

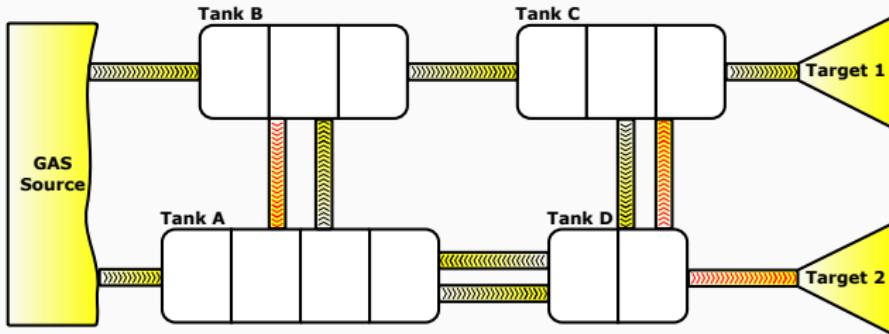
Systems Design Laboratory

Gas Tanks

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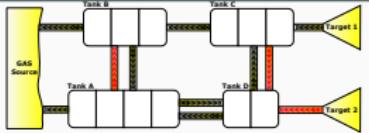
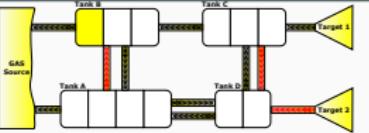
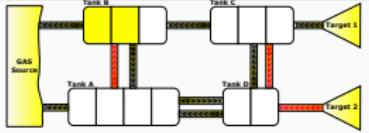
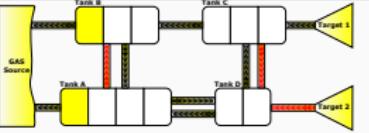
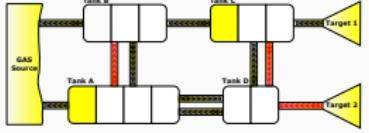
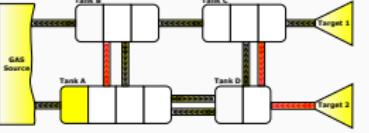
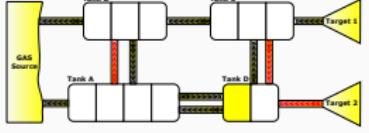
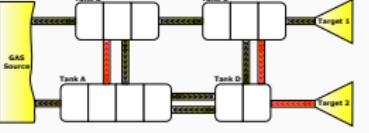
Plant



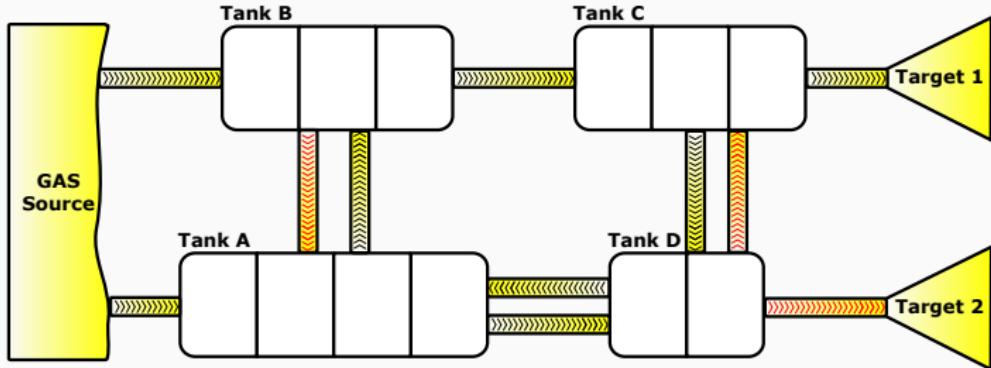
Main components:

- An infinite source of gas
- 2 extraction points (leftmost pipes)
- 4 tanks of different capacity (given in units of gas, e.g., m^3)
- 2 target points of infinite capacity
- Several other pipes to move single units of gas from one tank to another according to the direction of the arrows.
- Red pipes always allow to move gas (if any) from their source to their target.

Usecase example

1) IDLE	2) Source sends 1 gas unit to TankB
	
3) Source sends 1 gas unit to TankB	4) TankB sends 1 gas unit to TankA
	
5) TankB sends 1 gas unit to TankC	6) TankC sends 1 gas unit to Target 1
	
7) TankA sends 1 gas unit to TankD	8) TankD sends 1 gas unit to Target 2
	

The plant

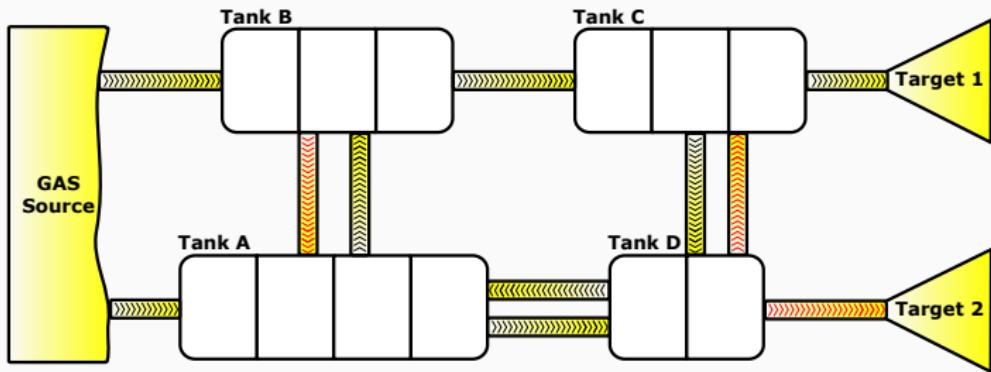


What should we model?



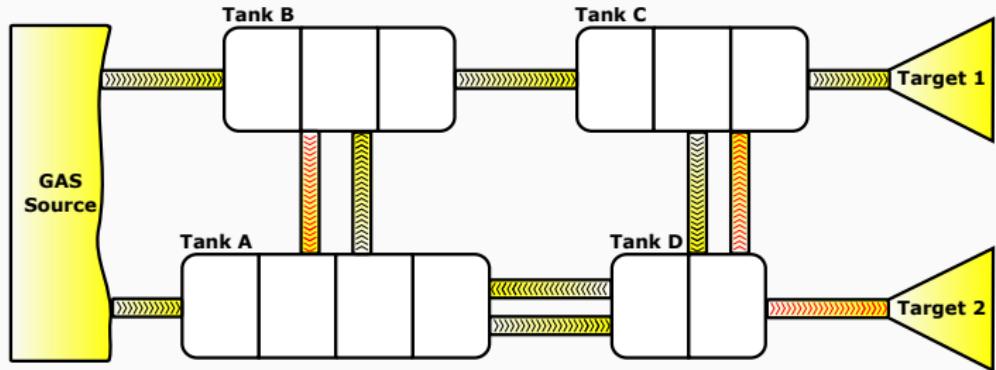
How many components?

Tank A

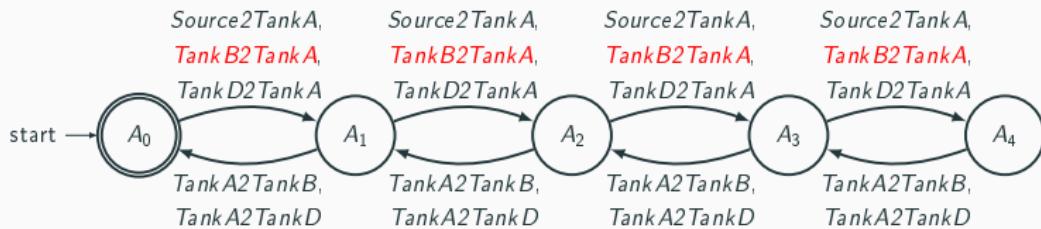
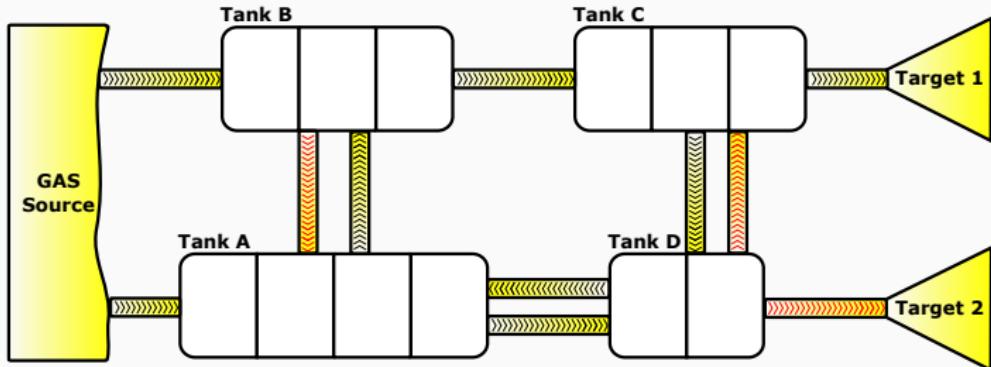


- States?
- Transitions?
- Event controllability?

