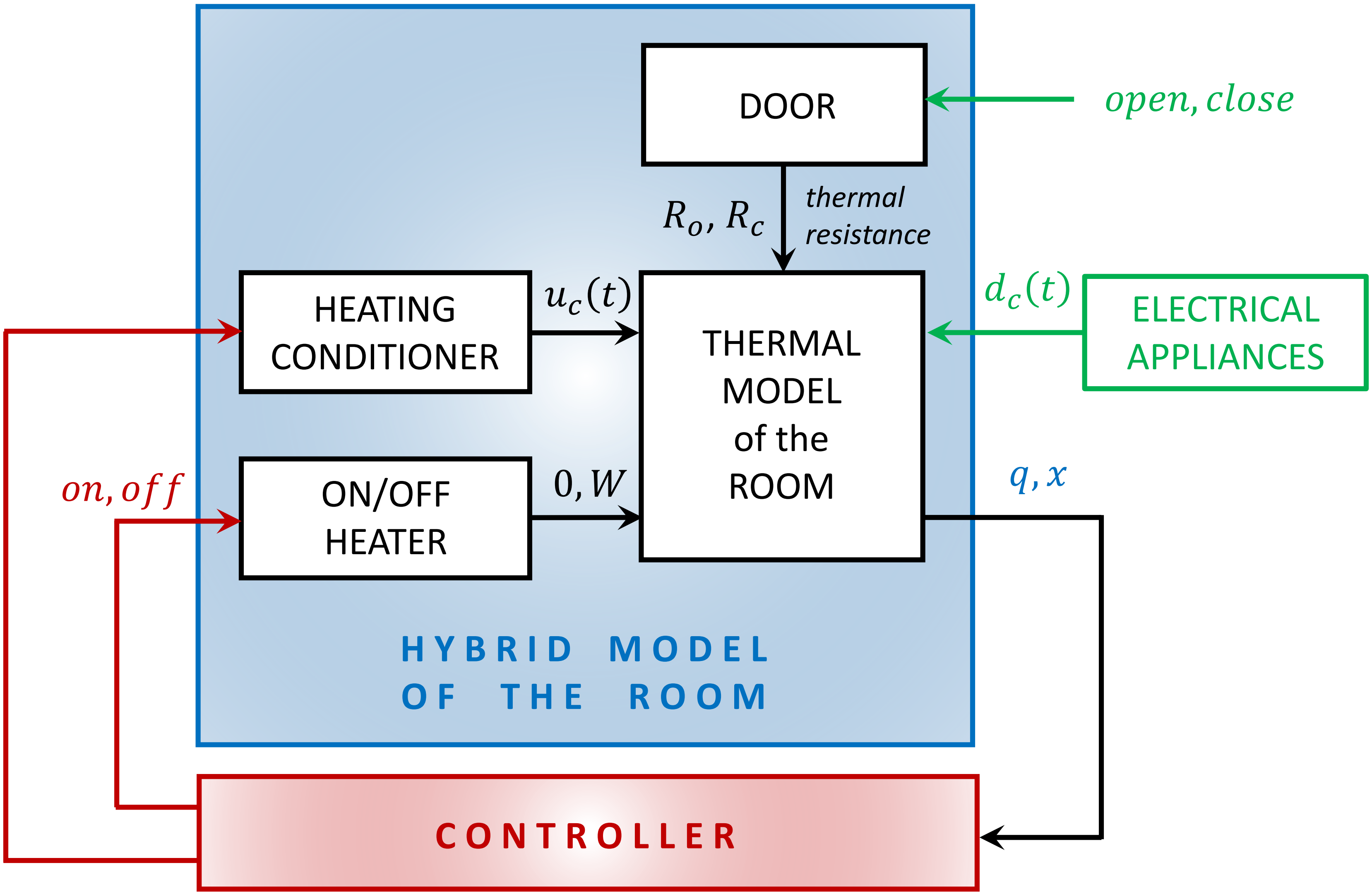
$d_d = open$

The thermal resistance decreases from R_c to R_o

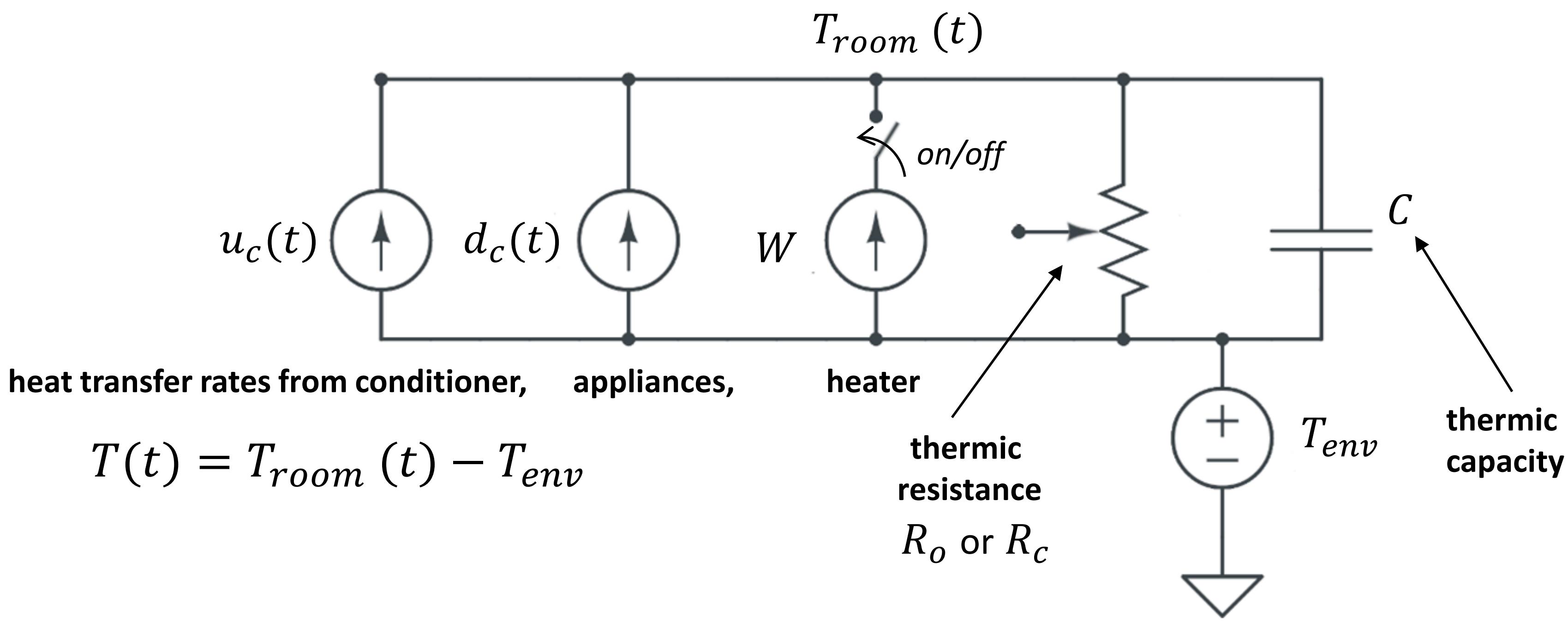
The temperature suddenly decreases ($r < 1$)



The control objective is to maintain the temperature $T_{room}(t)$ of the room in a given range $[T_{min}, T_{max}]$, whatever the disturbances happen to be.

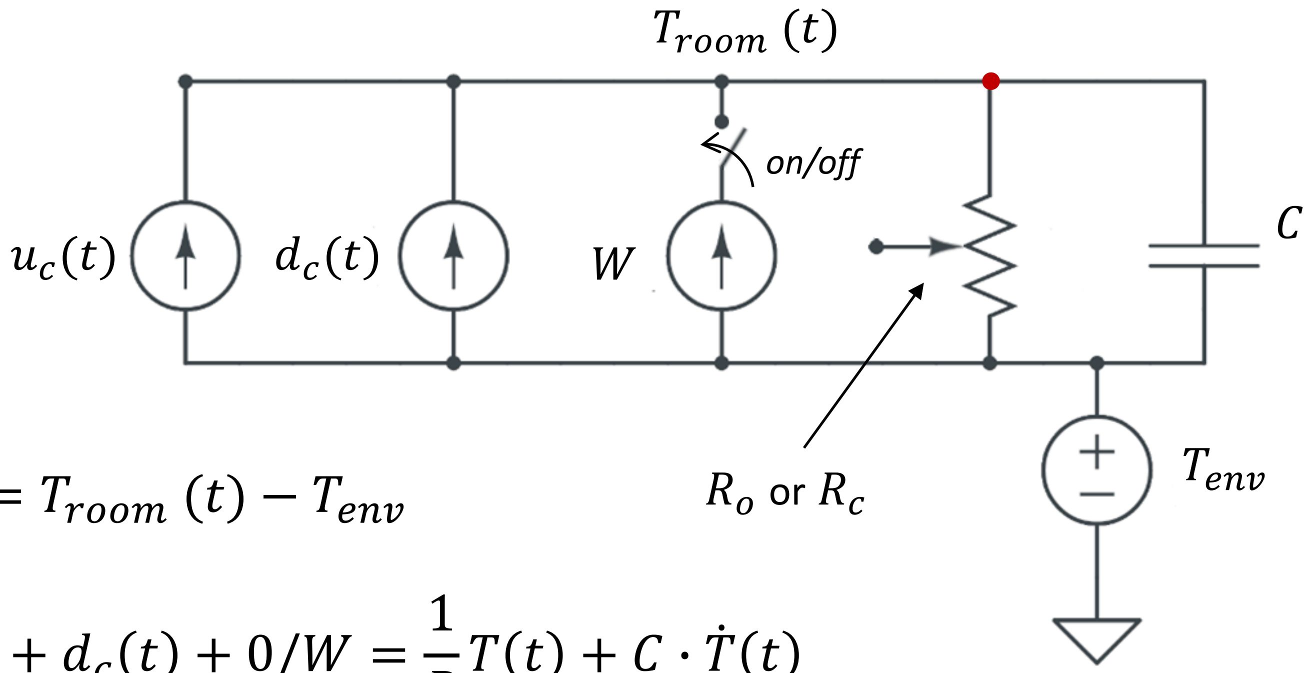

DOOR
open, close
R_o, R_c
thermal resistance
HEATING CONDITIONER
$u_c(t)$
THERMAL MODEL of the ROOM
$d_c(t)$
ELECTRICAL APPLIANCES
on, off
ON/OFF HEATER
$0, W$
q, x
HYBRID MODEL OF THE ROOM
CONTROLLER

Thermal model of the room



$$T(t) = T_{room}(t) - T_{env}$$

Thermal model of the room



$$T(t) = T_{room}(t) - T_{env}$$

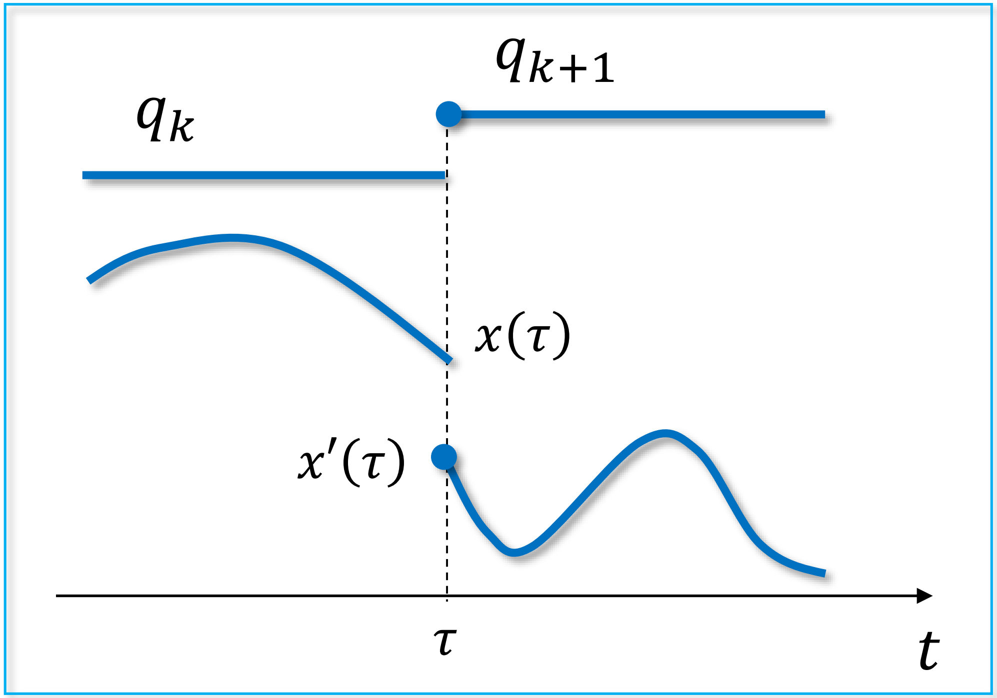
$$u_c(t) + d_c(t) + 0/W = \frac{1}{R} T(t) + C \cdot \dot{T}(t)$$

$$\dot{T}(t) = -\frac{1}{RC} T(t) + \frac{1}{C} u_c(t) + \frac{1}{C} d_c(t) + \frac{0/W}{C}$$

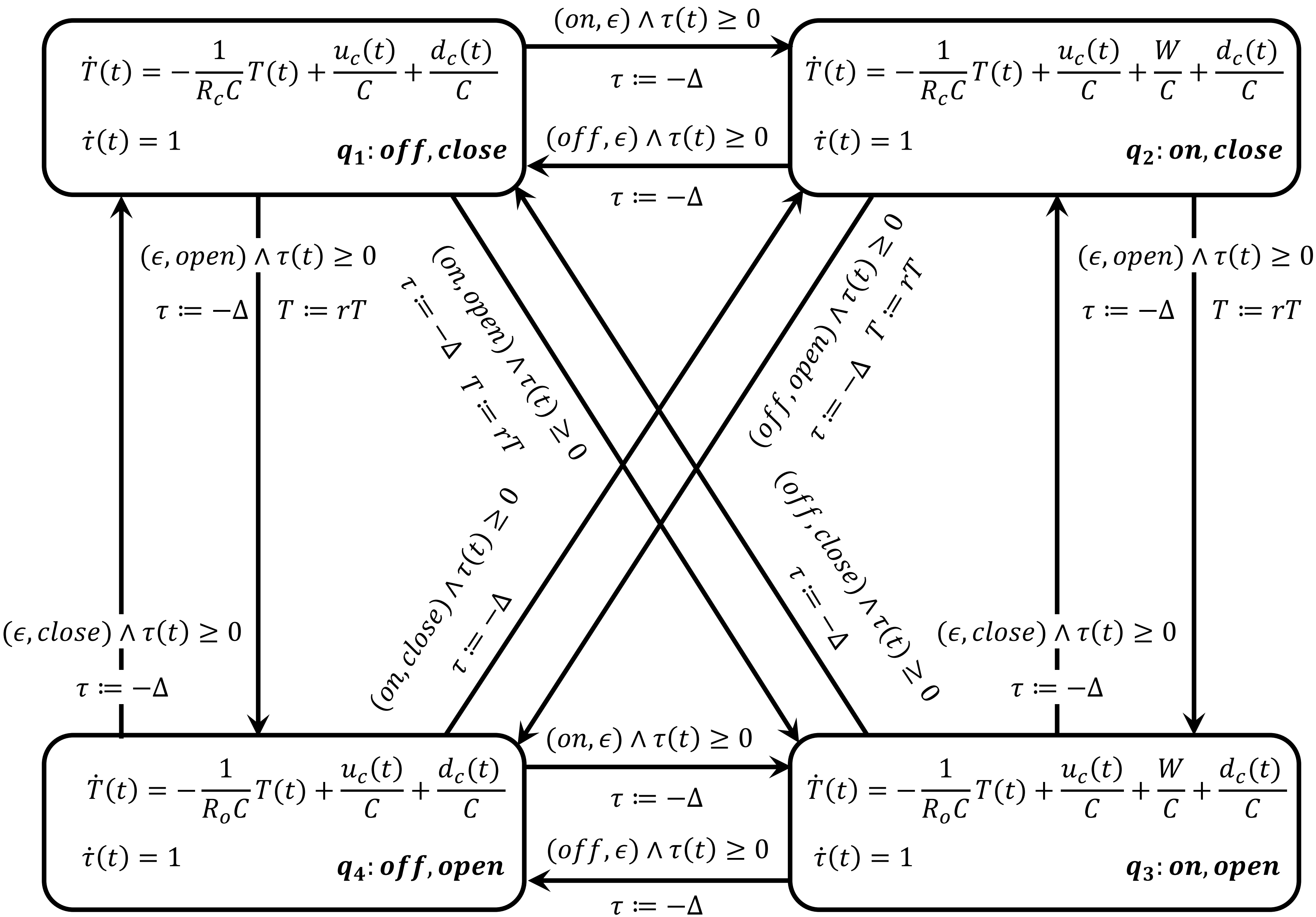
The value of the thermic resistance R depends on whether the door is open or closed

Temperature reset when opening the door

$(q_{k+1}, x'(\tau)) \in \delta(q_k, x(\tau), u_d, open)$ $\qquad T := rT$ with $r < 1$



To prevent the discrete disturbance from dropping the temperature by opening and closing the door over and over again in a short period of time, a minimum interval of time Δ is assumed between two consecutive transitions.



$\dot{T}(t) = -\frac{1}{R_c C}T(t) + \frac{u_c(t)}{C} + \frac{d_c(t)}{C}$
$\dot{\tau}(t) = 1$
$\boldsymbol{q_1}$: ***off, close***
$\dot{T}(t) = -\frac{1}{R_c C}T(t) + \frac{u_c(t)}{C} + \frac{W}{C} + \frac{d_c(t)}{C}$
$\dot{\tau}(t) = 1$
$\boldsymbol{q_2}$: ***on, close***
$\dot{T}(t) = -\frac{1}{R_o C}T(t) + \frac{u_c(t)}{C} + \frac{W}{C} + \frac{d_c(t)}{C}$
$\dot{\tau}(t) = 1$
$\boldsymbol{q_3}$: ***on, open***
$\dot{T}(t) = -\frac{1}{R_o C}T(t) + \frac{u_c(t)}{C} + \frac{d_c(t)}{C}$
$\dot{\tau}(t) = 1$
$\boldsymbol{q_4}$: ***off, open***
$(on, \epsilon) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$
$(off, \epsilon) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$
$(\epsilon, open) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$ $T \coloneqq rT$
$(on, open) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$ $T \coloneqq rT$
$(off, open) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$ $T \coloneqq rT$
$(\epsilon, open) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$ $T \coloneqq rT$
$(on, close) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$
$(off, close) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$
$(\epsilon, close) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$
$(\epsilon, close) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$
$(on, \epsilon) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$
$(off, \epsilon) \wedge \tau(t) \geq 0$
$\tau \coloneqq -\Delta$

A discrete game between control and disturbance

DISCRETE **UNCONTROLLABLE** PREDECESSORS (And the winner is … disturbance)

$$DUPre(S) = \{(q,x) \in Q \times X : \forall u_d, \exists d_d \mid (u_d, d_d) \neq (\epsilon, \epsilon) \wedge \delta\big((q,x),(u_d,d_d)\big) \nsubseteq S\}$$

is the set of configurations such that, for every controller discrete input, there exists a discrete disturbance input that forces the configuration outside S in one step.

DISCRETE **CONTROLLABLE** PREDECESSORS (And the winner is … control)

$$DCPre(S) = \{(q,x) \in Q \times X : \exists u_d \mid \forall d_d, (u_d, d_d) \neq (\epsilon, \epsilon) \wedge \delta\big((q,x),(u_d,d_d)\big) \subseteq S\}$$

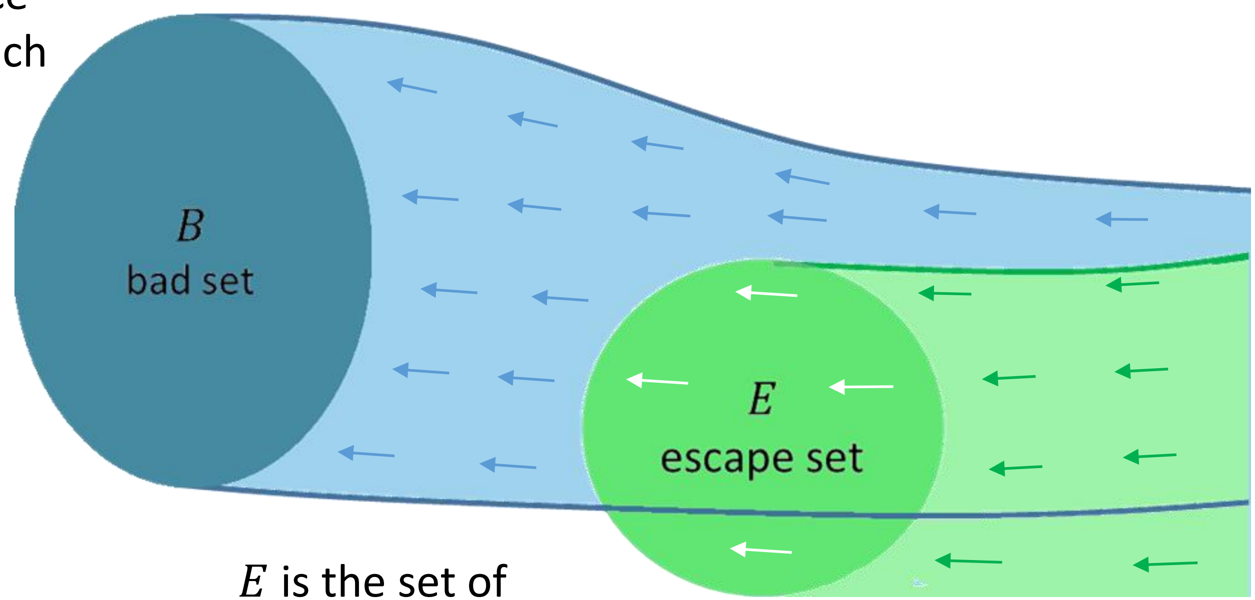
is the set of configurations that can be forced to remain into S in one step, whatever is the disturbance discrete input.

CONTINUOUS FLOW

CONTINUOUS **UNCONTROLLABLE** PREDECESSORS

$$CUPre(B,E) = \{\, (q,x) \in Q \times X : \forall\, u_c(t), \exists\, d_c(t)\ and\ \exists\, t^* > 0 |$$
$$for\ the\ corresponding\ trajectory\ x(t)$$
$$\forall\, t \in [0, t^*), \big(q, x(t)\big) \in Inv \cap \bar{E} \wedge \big(q, x(t^*)\big) \in B\}$$

B is the set of configurations the disturbance is trying to reach



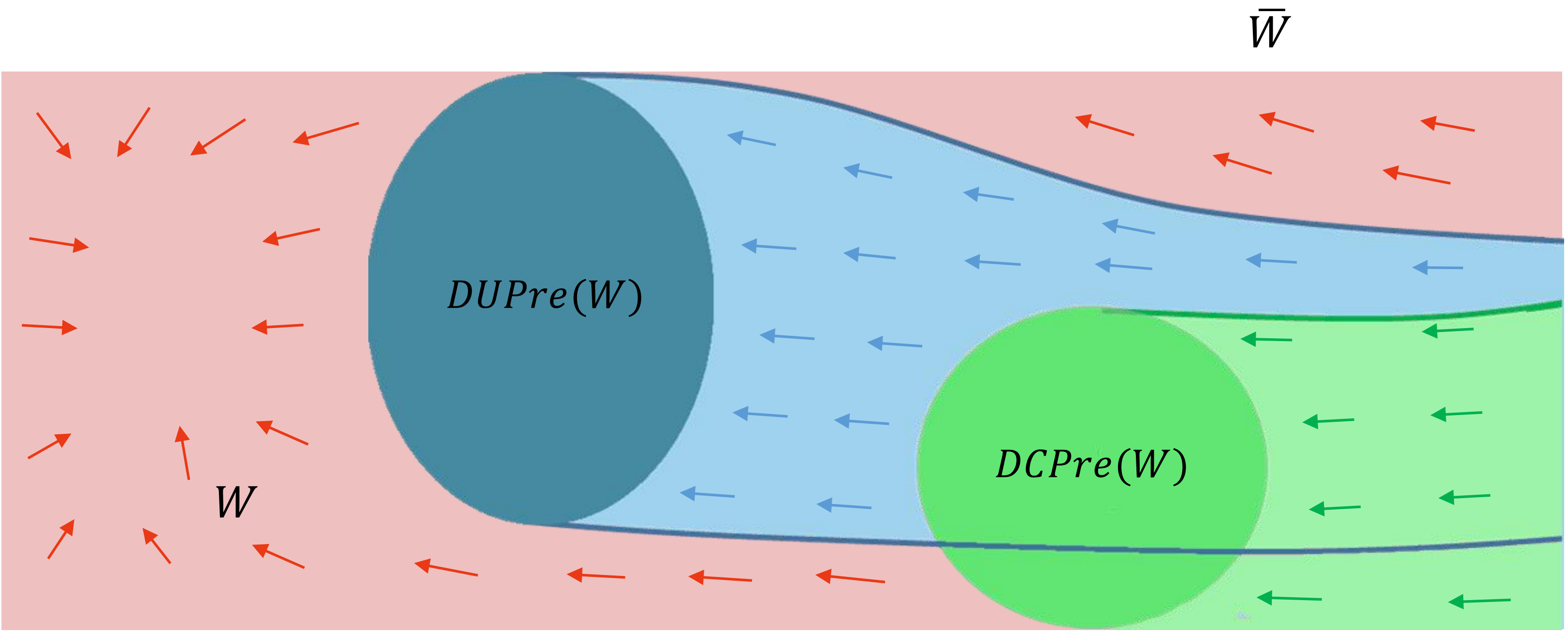
E is the set of configurations that must be avoided

A continuous game between control and disturbance

Given a set W,

$$CUPre(DUPre(W) \cup \overline{W}, DCPre(W))$$

is the set of states that, *whatever the continuous control is*, can be steered to the set $DUPre(W)$ or outside the set W while avoiding entering the set $DCPre(W)$.



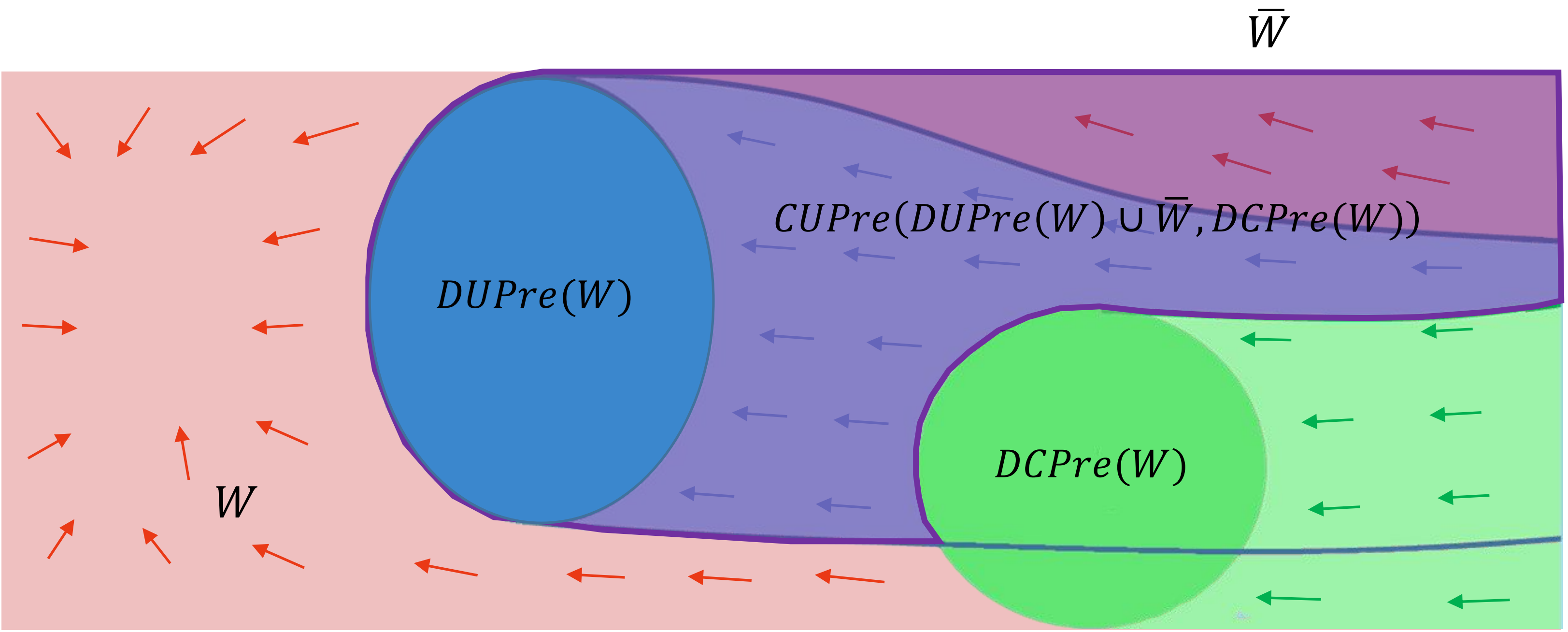
A continuous game between control and disturbance

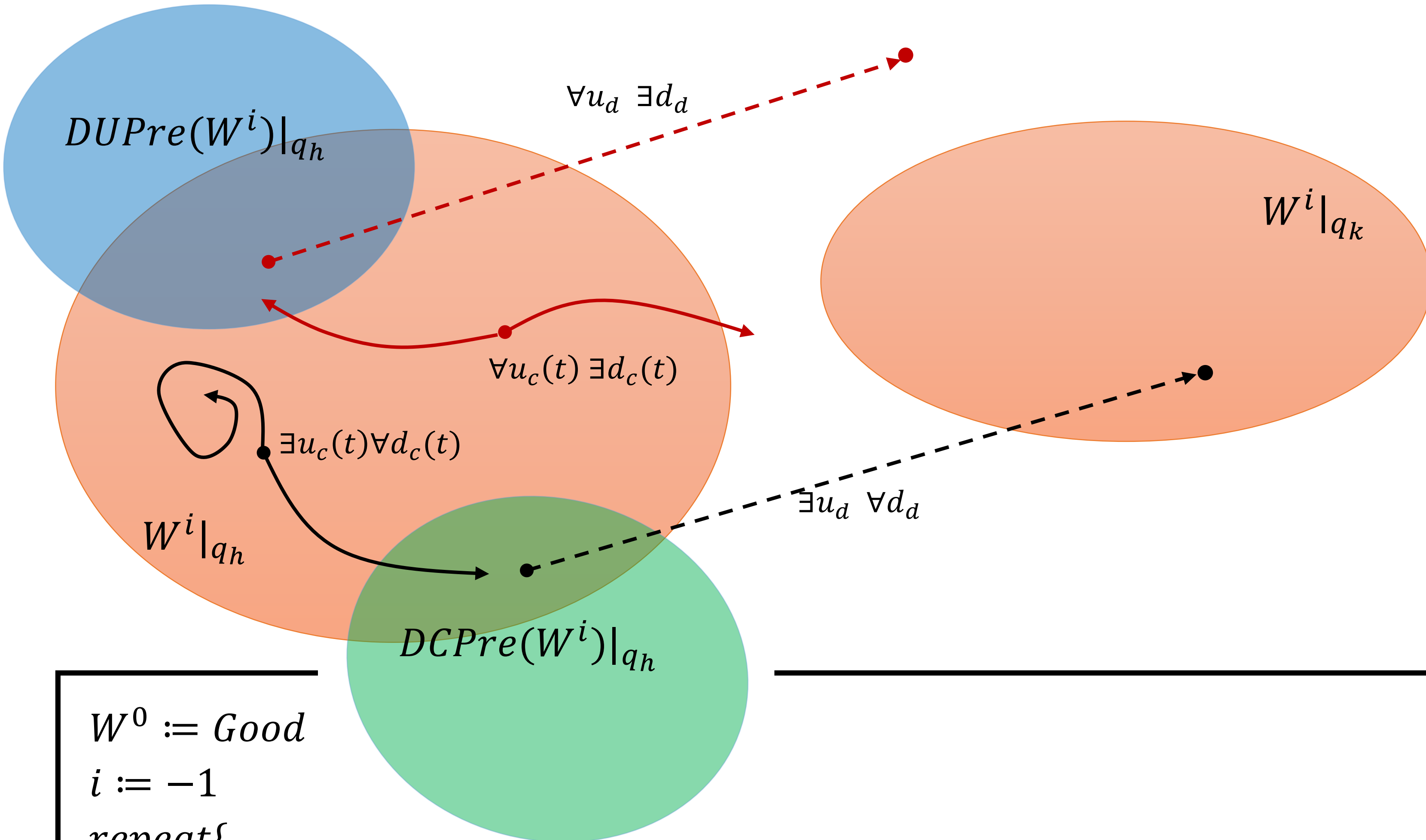
The «losing» states of the set W are those that belong to the set

$$DUPre(W)$$

or to the set

$$CUPre(DUPre(W) \cup \bar{W}, DCPre(W))$$





$W^0 := Good$

$i := -1$

$repeat\{$

$\quad i := i + 1$

$\quad W^{i+1} := W^i \backslash \left[DUPre\left(W^i\right) \cup CUPre\left(DUPre\left(W^i\right) \cup \overline{W^i}, DCPre\left(W^i\right)\right) \ \right]$