Tank A



Tank A



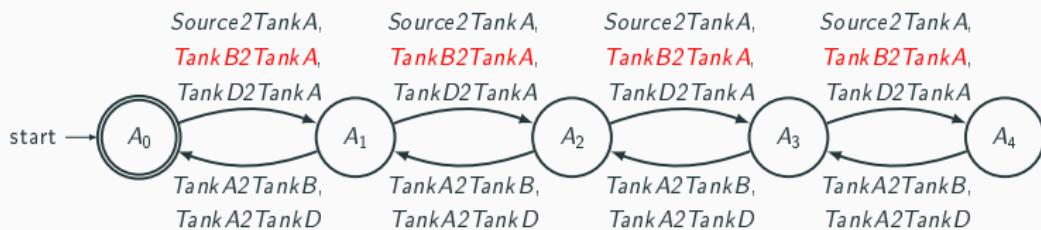
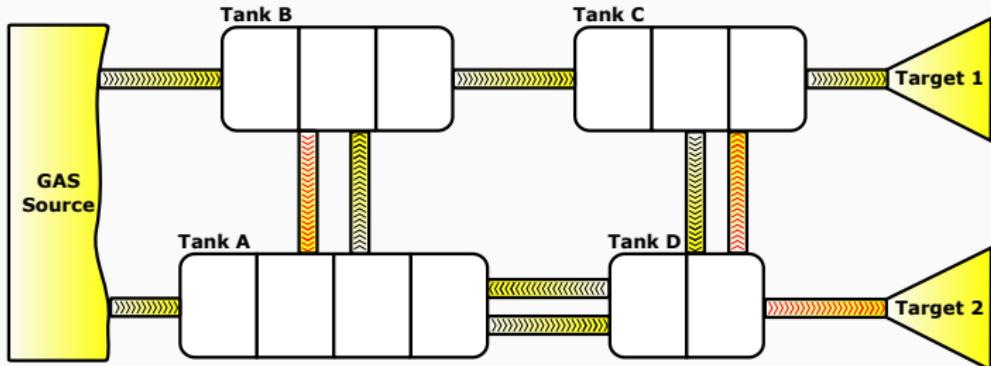
Event modeling intuition:

Source2TankA: source supplies 1 gas unit to TankA

TankB2TankA: TankB supplies 1 gas unit, by removing it from itself, to TankA

...

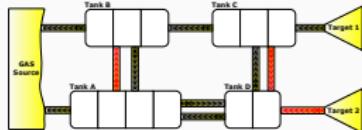
Tank A



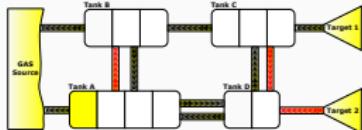
Realistic? Can't you spot a problem?

Realistically, without control might be like...

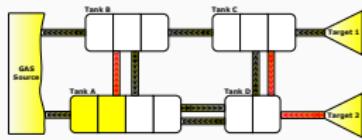
1) IDLE



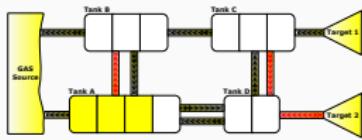
2) Source sends 1 gas unit to TankA



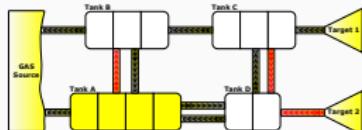
3) Source sends 1 gas unit to TankA



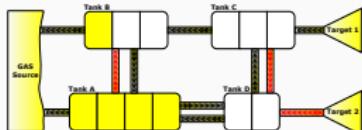
4) Source sends 1 gas unit to TankA



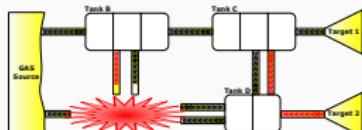
5) Source sends 1 gas unit to TankA



6) Source sends 1 gas unit to TankB

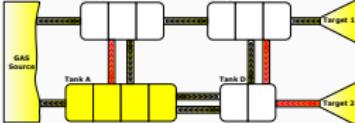
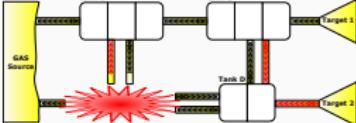
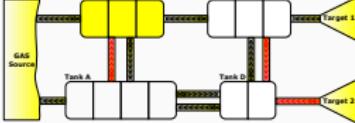
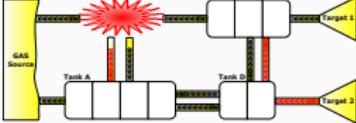
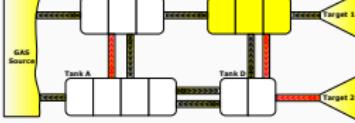
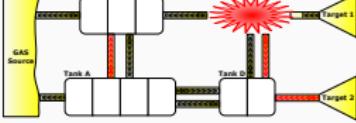
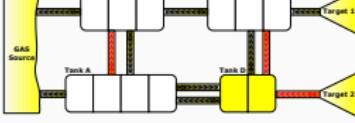
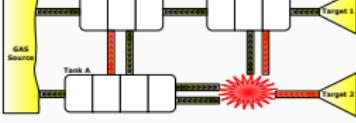


7) TankB sends 1 gas unit to TankA

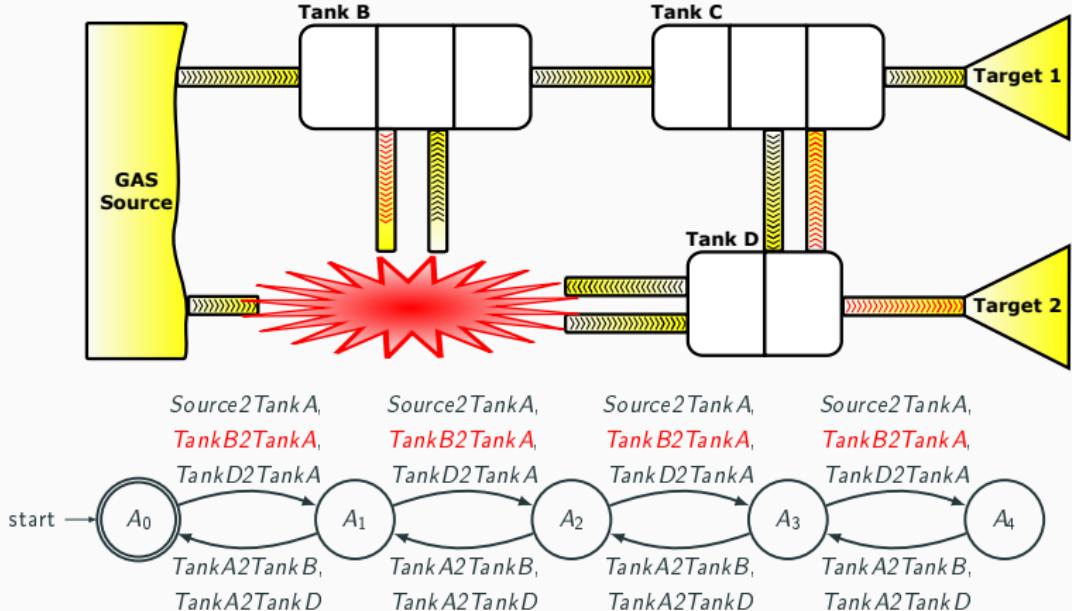


TankA blew up because it exceeded capacity. Note that *TankB2TankA* is uncontrollable.