$\}\ until\ (W^{i+1} = W^i)$

$Safe := W^i$

Result

$U = 0{,}5$

$D = 0{,}01$

$W = 0{,}2$

$r = 0{,}95$

$C = 1$

$R_o = 500$

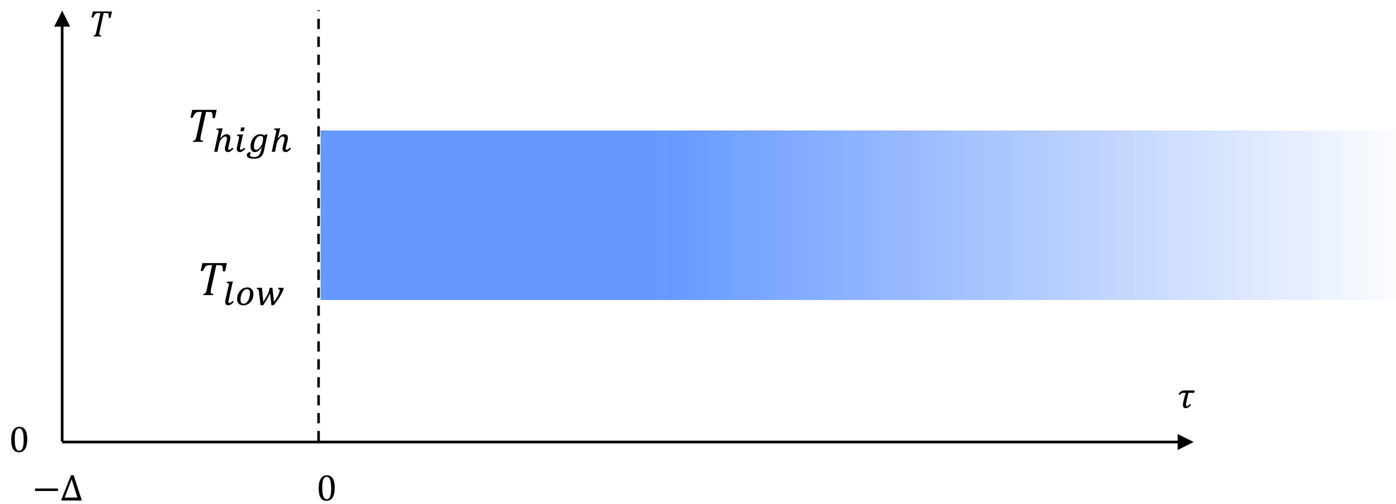
$R_c = 1000$

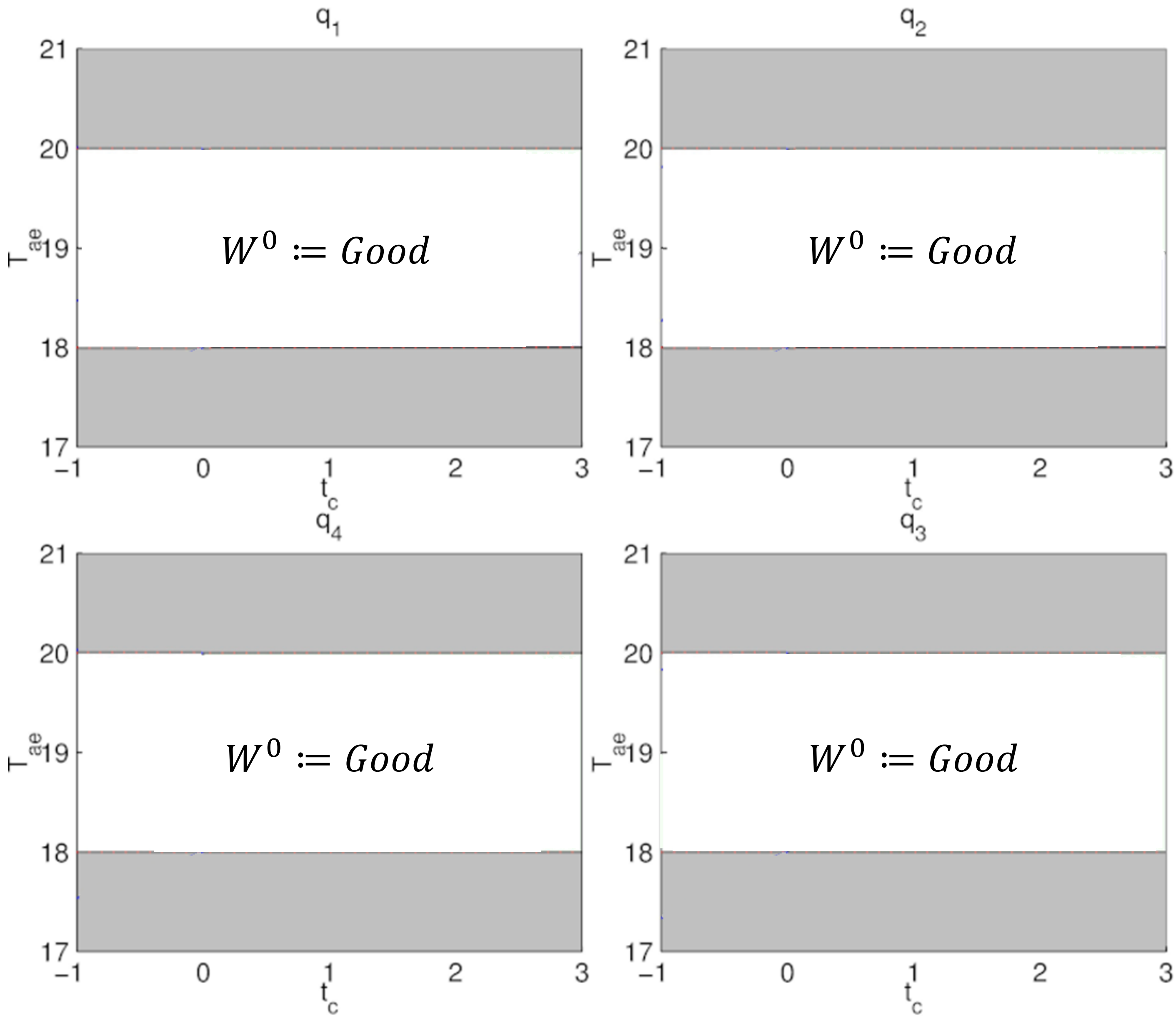
COMPUTATION OF DISCRETE PREDECESSORS

- No transitions may take place for $\tau < 0$
- Guard conditions do not depend on the value of T

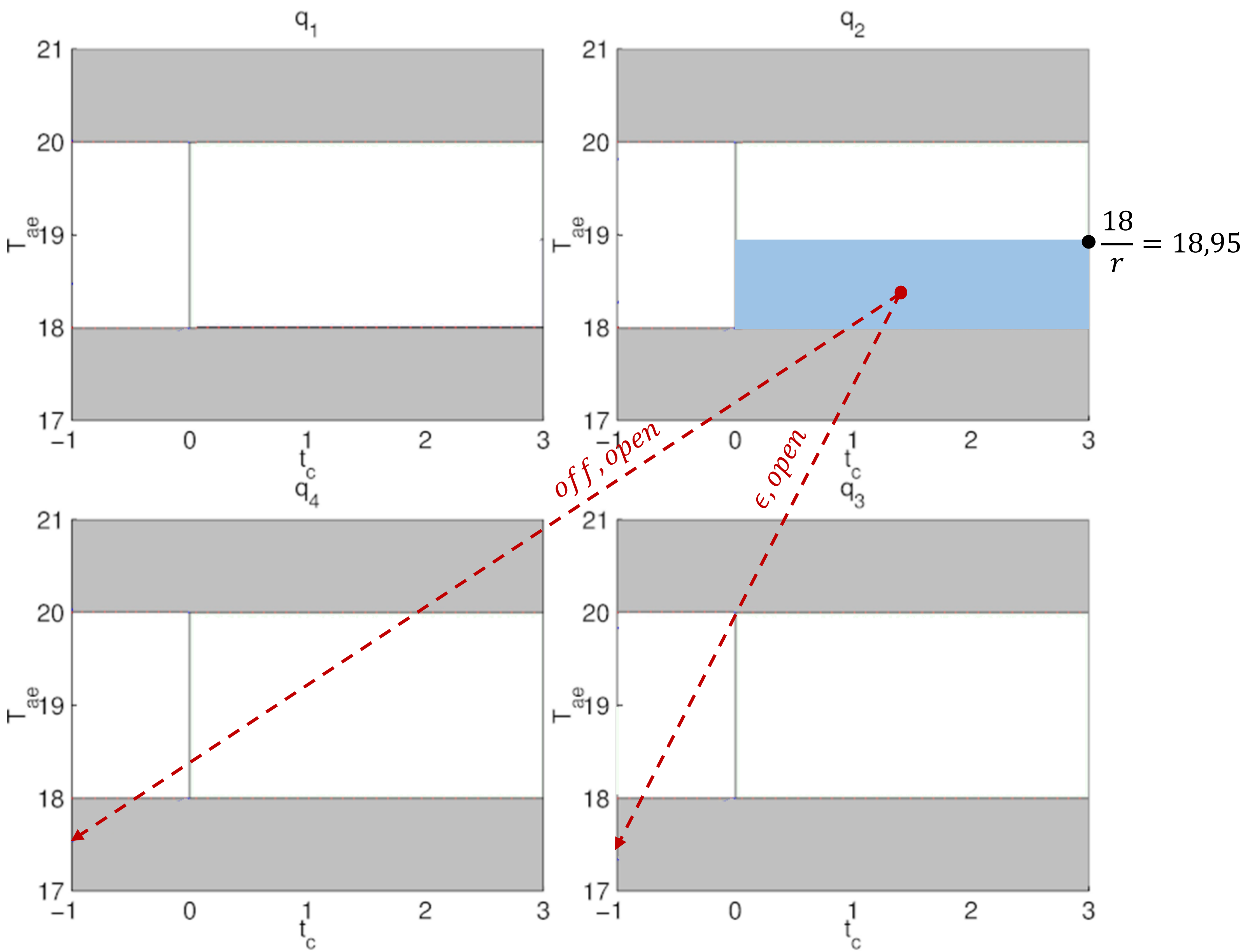
The discrete predecessors are sets of the form

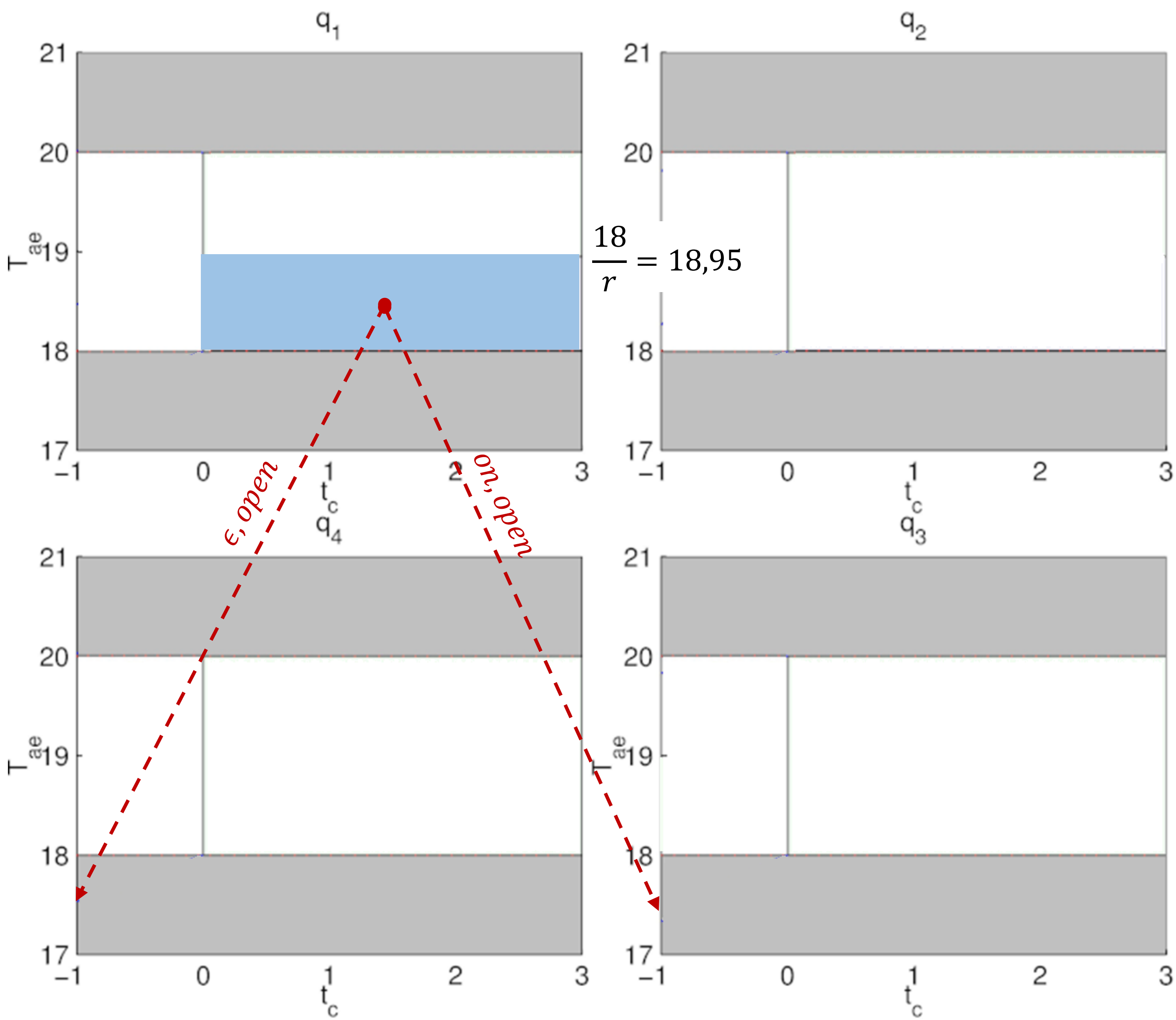
$$T \in [T_{low}, T_{high}], \qquad \tau \geq 0$$

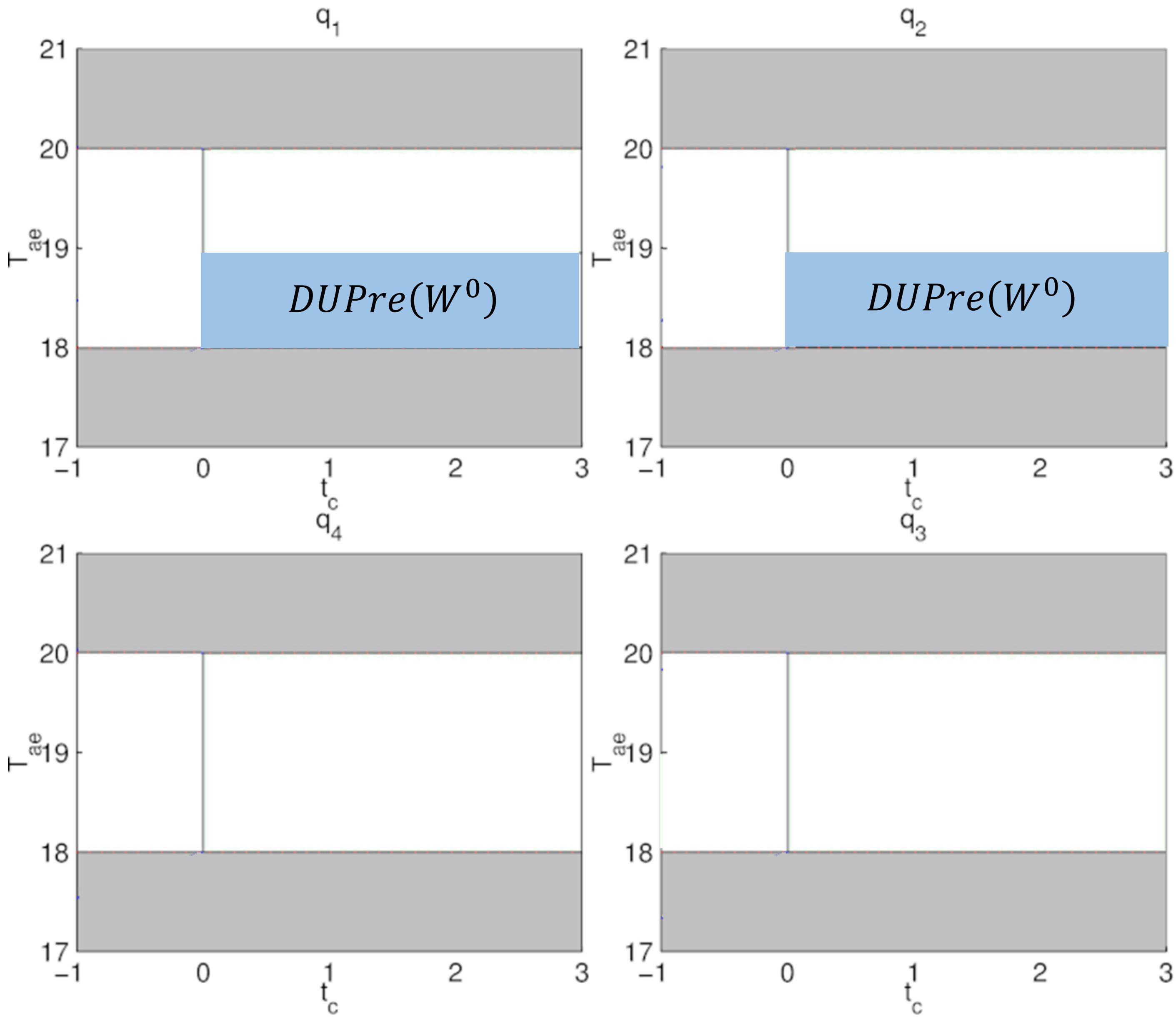




q_1
q_2
q_4
q_3
$W^0 \coloneqq Good$
$W^0 \coloneqq Good$
$W^0 \coloneqq Good$
$W^0 \coloneqq Good$
T_{ae}
t_c
21
20
19
18
17
−1
0
1
2
3


q1
q2
q4
q3
T_{ae}
t_c
$\frac{18}{r} = 18{,}95$
$off, open$
$\epsilon, open$


q1
q2
q4
q3
21
20
19
18
17
−1
0
1
2
3
Tae
tc
18/r = 18,95
ϵ, open
on, open


q1
q2
q4
q3
21
20
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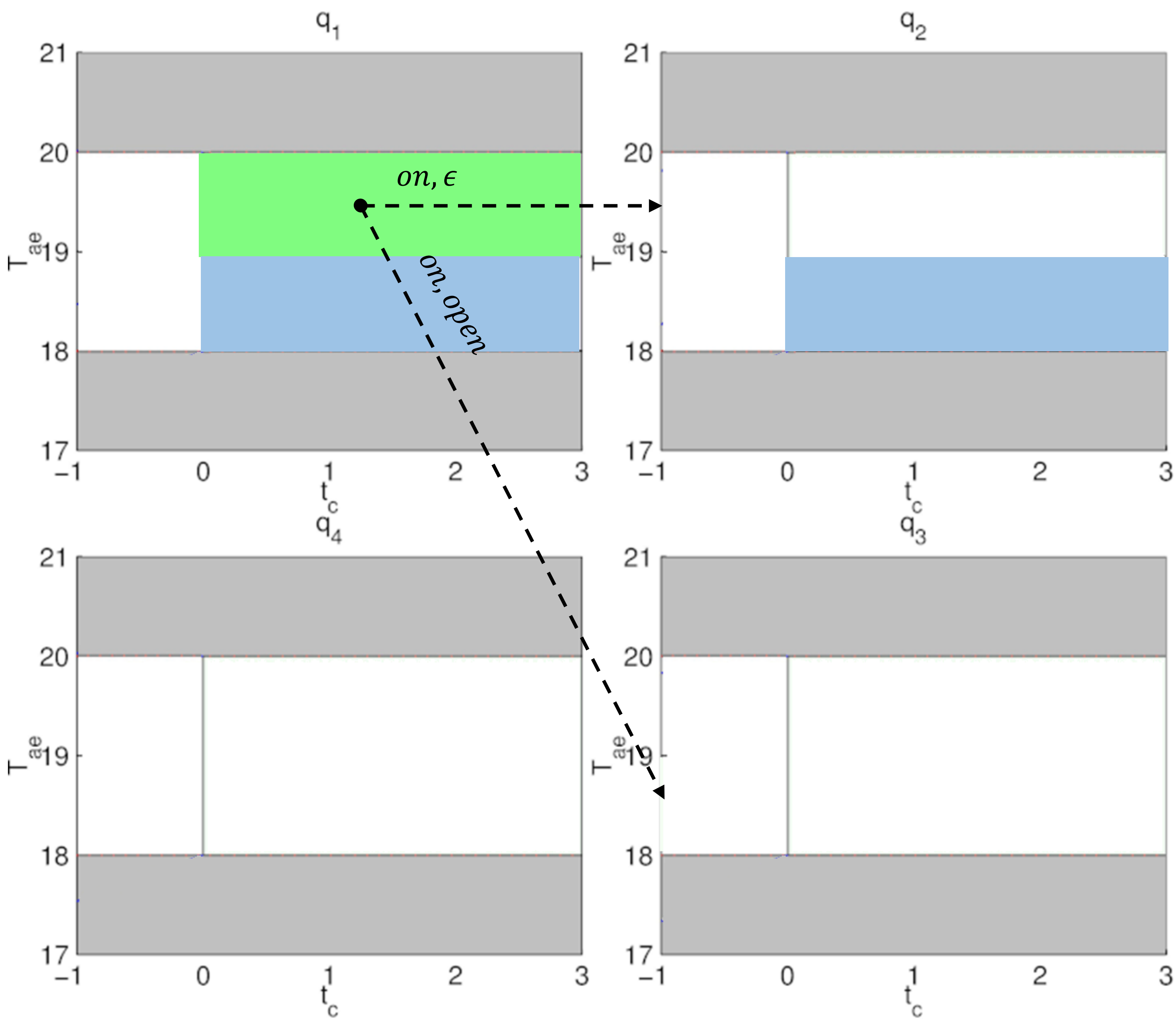
18
17
−1
0
1
2
3
T_{ae}
t_c
$DUPre(W^0)$
$DUPre(W^0)$


q1
q2
q4
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T_ae
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on, ϵ
on, open
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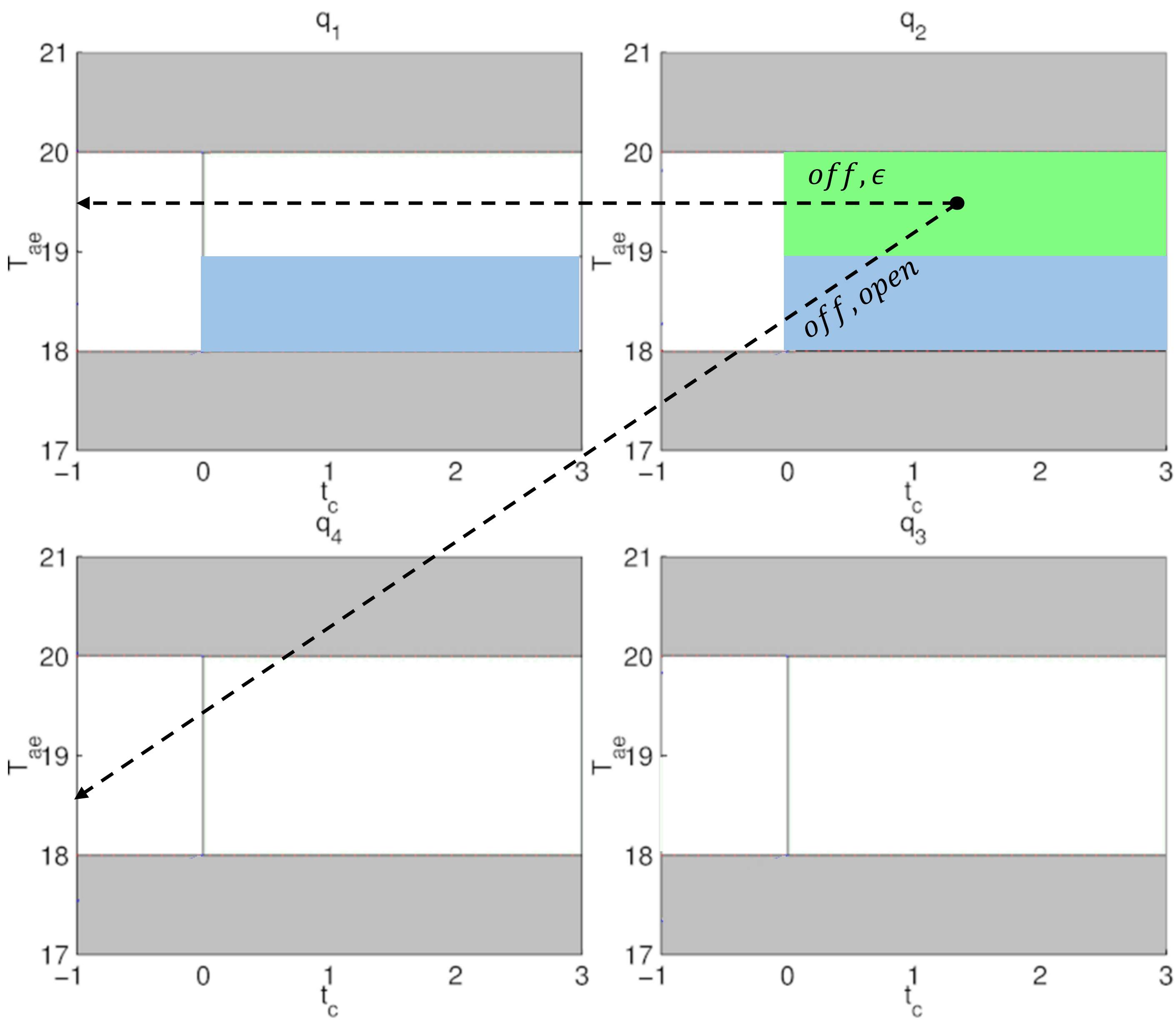

q1
q2
q4
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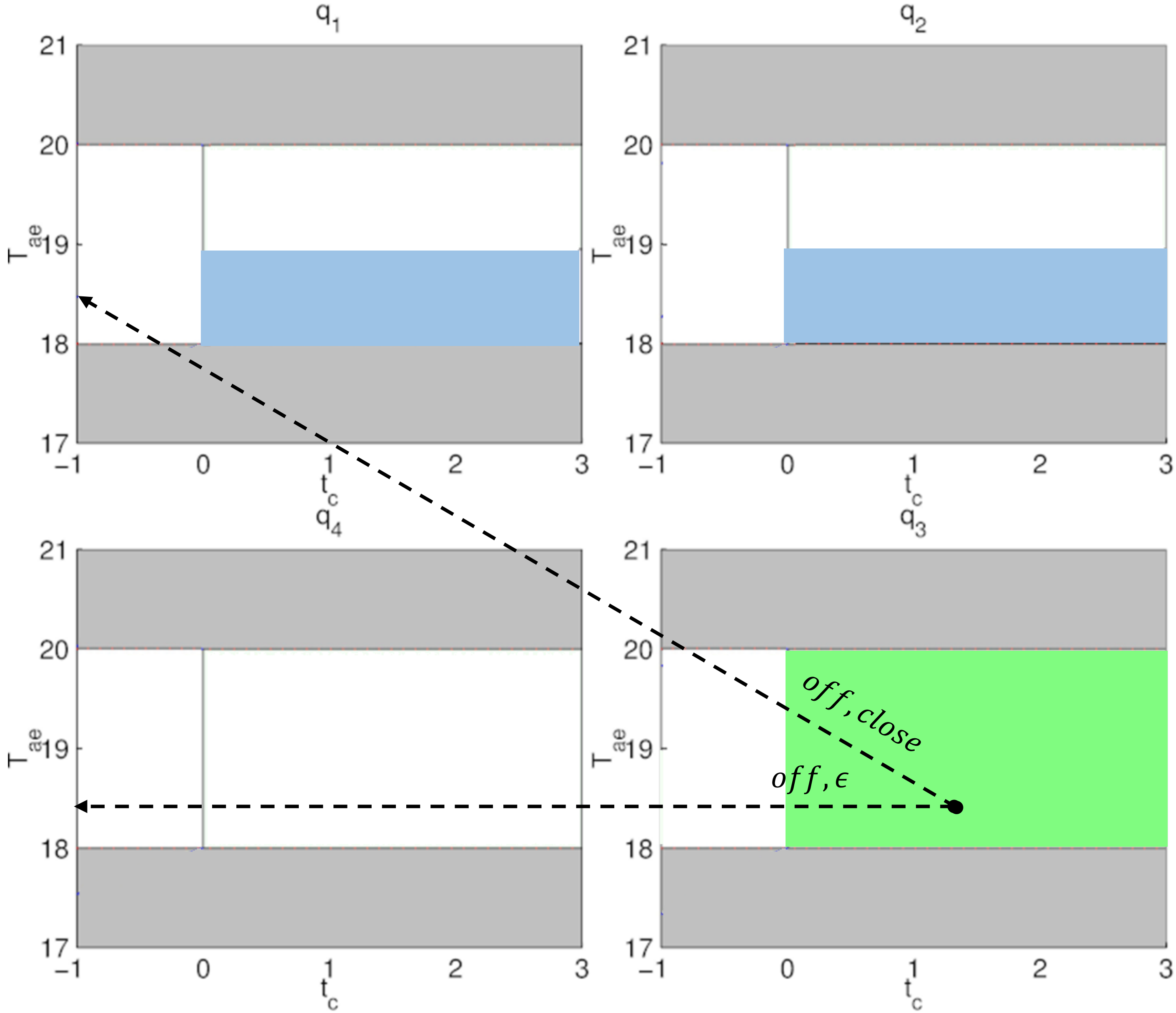
T_ae
t_c
off, ϵ
off, open


q1
q2
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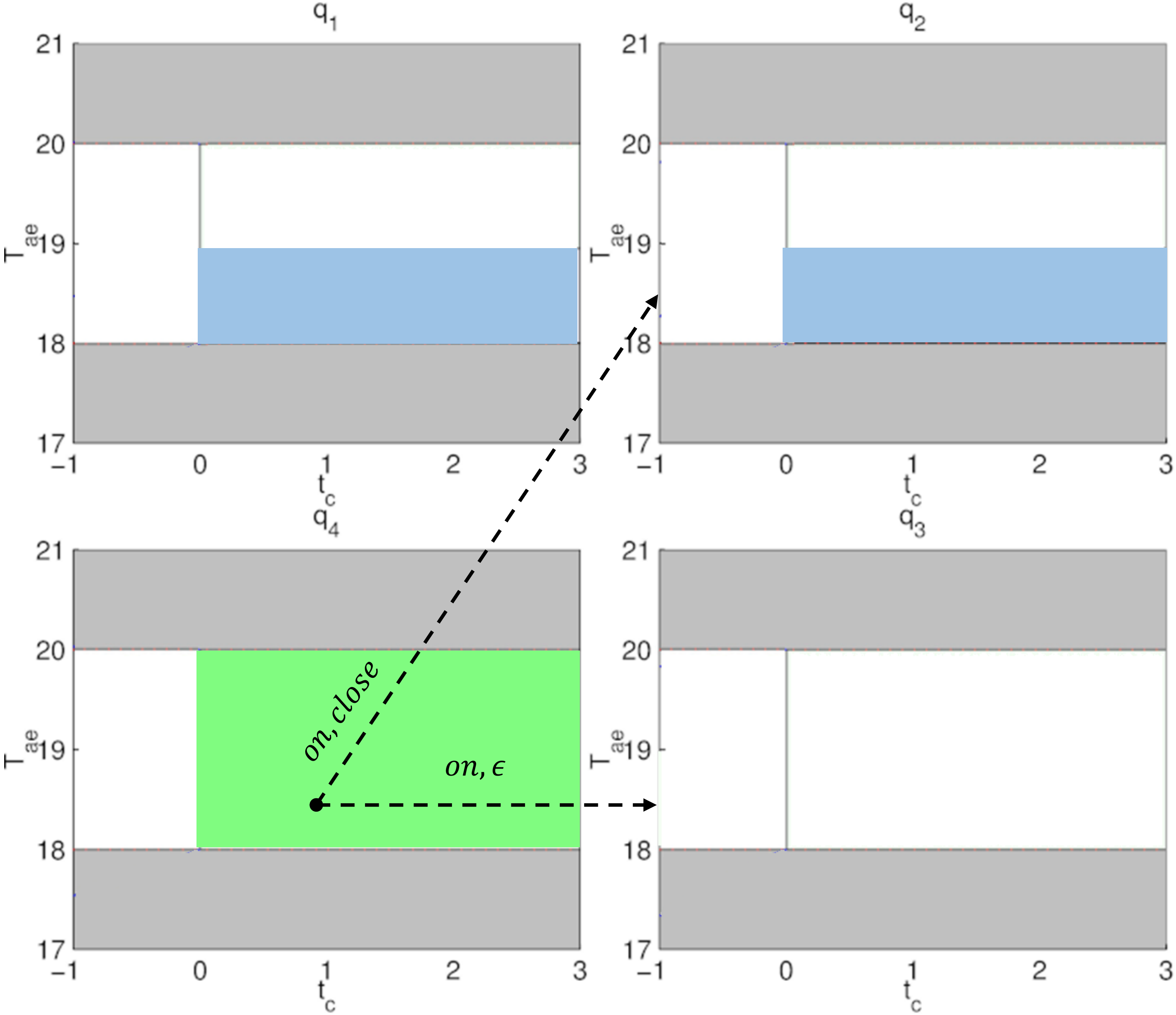
Tae
tc
off, close
off, ϵ


q1
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q4
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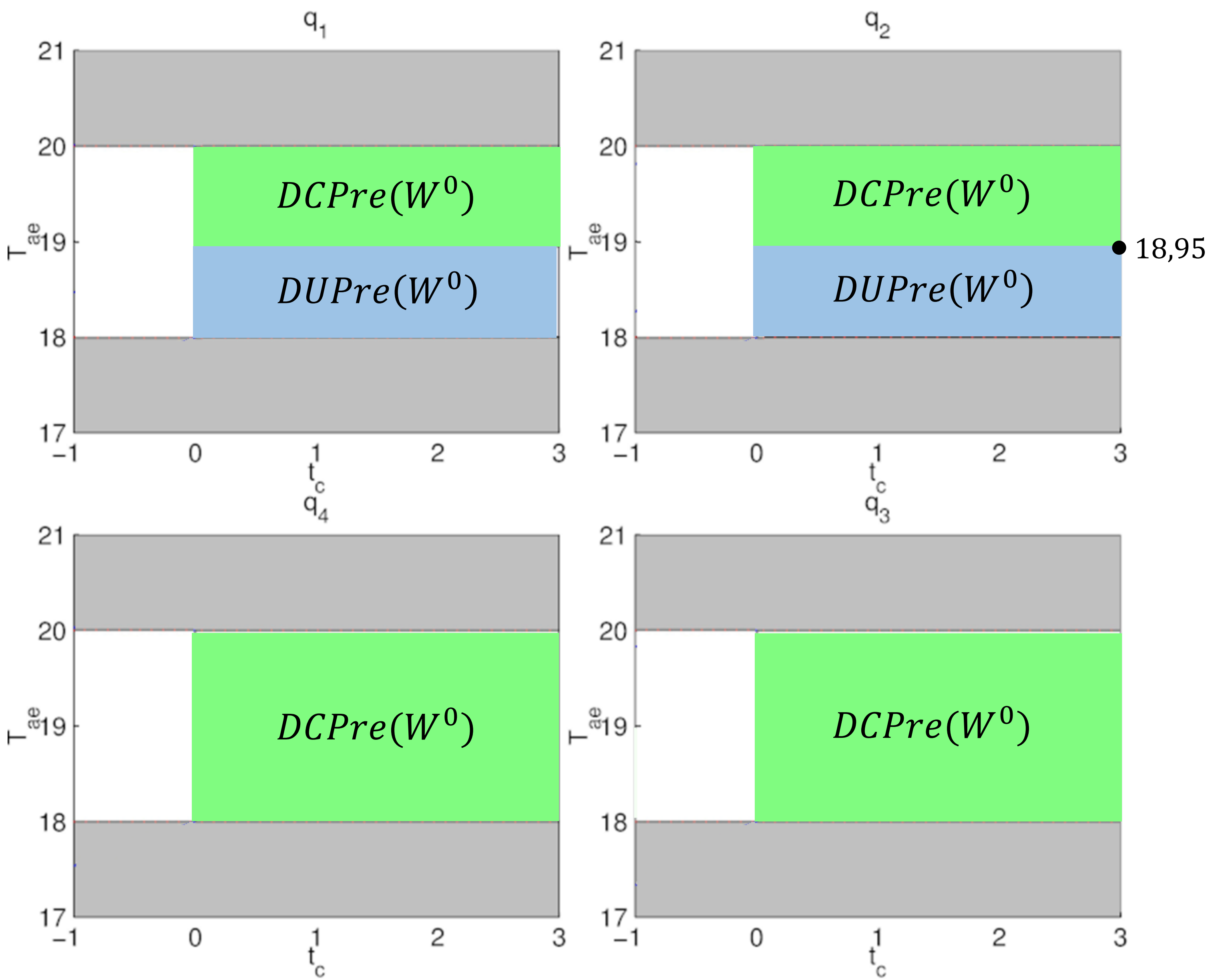
T_{ae}
t_c
on, close
on, ϵ