Problem: If a tank exceeds its capacity, it blows up

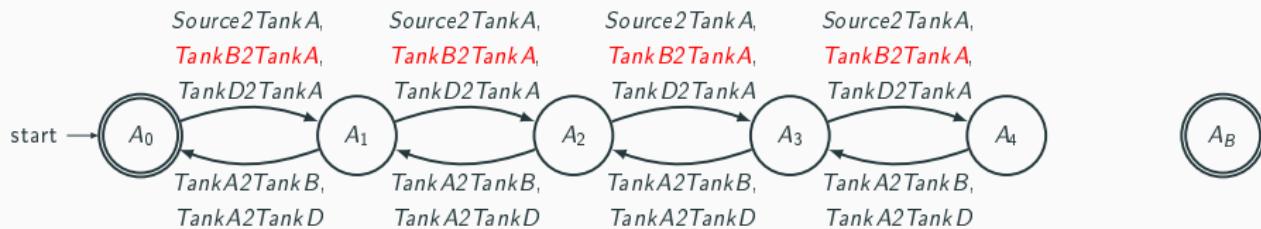
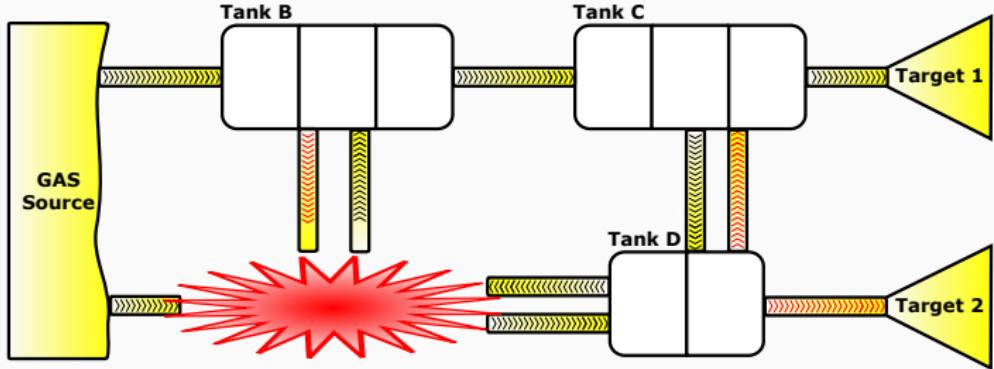
Critical state	Condition(s) leading to explosion
TankA  	If Source/TankB/TankD supplies TankA...
TankB  	If Source/TankA supplies TankB...
TankC  	If TankB/TankD supplies TankC...
TankD  	If TankA/TankC supplies TankD...

Tank A - Modeling the blow up situation



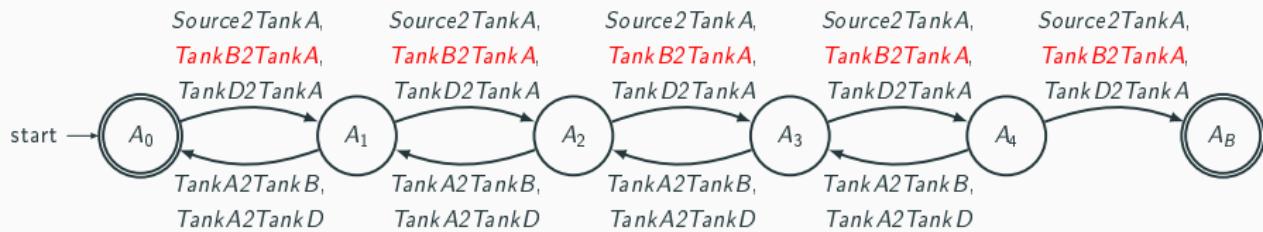
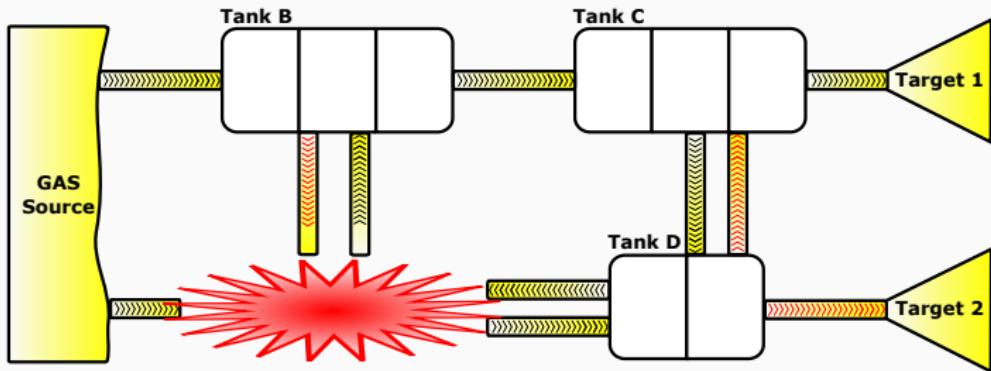
How can we model the fact that Tank A blows up?

Tank A



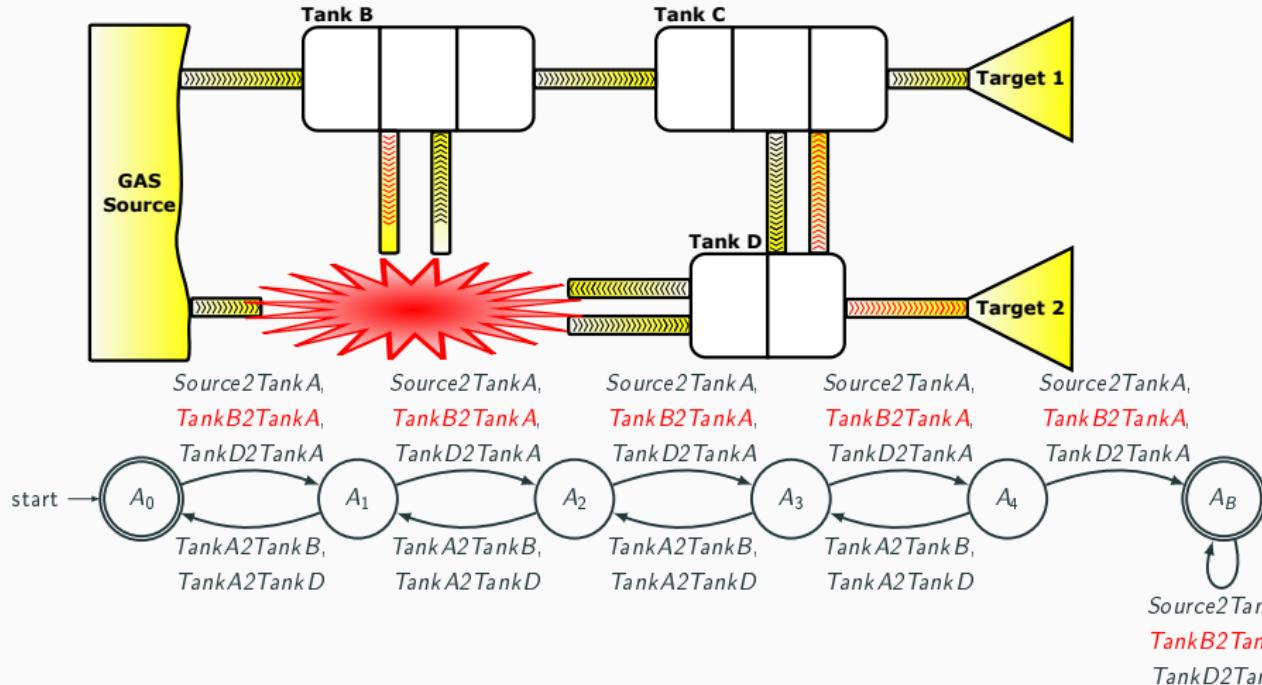
A new marked state to model “TankA has blown up”

Tank A



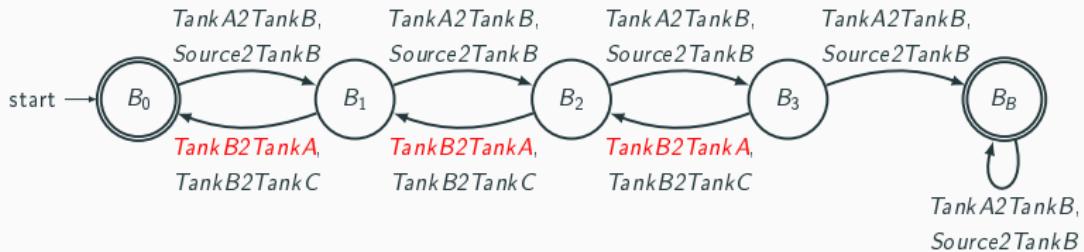
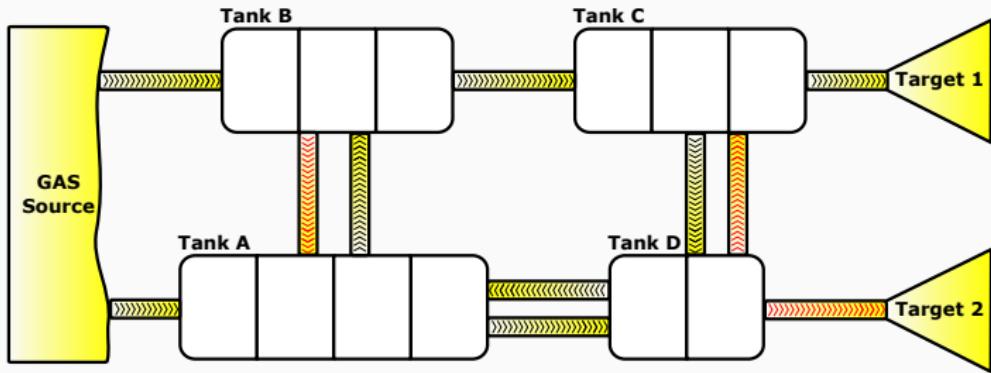
A_B is reachable if TankA exceeds its capacity.

Tank A

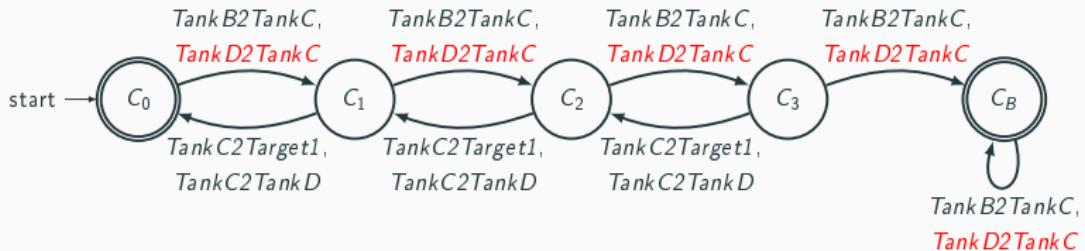
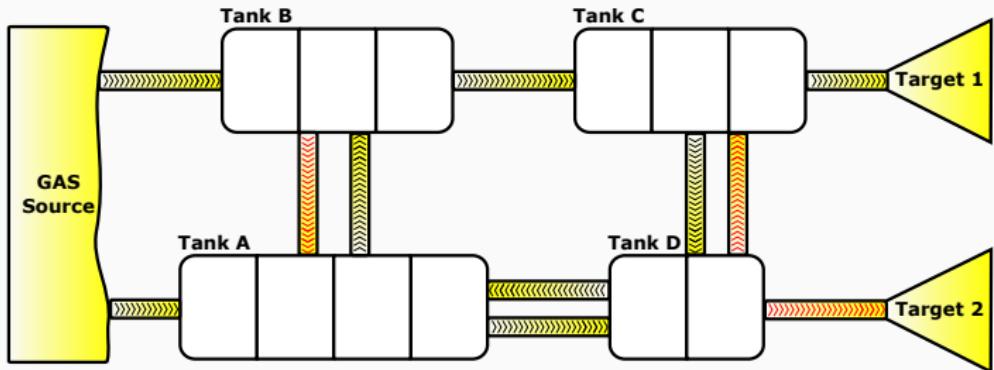


Once Tank A blows up, the pipes sending gas to it can still send gas. If this happens, the shipped units of gas are clearly lost.

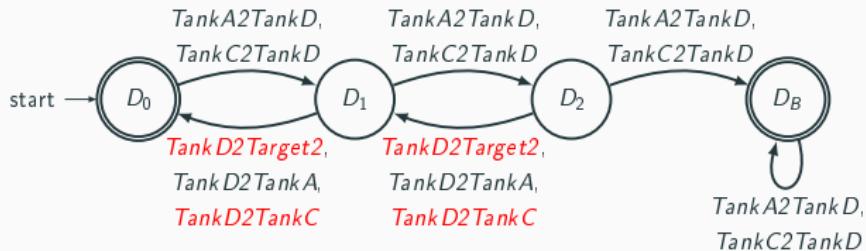
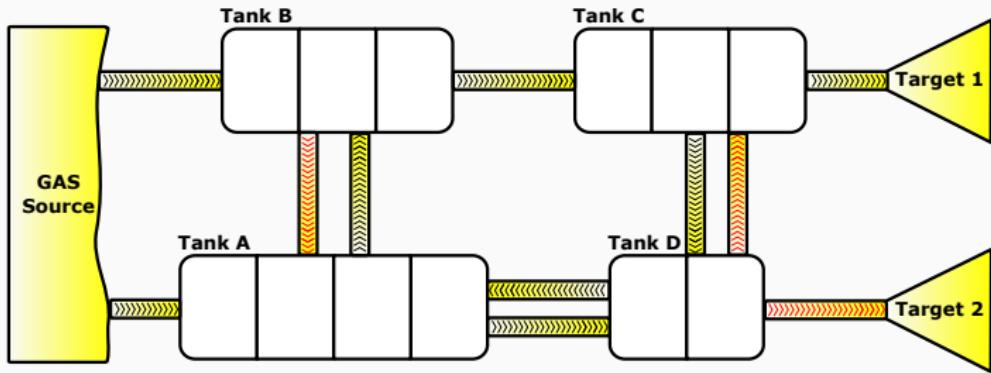
Tank B



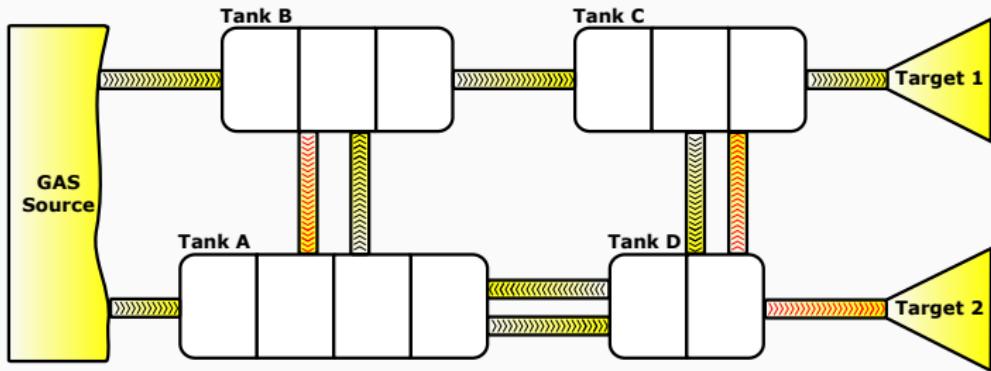
Tank C



Tank D

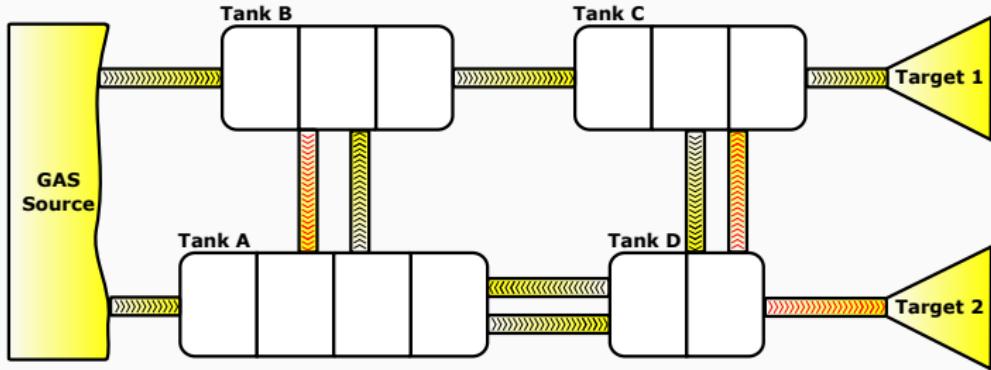


Source and Targets



- States?
- Transitions?
- Event controllability?