q1
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T_ae
DCPre(W^0)
DUPre(W^0)
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t_c
q2
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T_ae
DCPre(W^0)
DUPre(W^0)
18,95
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t_c
q4
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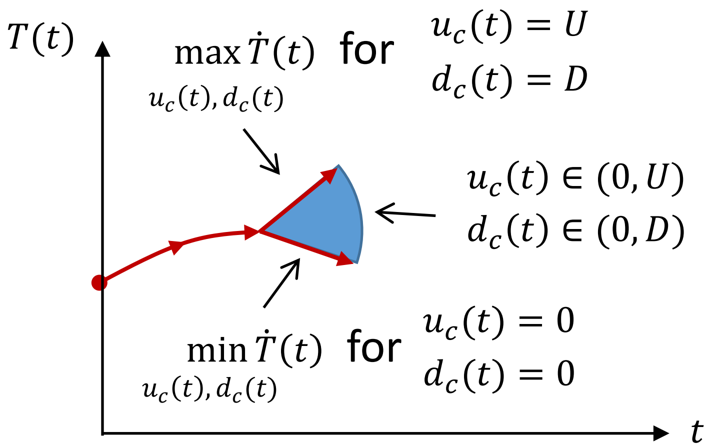
18
17
T_ae
DCPre(W^0)
−1
0
1
2
3
t_c
q3
21
20
19
18
17
T_ae
DCPre(W^0)
−1
0
1
2
3
t_c

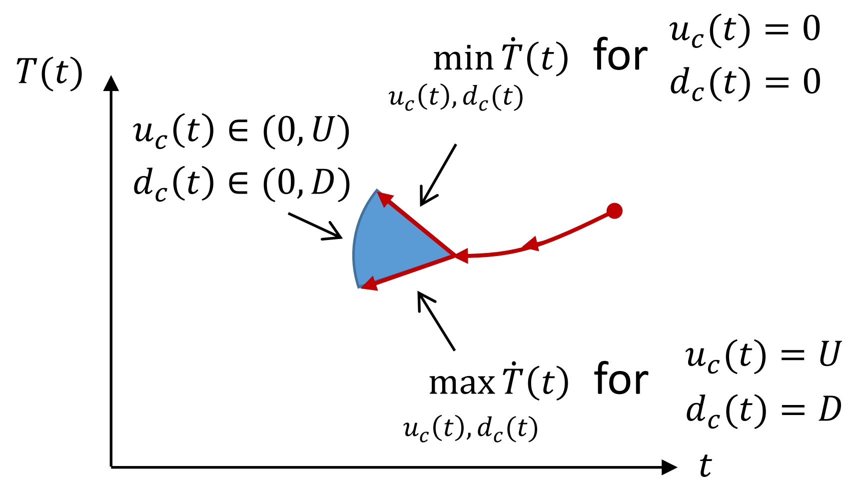
COMPUTATION OF CONTINUOUS PREDECESSORS

- Can be calculated one location q at a time
- Can be viewed as a game between the continuous control and the continuous disturbance
- The boundaries of $CUPre(W^i)|_{q_h}$ are obtained by solving a min-max problem

$$\dot{T}(t) = -\frac{1}{R_{o/c}C}T(t) + \frac{u_c(t)}{C} + \frac{W/0}{C} + \frac{d_c(t)}{C} \qquad \begin{array}{l} u_c(t) \in [0, U] \\ d_c(t) \in [0, D] \end{array}$$



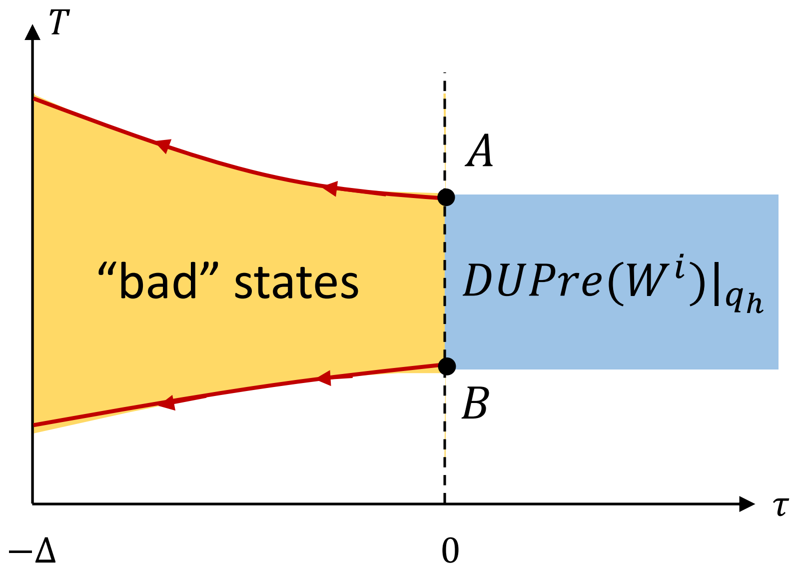
Integration forward in time



Integration backward in time

COMPUTATION OF CONTINUOUS PREDECESSORS

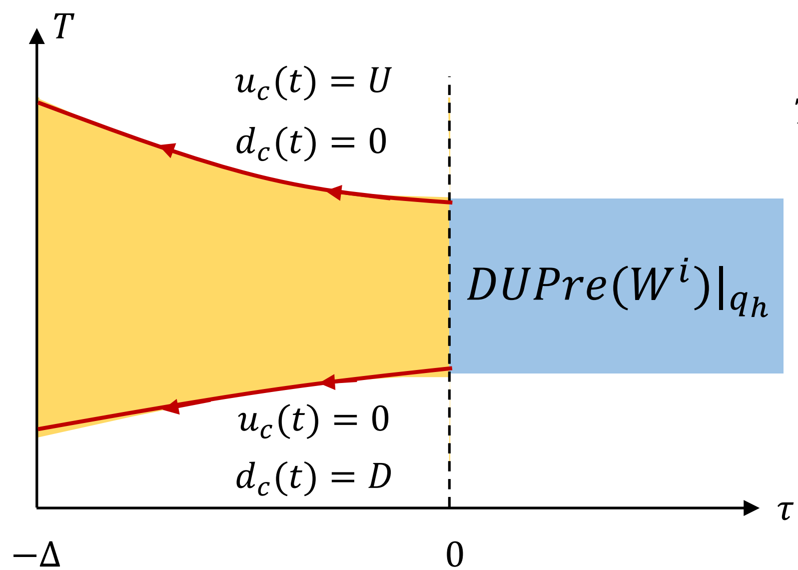
The states that can be steered to $DUPre(W^0)$ [while avoiding $DCPre(W^0)$] can be computed by integration backward in time from points A and B

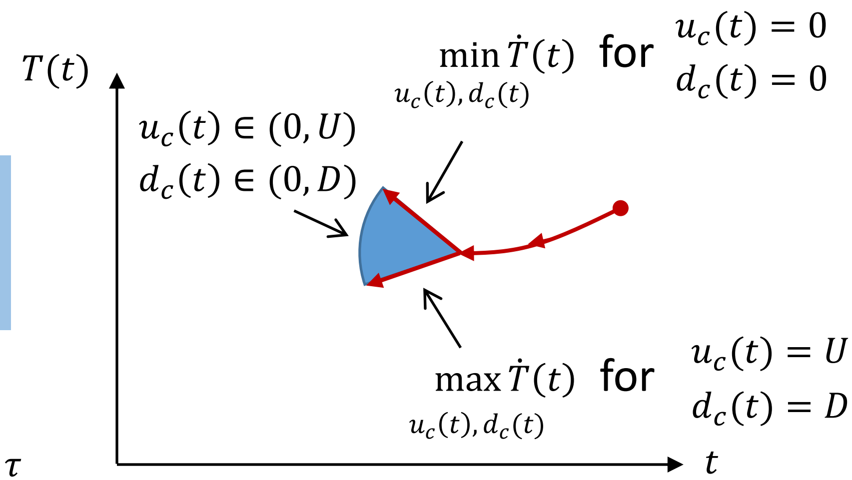


Which evolution of $u(t) \in [0, U]$ and $d(t) \in [0, D]$ should be considered while integrating?

COMPUTATION OF CONTINUOUS PREDECESSORS

States that can be steered to $DUPre(W^0)$ [while avoiding $DCPre(W^0)$] can be computed by integration backward in time



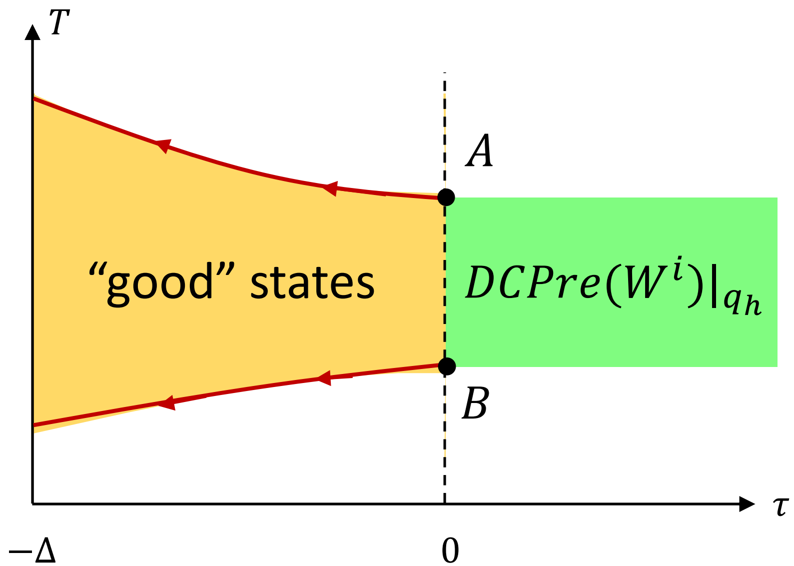


Integration backward in time

The continuous disturbance would like to maximize the yellow area while the continuous control would like to minimize it.

COMPUTATION OF CONTINUOUS PREDECESSORS

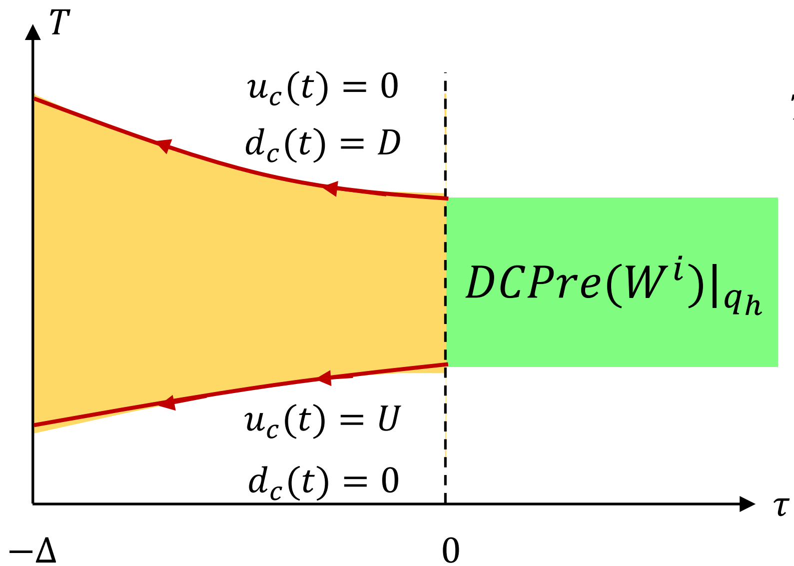
The states that can be steered to $DCPre(W^0)$ [while avoiding $DUPre(W^0)$] can be computed by integration backward in time from points A and B

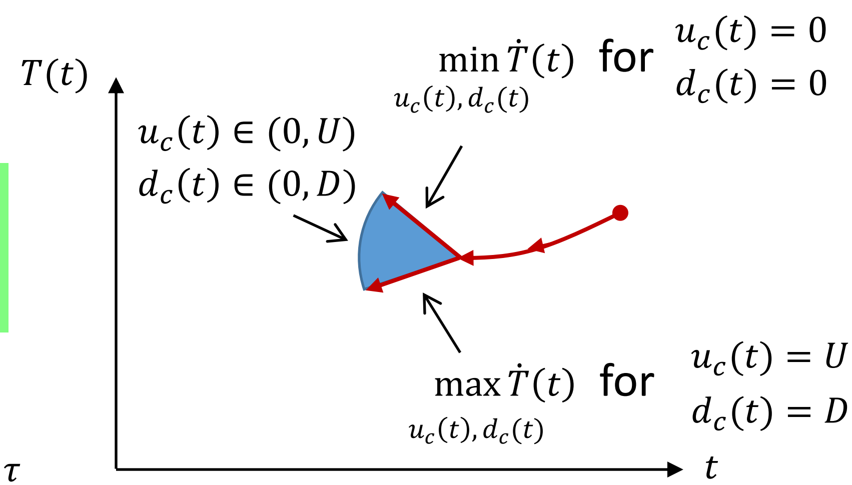


Which evolution of $u(t) \in [0, U]$ and $d(t) \in [0, D]$ should be considered while integrating?

COMPUTATION OF CONTINUOUS PREDECESSORS

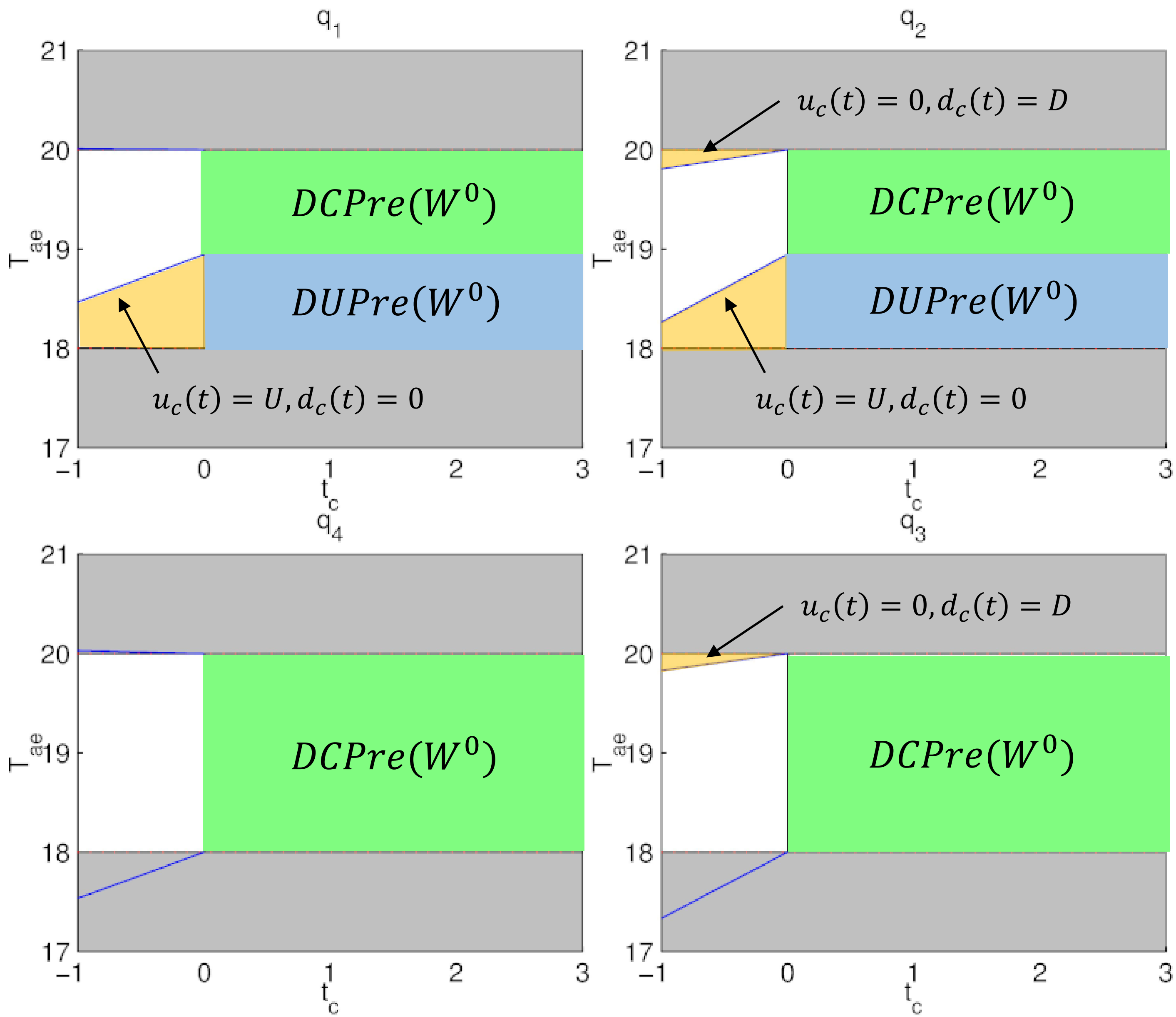
States that can be steered to $DCPre(W^0)$ [while avoiding $DUPre(W^0)$] can be computed by integration backward in time