Source and Targets



Source

Target 1

Target 2

Source2TankA, Source2TankB

TankC2Target1

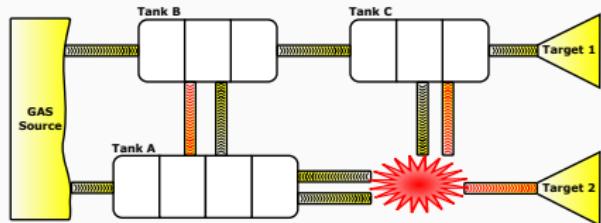
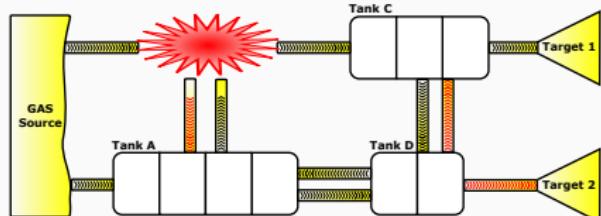
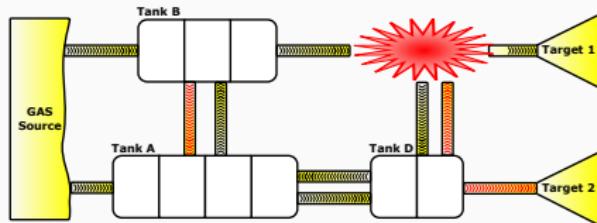
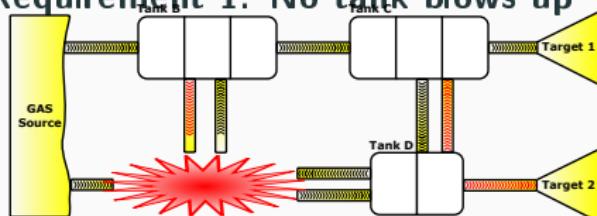
TankD2Target2



Not wrong, but quite useless.

Requirement 1

Requirement 1: No tank blows up



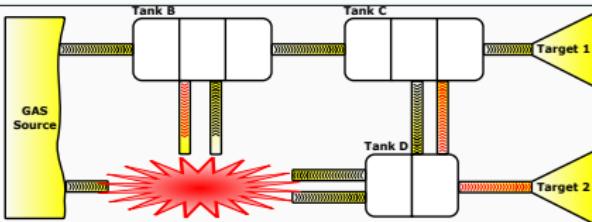
We may start from the parallel composition of all tanks ... (don't!)

Think: How many (separate) situations do we need to address?