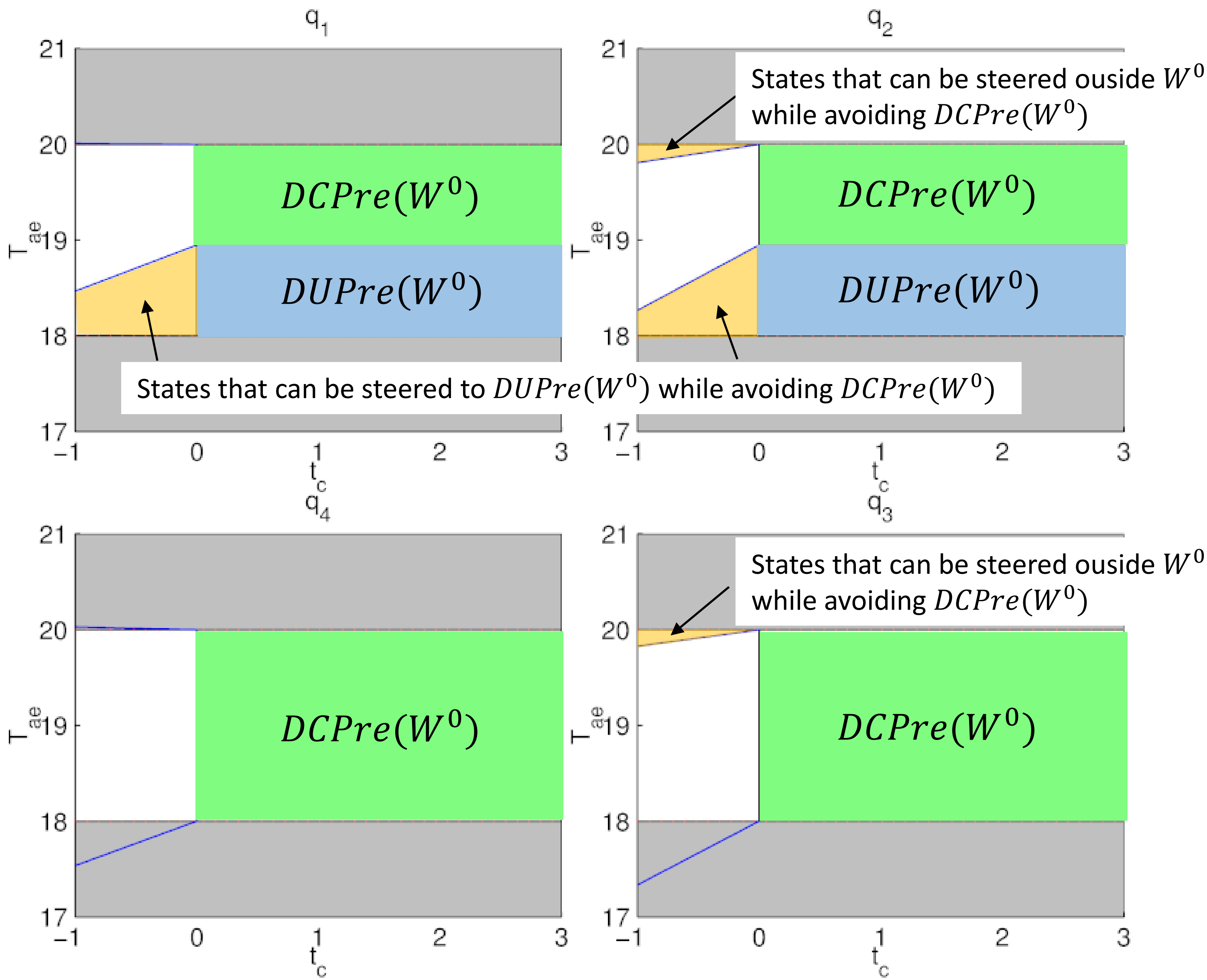


Integration backward in time

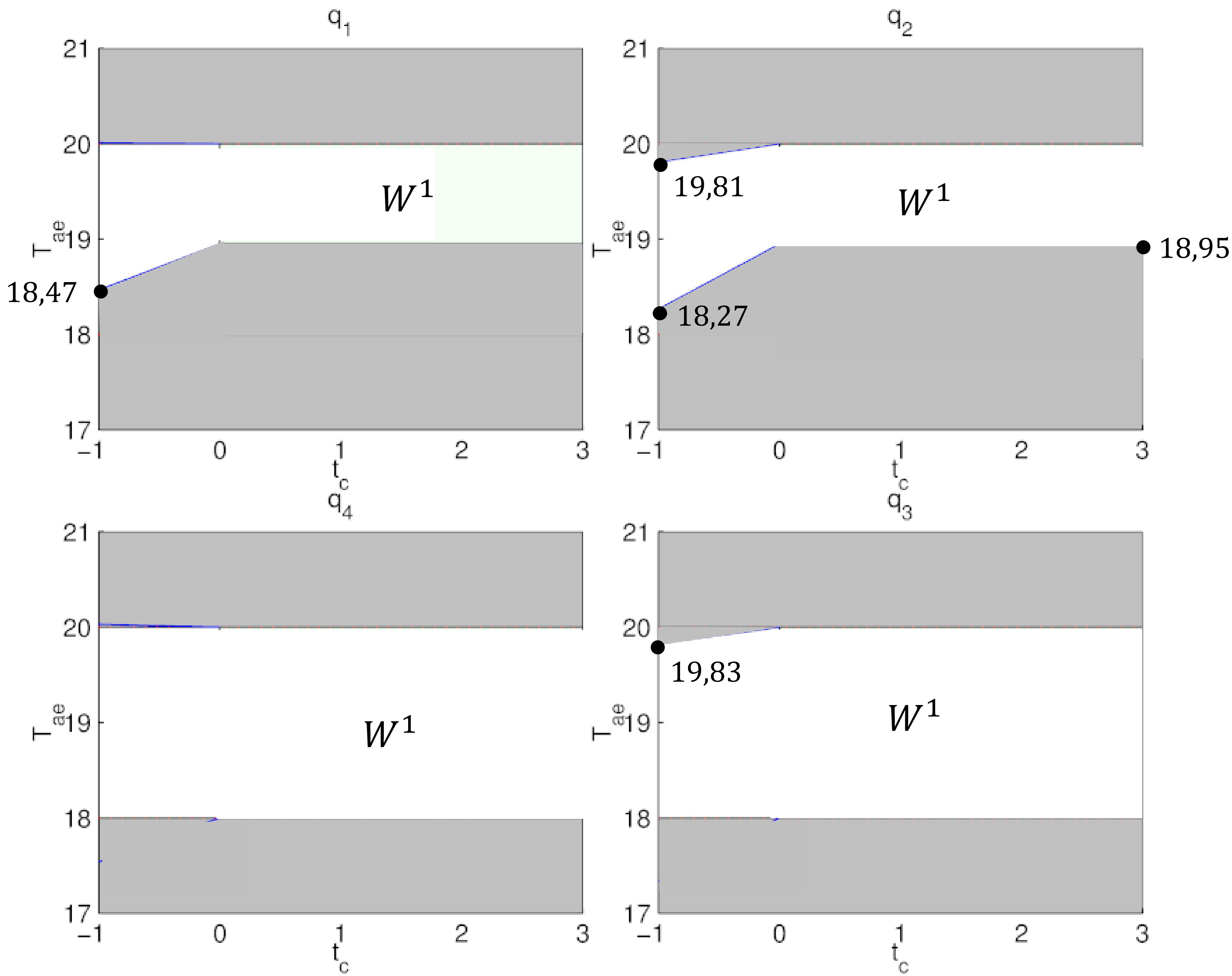
The continuous control would like to maximize the yellow area while the continuous disturbance would like to minimize it.



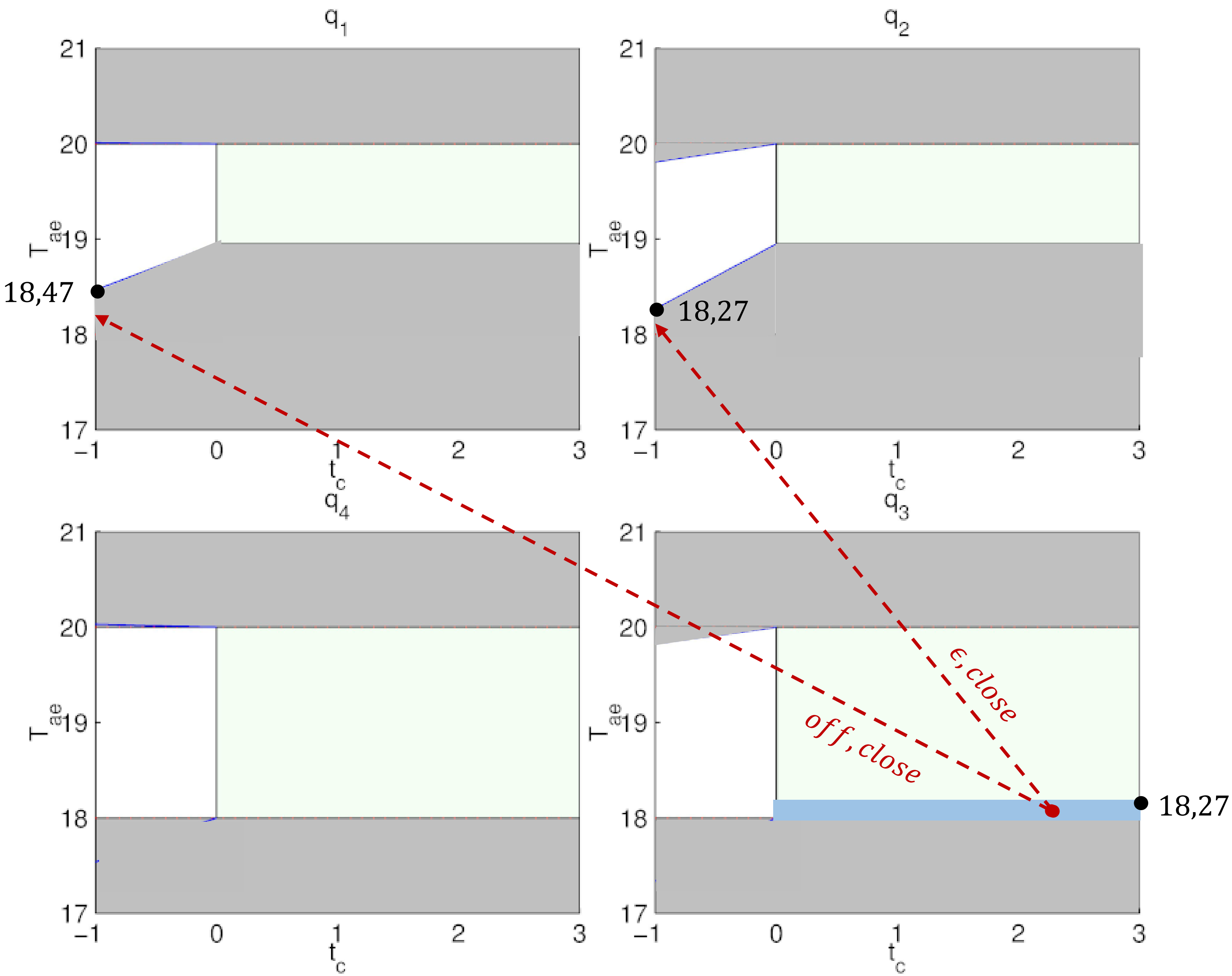
$$CUPre\left(DUPre\left(W^i\right) \cup \overline{W^i}, DCPre\left(W^i\right)\right)$$

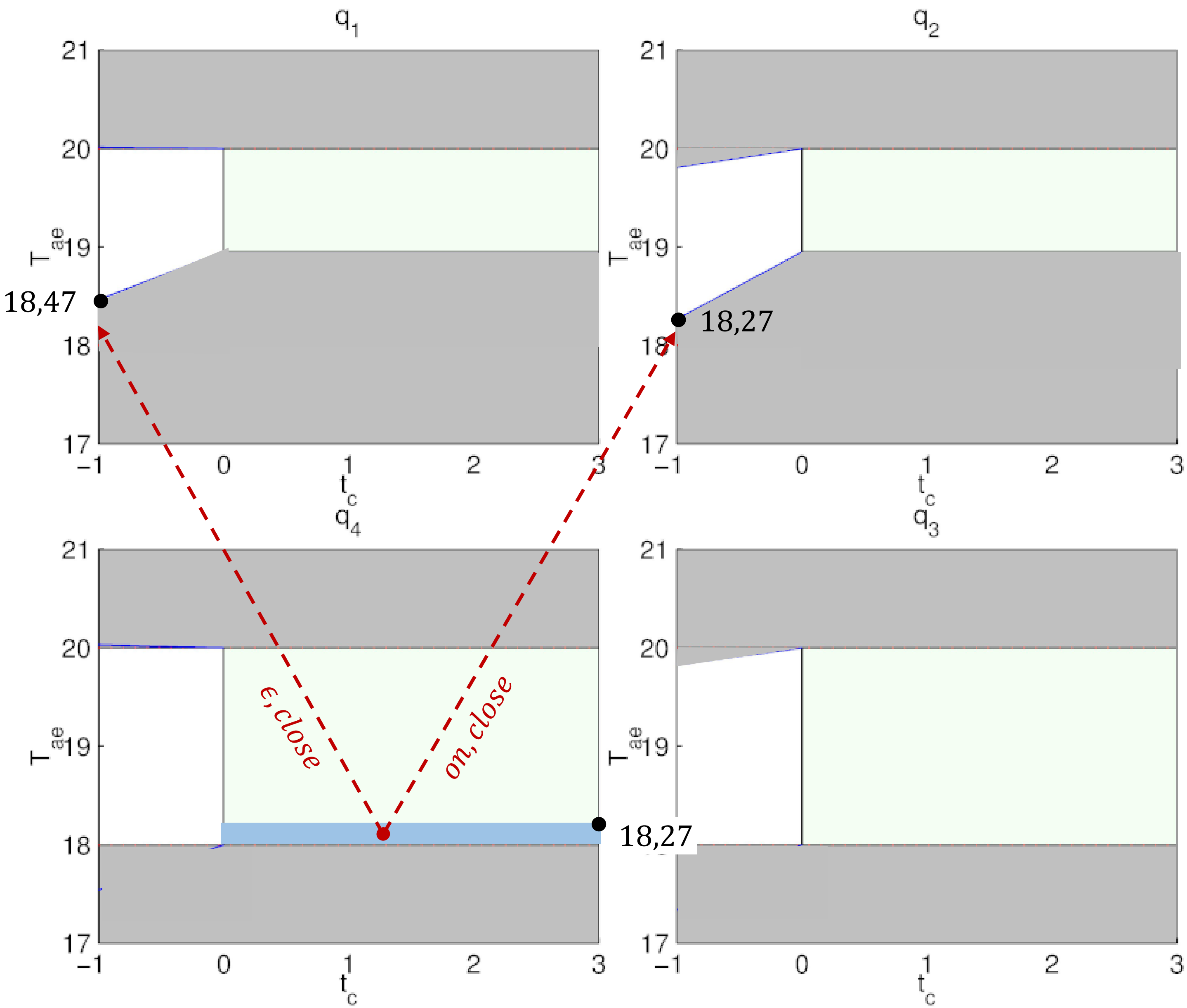


$$CUPre\left(DUPre\left(W^i\right) \cup \overline{W^i}, DCPre\left(W^i\right)\right)$$