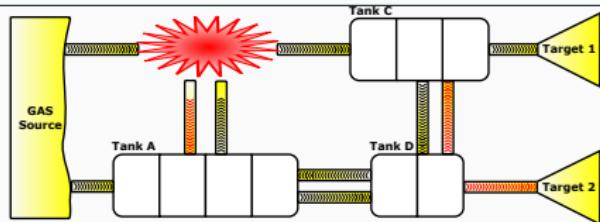
Requirement 1 - Decomposition

Requirement 1: No tank blows up

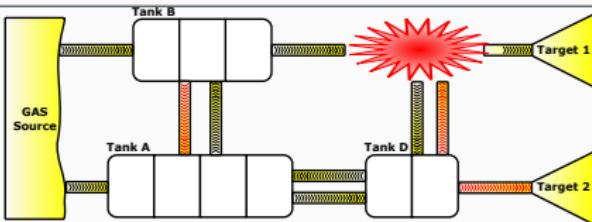
R_{1A} : Tank A does not blow up



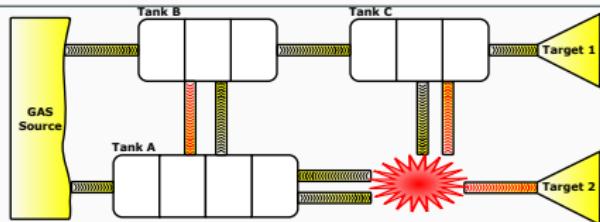
R_{1B} : Tank B does not blow up



R_{1C} : Tank C does not blow up

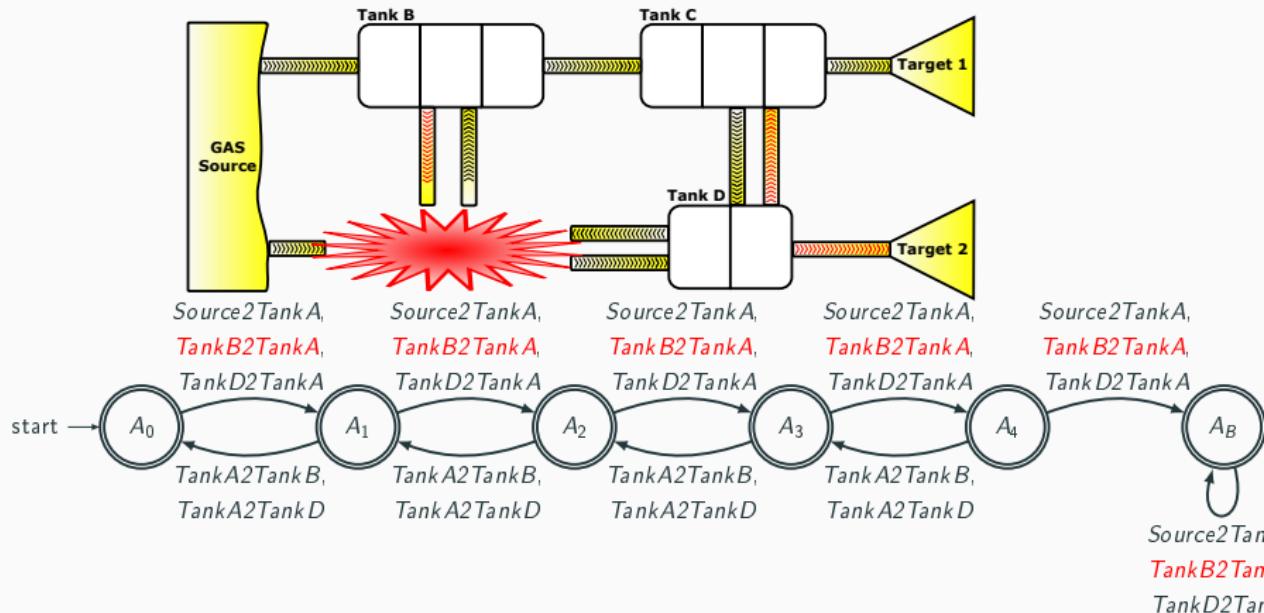


R_{1D} : Tank D does not blow up



Requirement 1A

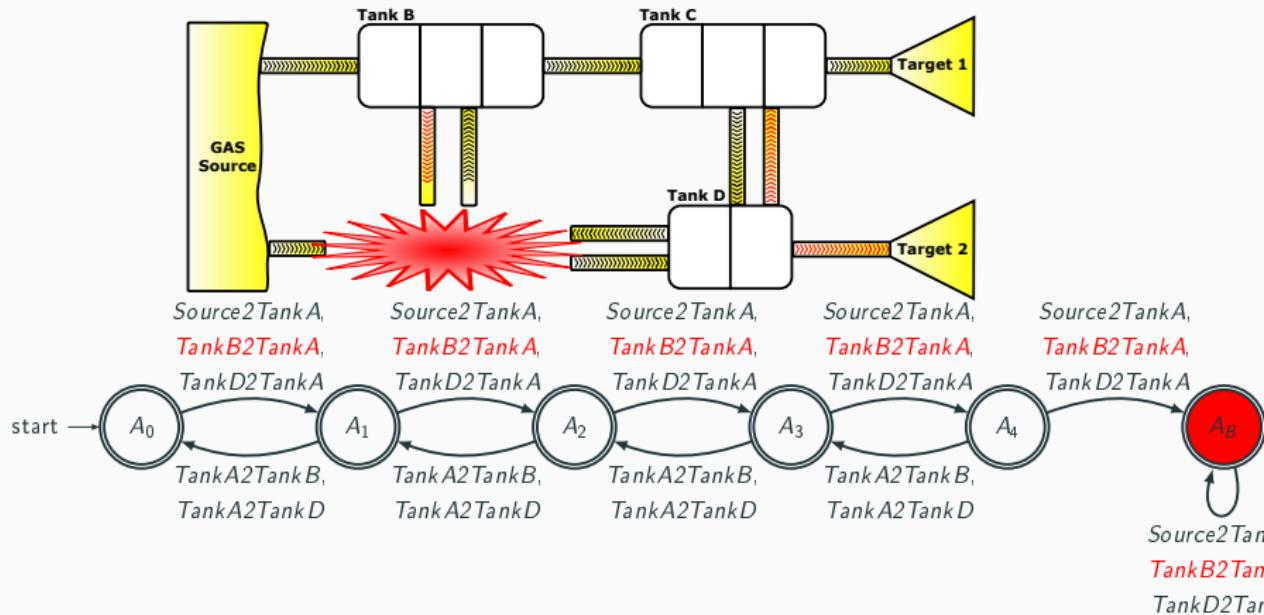
Requirement 1A: TankA does not blow up



Step 1: We need to keep track of the level of TankA

Requirement 1A

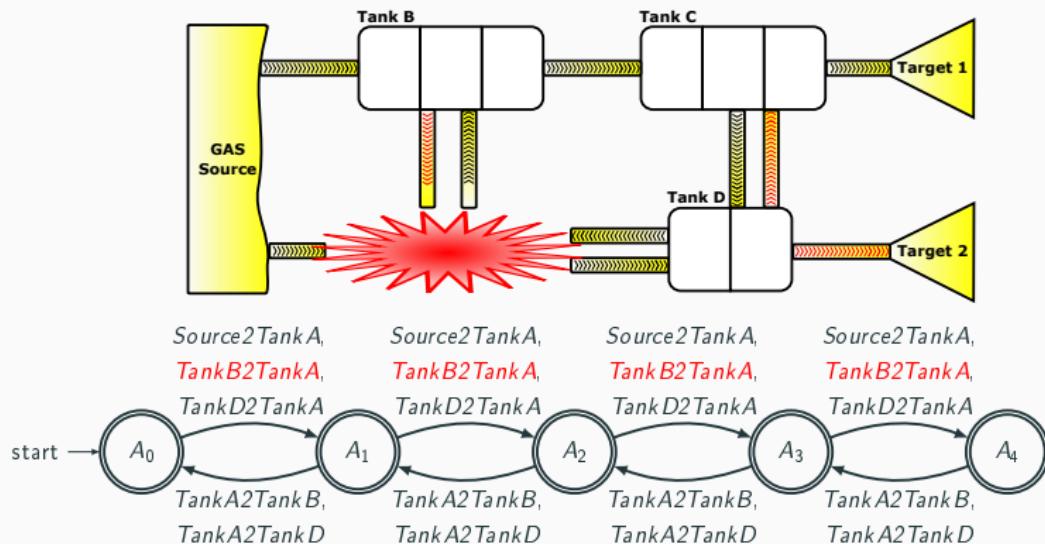
Requirement 1A: TankA does not blow up



Step 2: Detect and remove the illegal state A_B .

Requirement 1A

Requirement 1A: TankA does not blow up



Final requirement.

Requirements 1A, 1B, 1C, and 1D - Formalization

	R_{1A}
	<p>start — A_1 —> A_1: $TankA2\,TankB,\,TankB2\,TankA,\,TankD2\,TankA$ A_1 —> A_1: $TankA2\,TankB,\,TankB2\,TankD$ A_1 —> A_2: $TankA2\,TankB,\,TankB2\,TankA,\,TankD2\,TankA$ A_2 —> A_2: $TankA2\,TankB,\,TankB2\,TankD$ A_2 —> A_3: $TankA2\,TankB,\,TankB2\,TankA,\,TankD2\,TankA$ A_3 —> A_3: $TankA2\,TankB,\,TankB2\,TankD$ A_3 —> A_4: $TankA2\,TankB,\,TankB2\,TankA,\,TankD2\,TankA$ A_4 —> A_4: $TankA2\,TankB,\,TankB2\,TankD$ A_4 —> A_B: $TankA2\,TankB,\,TankB2\,TankA,\,TankD2\,TankA$ A_B —> A_B: $TankA2\,TankB,\,TankB2\,TankA,\,TankD2\,TankA$</p>
	R_{1B} : TankB does not blow up.
	<p>start — B_1 —> B_1: $TankA2\,TankB,\,Source2\,TankB$ B_1 —> B_1: $TankB2\,TankA,\,TankB2\,TankC$ B_1 —> B_2: $TankA2\,TankB,\,Source2\,TankB$ B_2 —> B_2: $TankB2\,TankA,\,TankB2\,TankC$ B_2 —> B_3: $TankA2\,TankB,\,Source2\,TankB$ B_3 —> B_3: $TankB2\,TankA,\,TankB2\,TankC$ B_3 —> B_B: $TankA2\,TankB,\,Source2\,TankB$ B_B —> B_B: $TankA2\,TankB,\,Source2\,TankB$</p>
	R_{1C} : TankC does not blow up.
	<p>start — C_1 —> C_1: $TankB2\,TankC,\,TankD2\,TankC$ C_1 —> C_1: $TankC2\,Target1,\,TankC2\,TankD$ C_1 —> C_2: $TankB2\,TankC,\,TankD2\,TankC$ C_2 —> C_2: $TankC2\,Target1,\,TankC2\,TankD$ C_2 —> C_3: $TankB2\,TankC,\,TankD2\,TankC$ C_3 —> C_3: $TankC2\,Target1,\,TankC2\,TankD$ C_3 —> C_B: $TankB2\,TankC,\,TankD2\,TankC$ C_B —> C_B: $TankB2\,TankC,\,TankD2\,TankC$</p>
	R_{1D} : TankD does not blow up.
	<p>start — D_1 —> D_1: $TankA2\,TankD,\,TankC2\,TankD$ D_1 —> D_1: $TankD2\,Target2,\,TankD2\,TankA,\,TankD2\,TankC$ D_1 —> D_2: $TankA2\,TankD,\,TankC2\,TankD$ D_2 —> D_2: $TankD2\,Target2,\,TankD2\,TankA,\,TankD2\,TankC$ D_2 —> D_3: $TankA2\,TankD,\,TankC2\,TankD$ D_3 —> D_3: $TankD2\,Target2,\,TankD2\,TankA,\,TankD2\,TankC$ D_3 —> D_B: $TankA2\,TankD,\,TankC2\,TankD$ D_B —> D_B: $TankA2\,TankD,\,TankC2\,TankD$</p>

Step 1: Start with marked copies of TankA, TankB, TankC, and TankD

Requirements 1A, 1B, 1C, and 1D - Formalization

	R_{1A}
	<p>Formalization:</p> <pre> graph LR start((start)) -- "TankA2.TankB, Source2.TankA, TankB2.TankA, TankD2.TankA" --> A1((A1)) A1 -- "TankA2.TankB, TankA2.TankD" --> A2((A2)) A2 -- "TankA2.TankB, TankA2.TankD" --> A3((A3)) A3 -- "TankA2.TankB, TankA2.TankD" --> A4((A4)) A4 -- "TankA2.TankB, TankA2.TankD" --> A5((A5)) A5 -- "TankA2.TankB, TankA2.TankD" --> A5 </pre>
R_{1B} : TankB does not blow up.	<p>Formalization:</p> <pre> graph LR start((start)) -- "TankA2.TankB, Source2.TankB, TankB2.TankA, TankD2.TankA" --> B1((B1)) B1 -- "TankA2.TankB, Source2.TankB, TankB2.TankC" --> B2((B2)) B2 -- "TankA2.TankB, Source2.TankB, TankB2.TankC" --> B3((B3)) B3 -- "TankA2.TankB, Source2.TankB, TankB2.TankC" --> B4((B4)) B4 -- "TankA2.TankB, Source2.TankB, TankB2.TankC" --> B4 </pre>
R_{1C} : TankC does not blow up.	<p>Formalization:</p> <pre> graph LR start((start)) -- "TankA2.TankC, TankD2.TankC, TankB2.TankC, TankD2.TankC" --> C1((C1)) C1 -- "TankC2.Target1, TankC2.TankD" --> C2((C2)) C2 -- "TankC2.Target1, TankC2.TankD" --> C3((C3)) C3 -- "TankC2.Target1, TankC2.TankD" --> C4((C4)) C4 -- "TankC2.Target1, TankC2.TankD" --> C4 </pre>
R_{1D} : TankD does not blow up.	<p>Formalization:</p> <pre> graph LR start((start)) -- "TankA2.TankD, TankC2.TankD, TankB2.TankD, TankC2.TankD" --> D1((D1)) D1 -- "TankD2.Target2, TankD2.TankA, TankD2.TankC" --> D2((D2)) D2 -- "TankD2.Target2, TankD2.TankA, TankD2.TankC" --> D3((D3)) D3 -- "TankD2.Target2, TankD2.TankA, TankD2.TankC" --> D4((D4)) D4 -- "TankD2.Target2, TankD2.TankA, TankD2.TankC" --> D4 </pre>

Step 2: Identify and remove the illegal states A_B , B_B , C_B , and D_B .

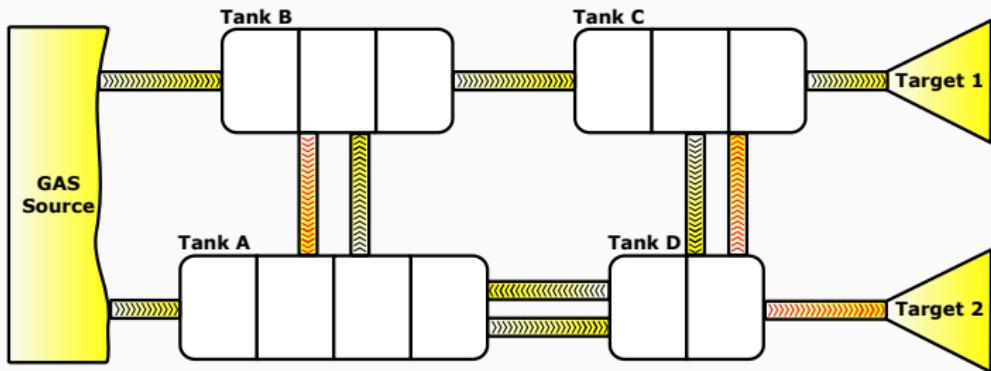
Requirements 1A, 1B, 1C, and 1D - Formalization

	<p>R_{1A}</p>
	<p>R_{1B}: TankB does not blow up.</p>
	<p>R_{1C}: TankC does not blow up.</p>
	<p>R_{1D}: TankD does not blow up.</p>

Final sub-requirements. $R_1 := R_{1A} \parallel R_{1B} \parallel R_{1C} \parallel R_{1D}$.

Requirement 2

Requirement 2: The plant must deliver at least 1 gas unit to each Target



How many automata?

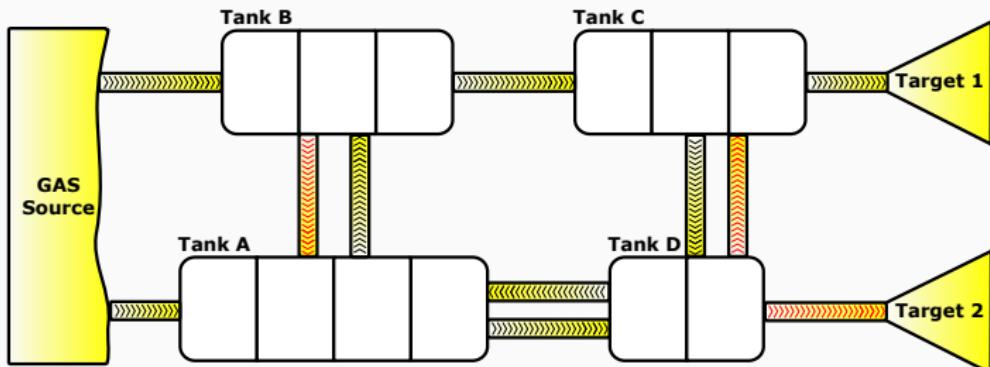
Requirement 2 - Decomposition

Requirement 2: The plant must deliver at least 1 gas unit to each Target



R_{2A} : The plant must deliver at least 1 gas unit to Target 1

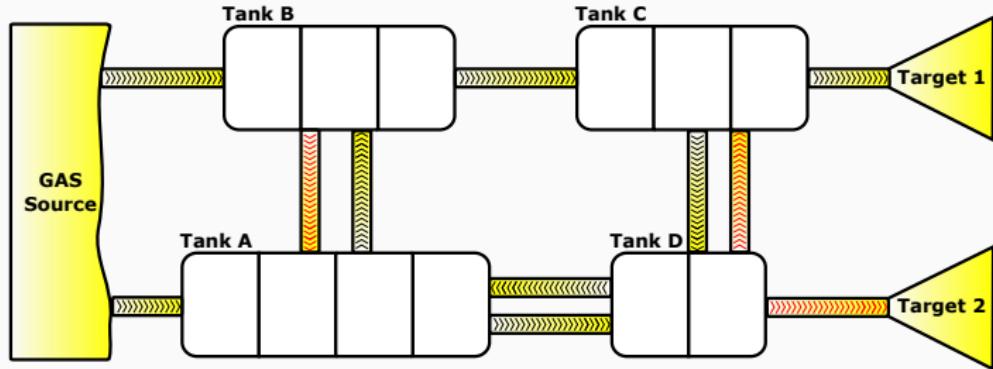
R_{2B} : The plant must deliver at least 1 gas unit to Target 2



Hint: some task must be completed

Requirement 2 - Target 1

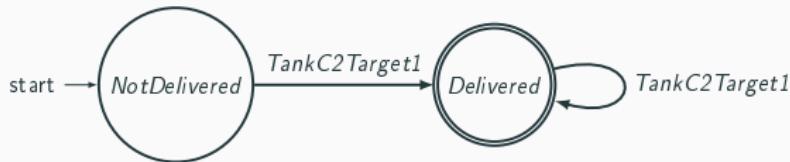
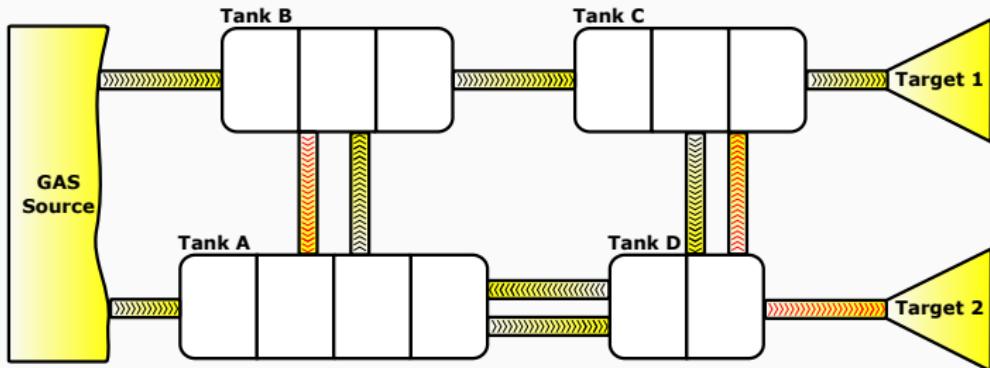
Requirement R_{2A} : The plant must deliver at least 1 gas unit to Target 1



- States?
- Marking?
- Transitions?
- Event controllability?

Requirement 2 - Target 1

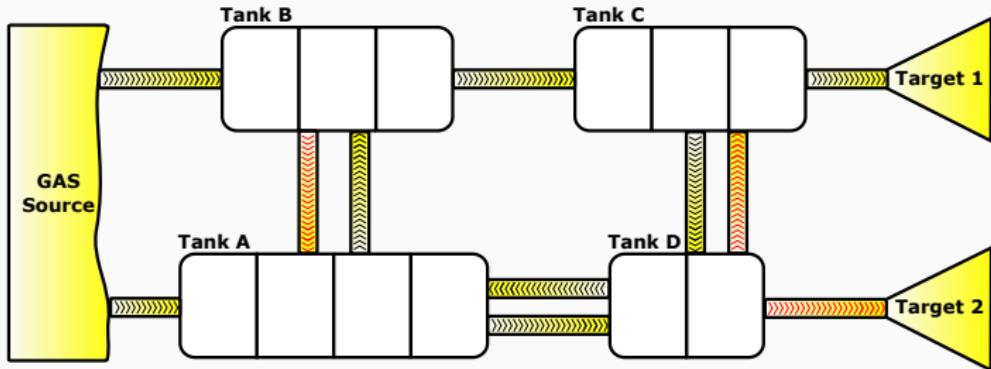
Requirement R_{2A} : The plant must deliver at least 1 gas unit to Target 1



Take a moment to reason on the marking.

Requirement 2 - Target 2

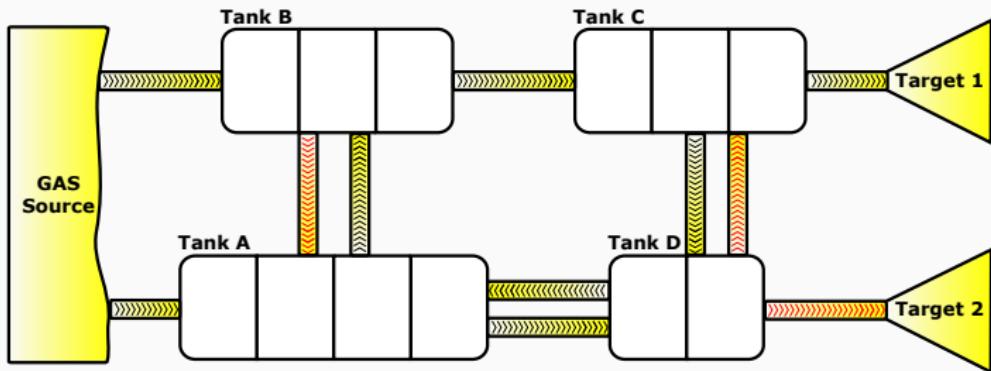
Requirement R_{2B} : The plant must deliver at least 1 gas unit to Target 2