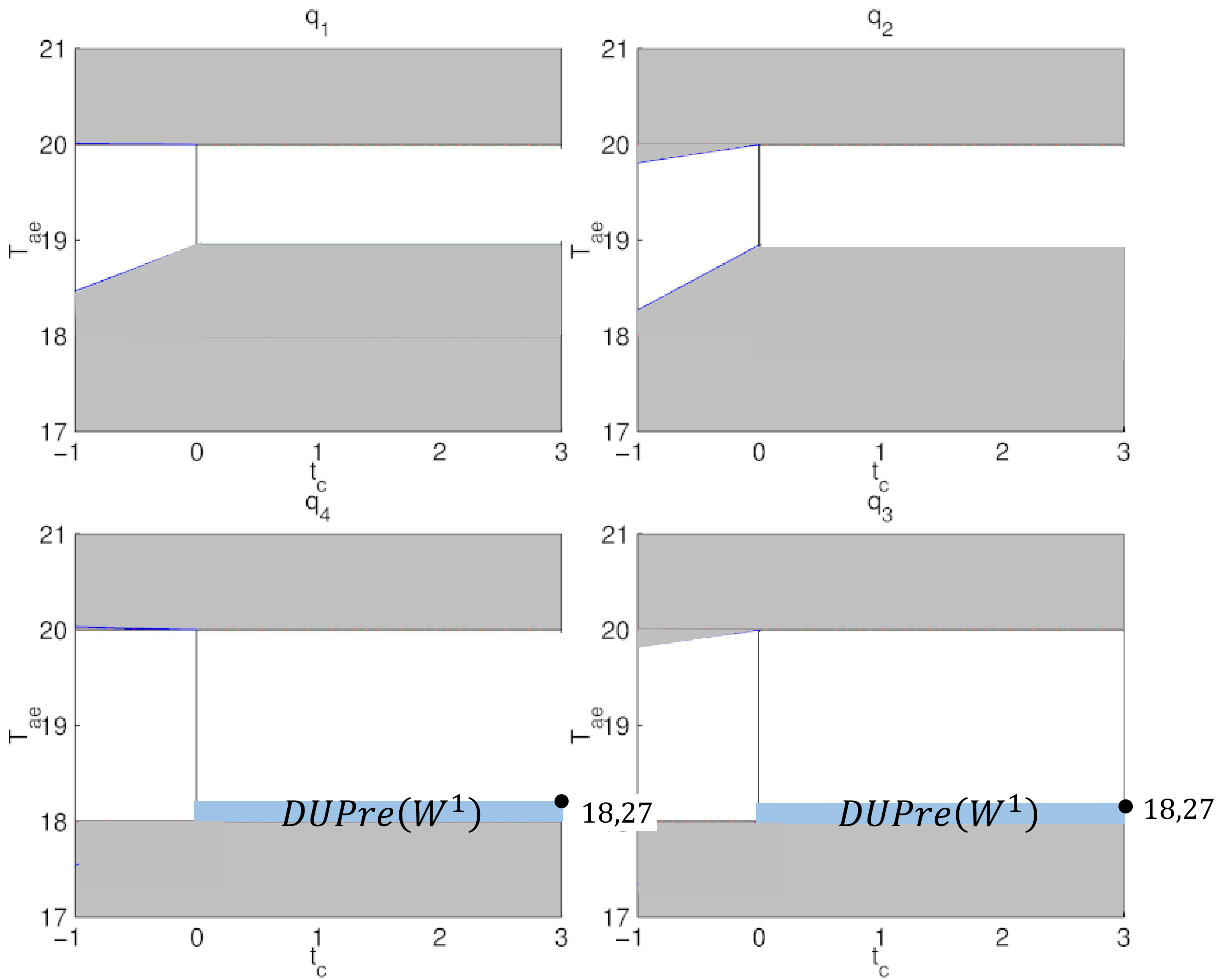


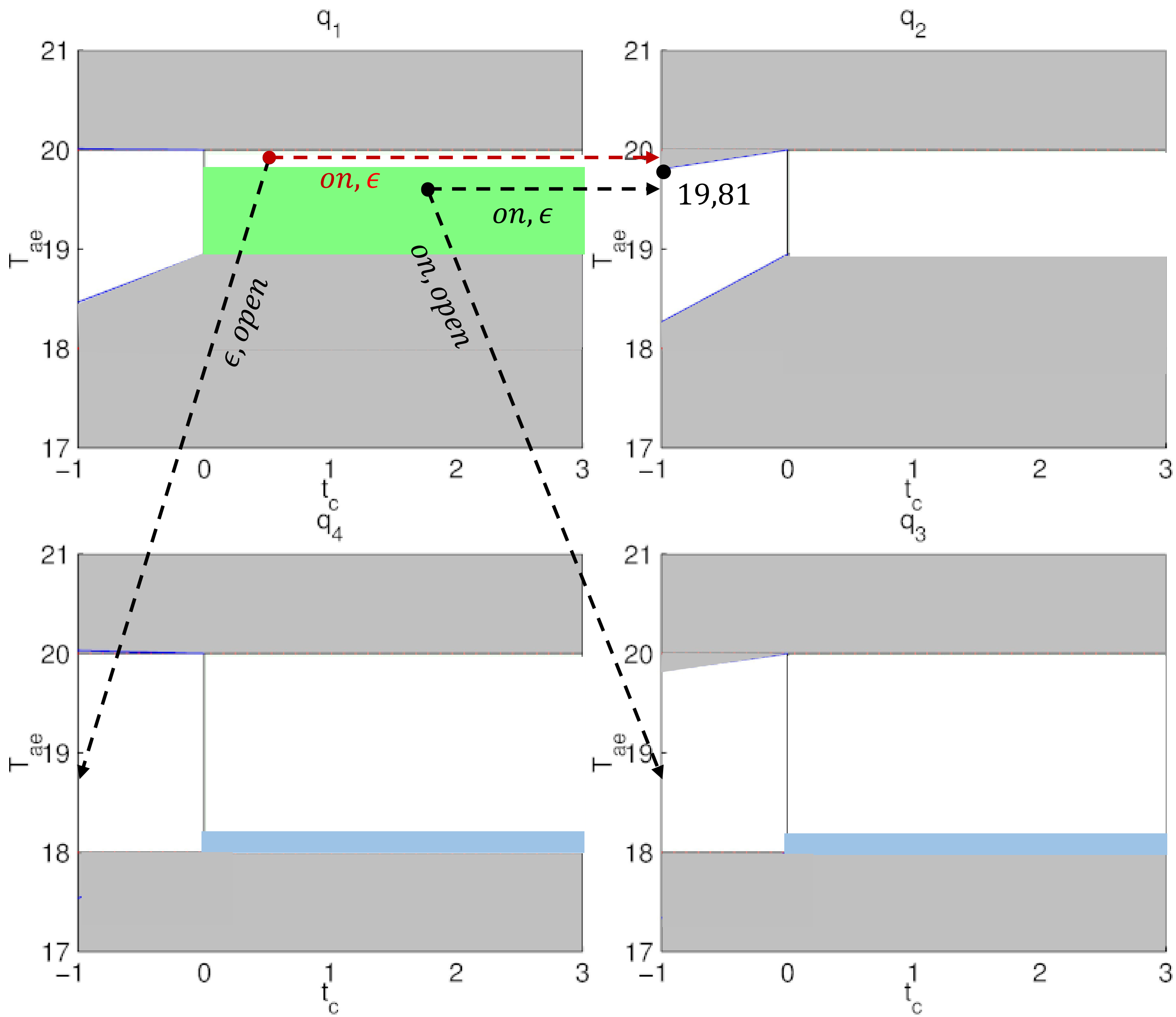
q1
21
20
19
18
17
−1
0
1
2
3
tc
Tae
W1
18,47
q2
19,81
W1
18,95
18,27
q4
W1
q3
19,83
W1

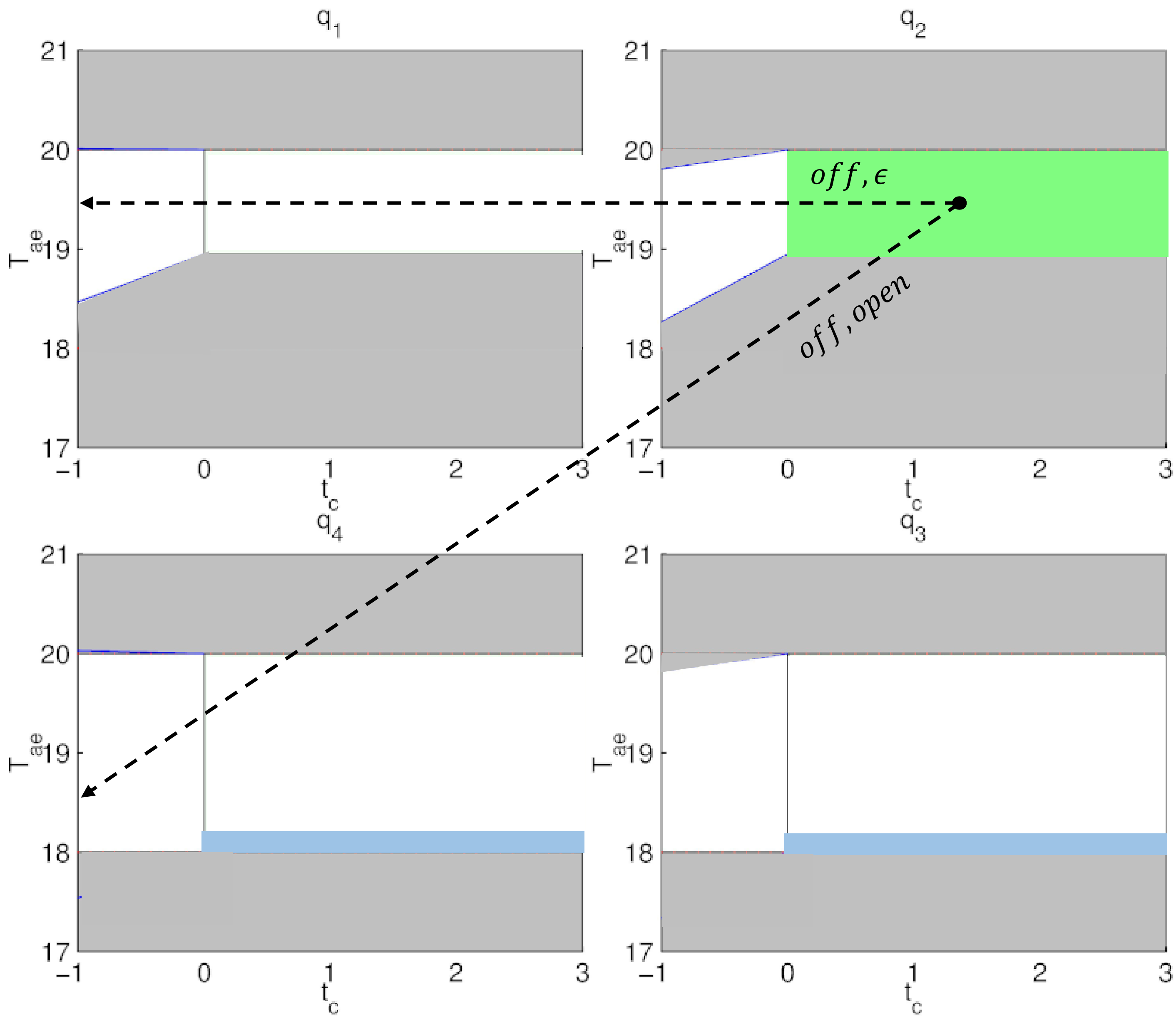

q1
q2
q4
q3
T_ae
t_c
18,47
18,27
18,27
ϵ, close
off, close



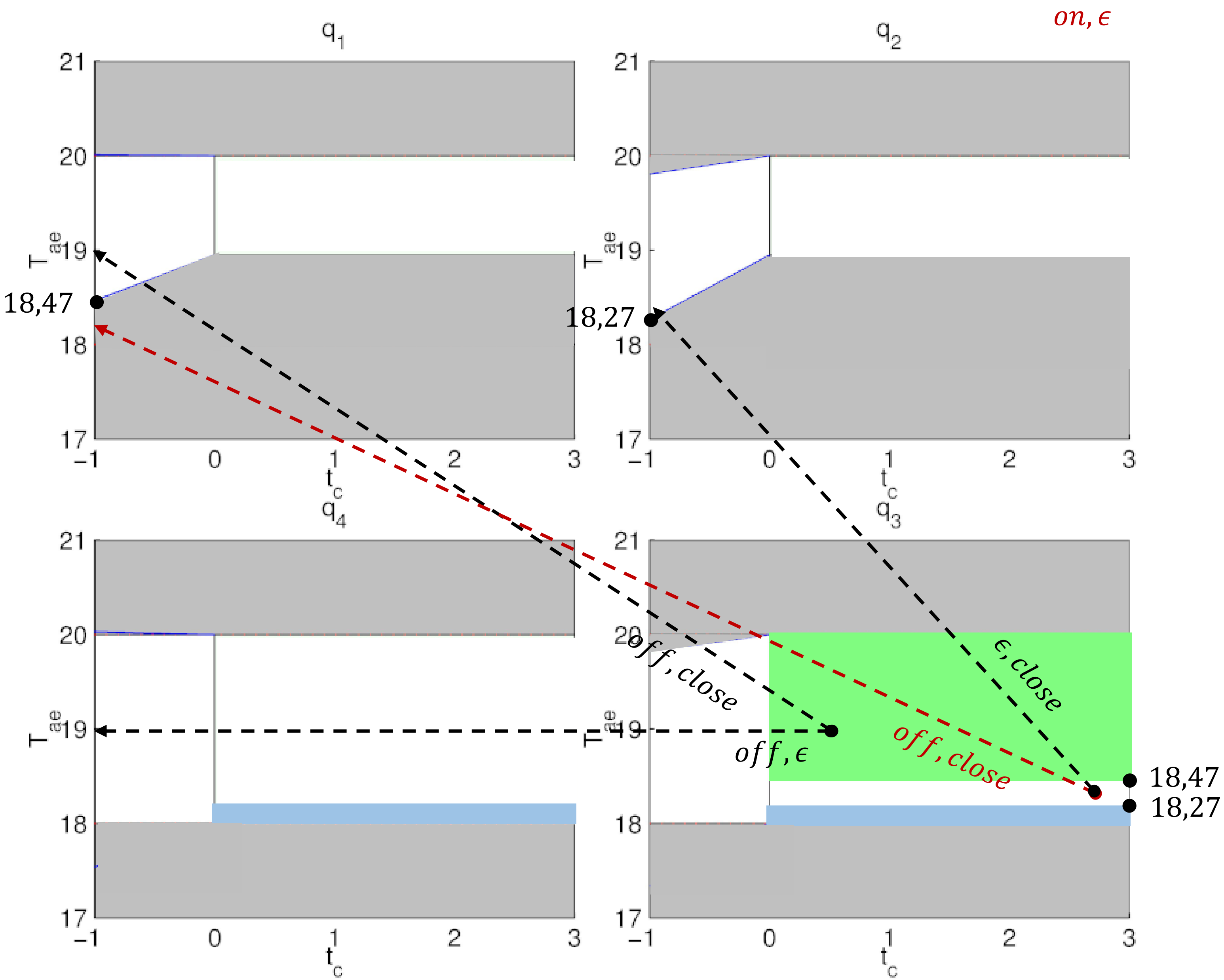
q1
q2
q4
q3
21
20
19
18
17
T_{ae}
t_c
−1
0
1
2
3
18,47
18,27
18,27
ϵ, close
on, close


q1
q2
q4
q3
21
20
19
18
17
T_ae
−1
0
1
2
3
t_c
DUPre(W^1)
18,27
DUPre(W^1)
18,27

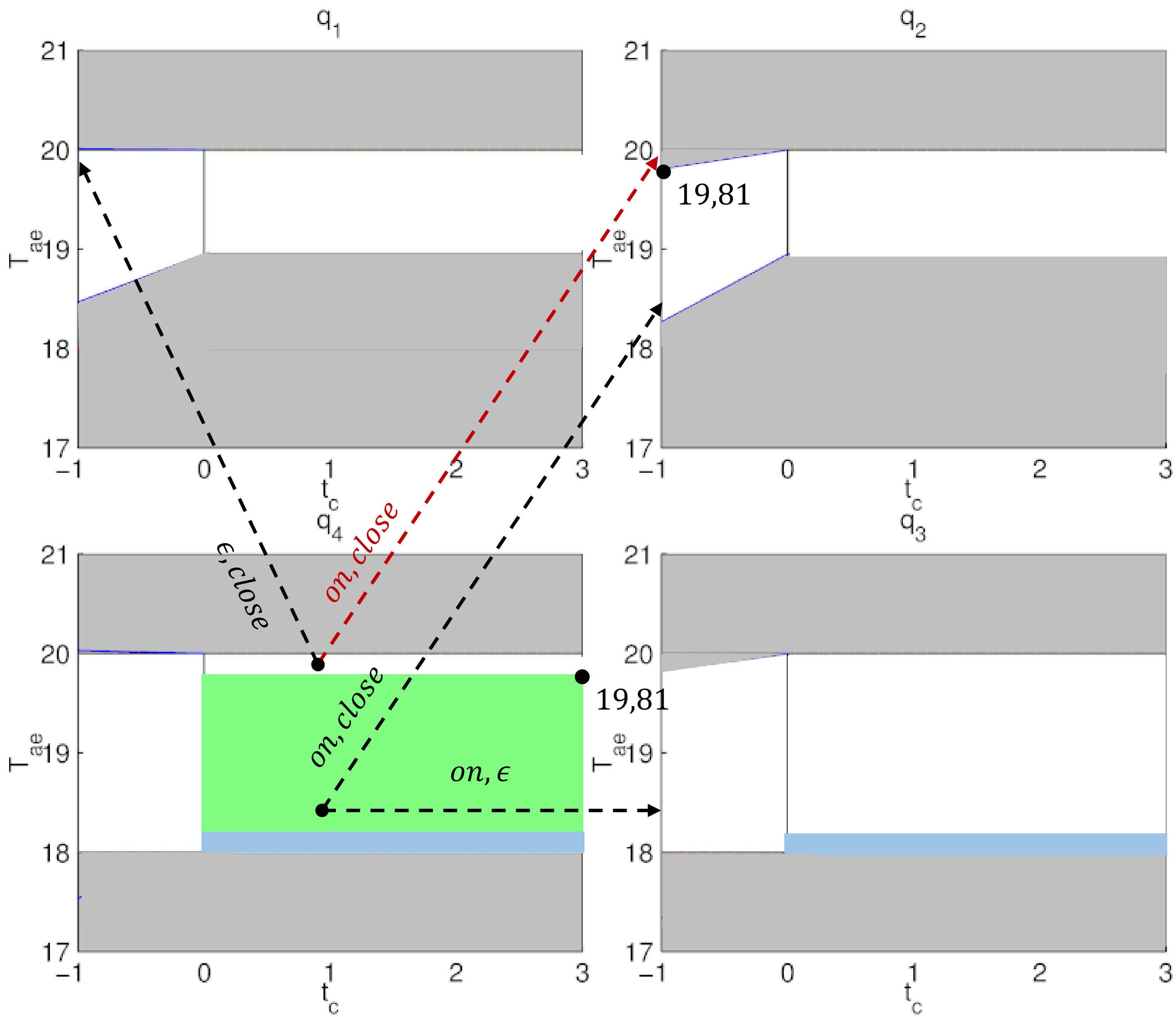

q1
q2
q4
q3
T_ae
t_c
on, ϵ
on, ϵ
ϵ, open
on, open
19,81

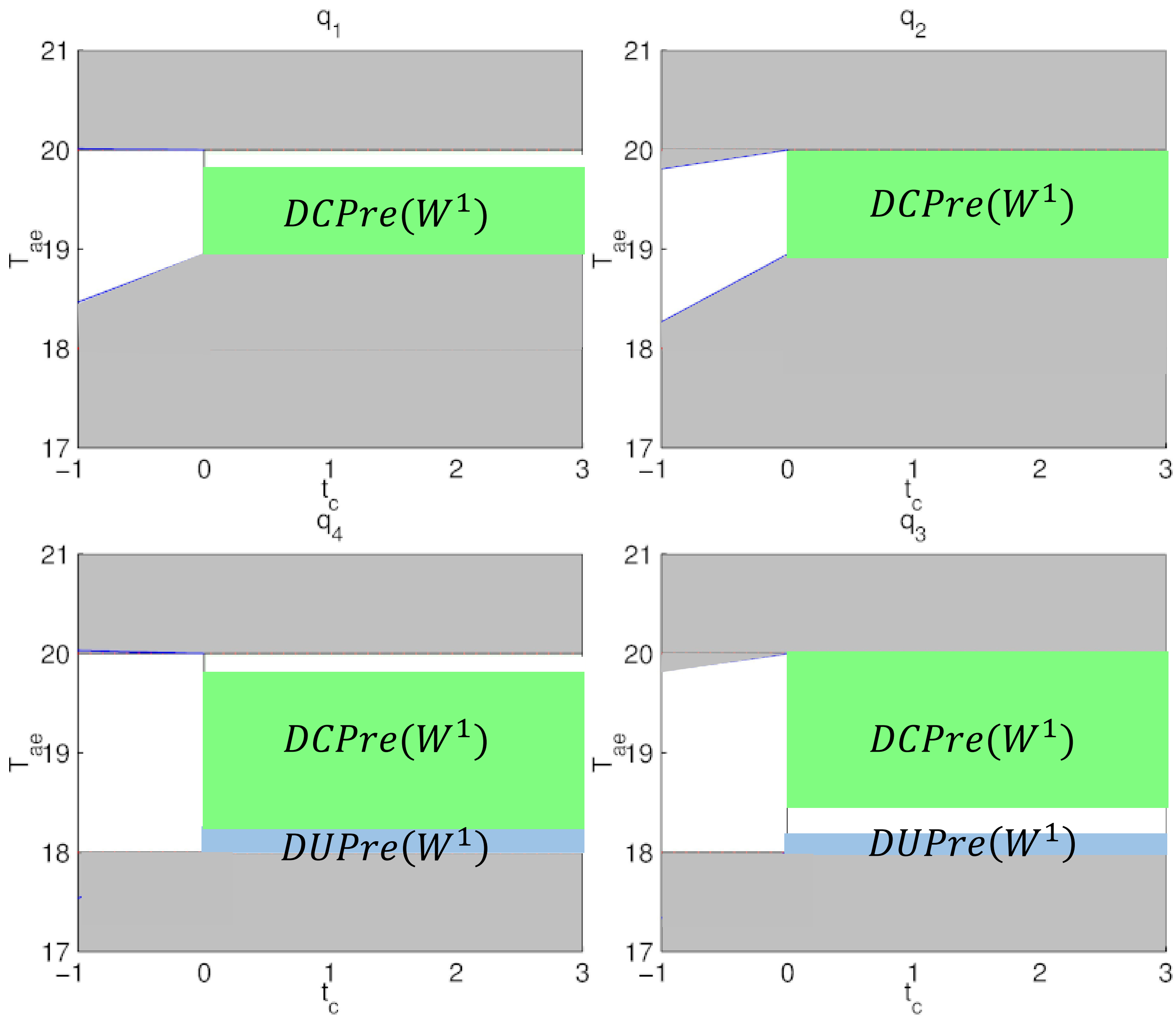


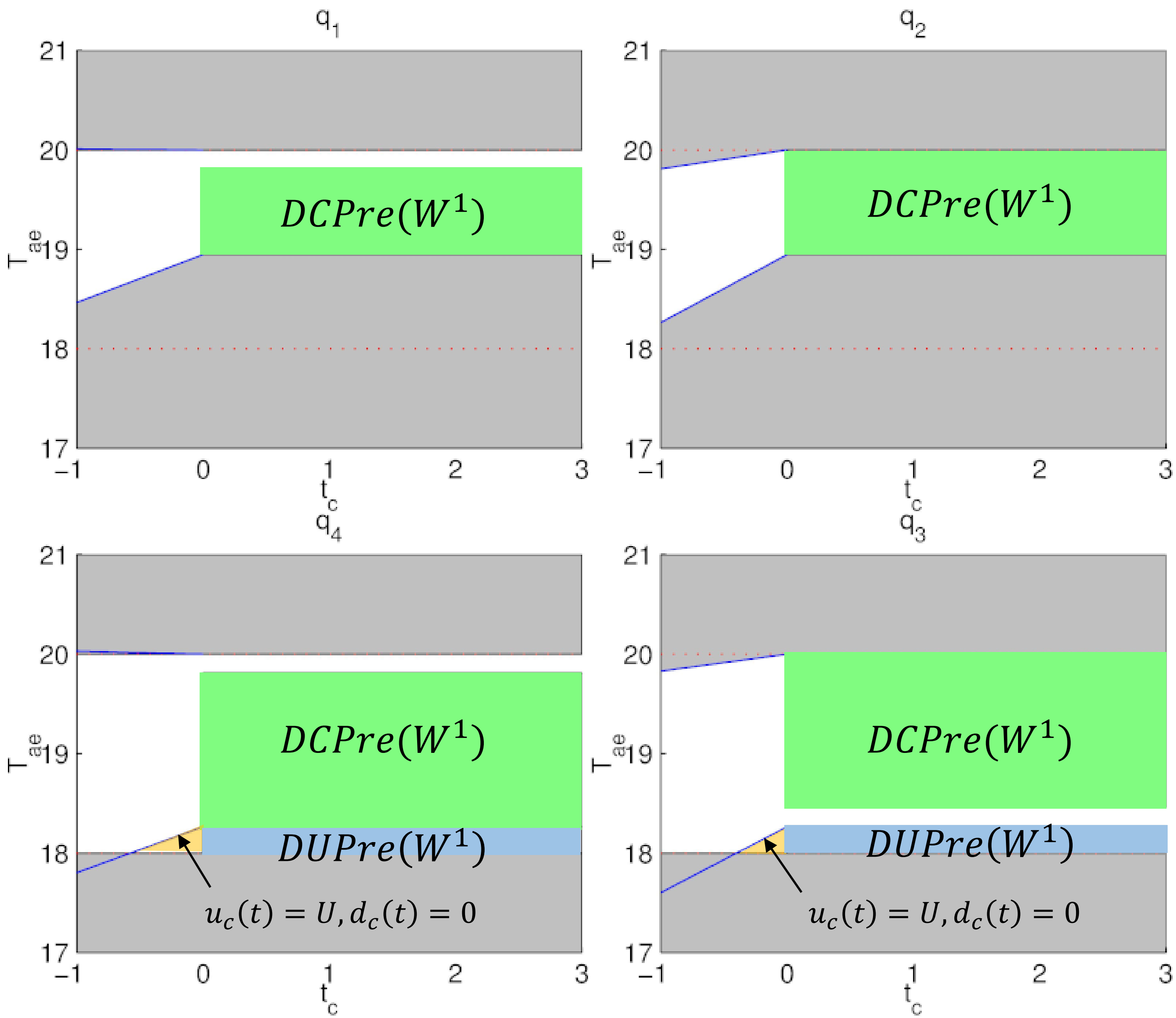
q1
q2
q4
q3
off, ϵ
$off, open$
T_{ae}
t_c

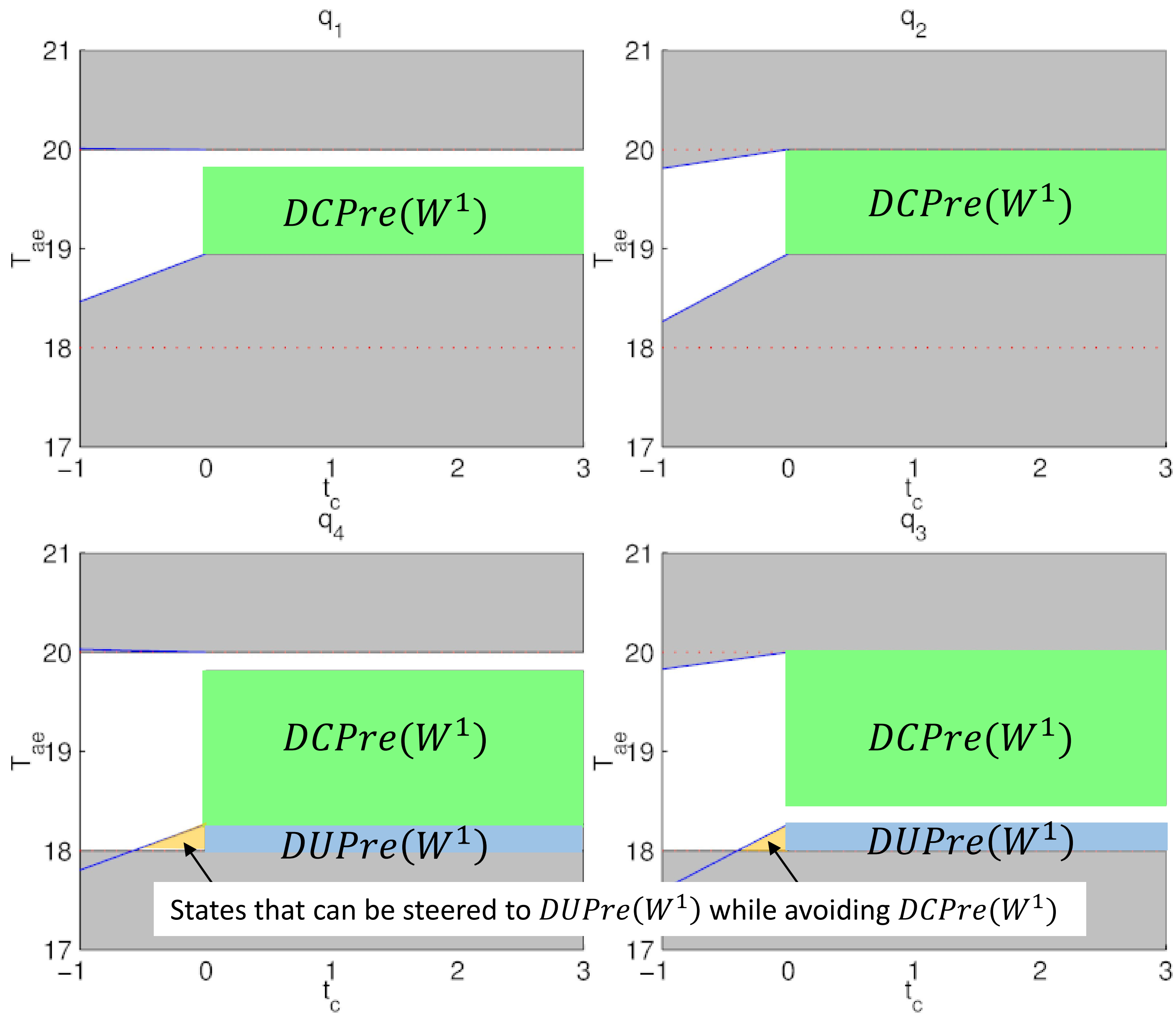


on, ϵ
q1
q2
q4
q3
21
20
19
18
17
Tae
−1
0
1
2
3
tc
18,47
18,27
off, close
ϵ, close
off, ϵ
off, close
18,47
18,27

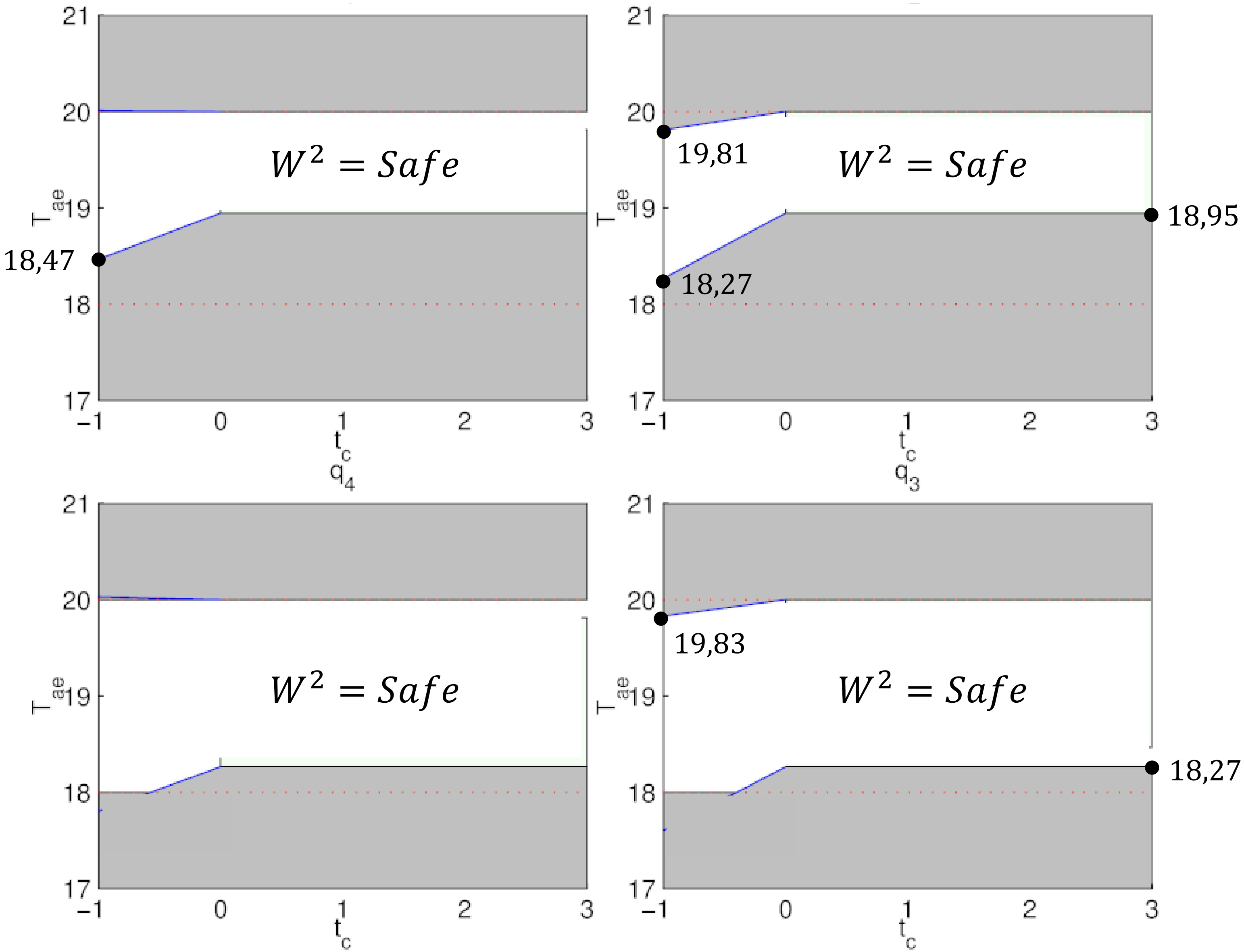

q1
q2
q4
q3
21
20
19
18
17
−1
0
1
2
3
t_c
T_ae
19,81
19,81
ε, close
on, close
on, close
on, ε


q1
q2
q4
q3
T_ae
t_c
DCPre(W^1)
DUPre(W^1)


q1
q2
q4
q3
21
20
19
18
17
−1
0
1
2
3
tc
Tae
DCPre(W1)
DUPre(W1)
uc(t) = U, dc(t) = 0


q1
q2
q4
q3
T_{ae}
t_c
$DCPre(W^1)$
$DUPre(W^1)$
States that can be steered to $DUPre(W^1)$ while avoiding $DCPre(W^1)$

Maximal Safe Set



Maximal Controller design