So, $R_2 := R_{2A} \parallel R_{2B}$

Requirement 3

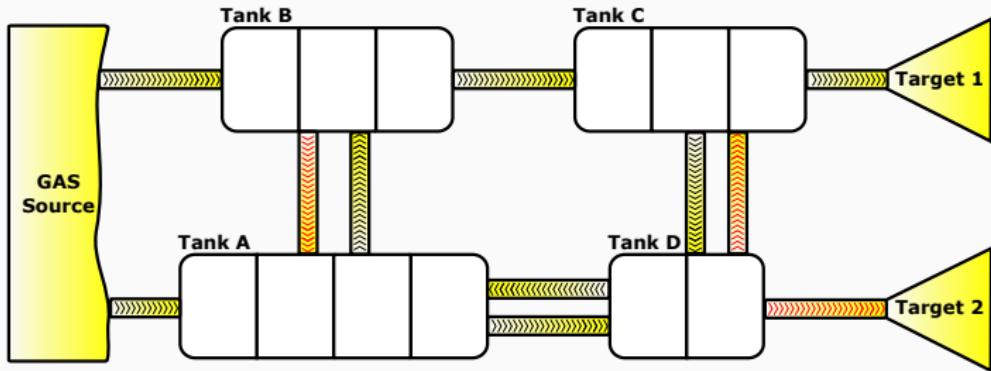
Requirement 3: Gas deliveries to Target 1 and Target 2 must always alternate



Straightforward but there is something to pay attention to... what?

Requirement 3

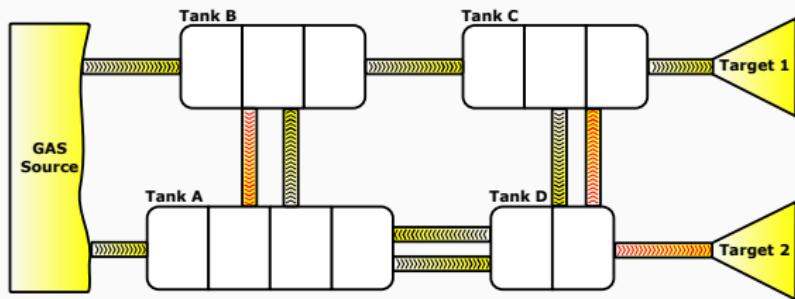
Requirement 3: Gas deliveries to Target 1 and Target 2 must always alternate



Which target will be supplied first? That's up to you.. but the requirement doesn't actually impose such a condition...So?

Requirement 3 - Attempt 1 - Non-deterministic

Requirement 3: Gas deliveries to Target 1 and Target 2 must always alternate



R_{3A} : If Target 1 is supplied first...

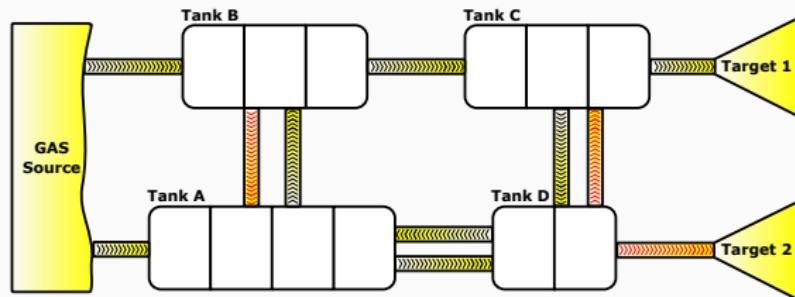
???

R_{3B} : If Target 2 is supplied first...

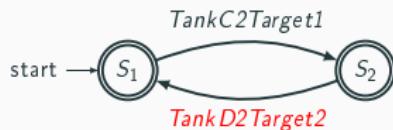
???

Requirement 3 - Attempt 1 - Non-deterministic

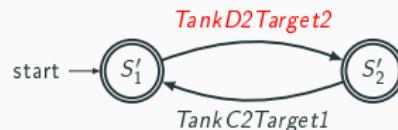
Requirement 3: Gas deliveries to Target 1 and Target 2 must always alternate



R_{3A} : If Target 1 is supplied first...



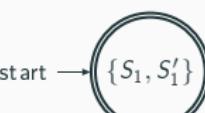
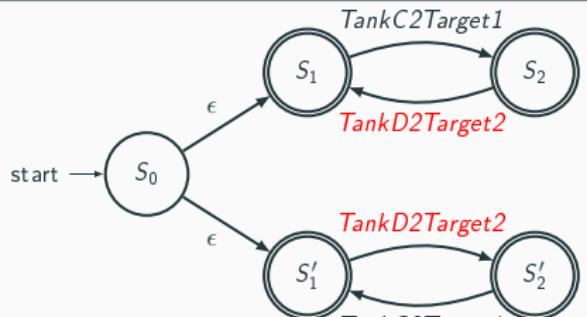
R_{3B} : If Target 2 is supplied first...



Not certainly an AND of the two automata.
We need the UNION of these two automata!

Requirement 3 - Attempt 1 - Nondeterministic

Requirement 3: Gas deliveries to Target 1 and Target 2 must always alternate

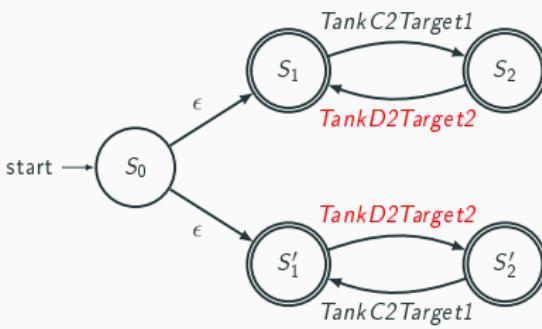
Requirement R_{3A}	Requirement R_{3B}
	
$R_{3A} \wedge R_{3B} := R_{3A} \ R_{3B} = R_{3A} \times R_{3B}$	$R_{3A} \vee R_{3B}$
	

Homework: synthesize a supervisor that (also) takes into consideration requirement $R_{3A} \wedge R_{3B}$. What effect does it have on the plant?

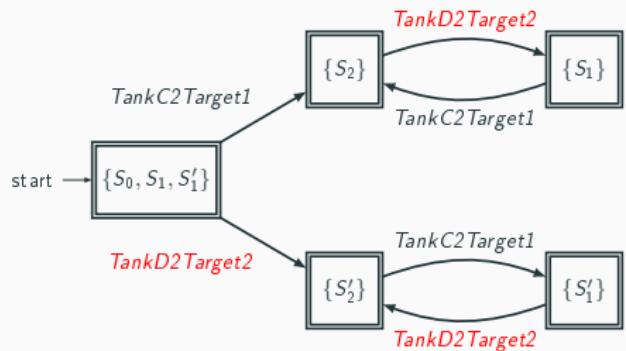
Requirement 3 - Attempt 1 - Nondeterministic

Requirement 3: Gas deliveries to Target 1 and Target 2 must always alternate

NFA



DFA



Requirement 3 - Attempt 2 - Deterministic

Requirement 3: Gas deliveries to Target 1 and Target 2 must always alternate



3A) If Target 1 is supplied, then Target 2 must be supplied at least once before Target 1 is supplied again.

3B) If Target 2 is supplied, then Target 1 must be supplied at least once before Target 2 is supplied again.

...

...

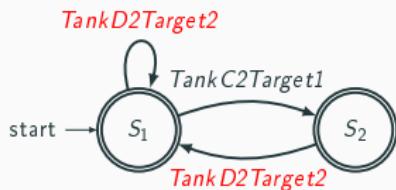
If Target $i = 1, 2$ is supplied, then Target $(i \bmod 2) + 1$ must be supplied at least once before Target i is supplied again.

Requirement 3 - Attempt 2 - Deterministic

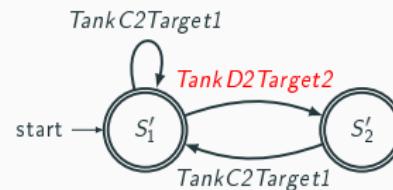
Requirement 3: Gas deliveries to Target 1 and Target 2 must always alternate



3A) If Target 1 is supplied, then Target 2 must be supplied at least once before Target 1 is supplied again.



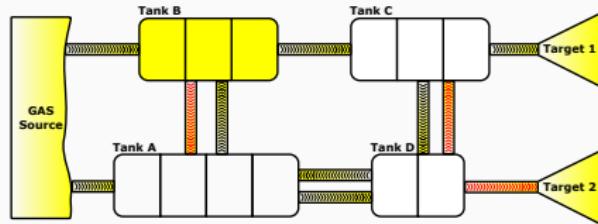
3B) If Target 2 is supplied, then Target 1 must be supplied at least once before Target 2 is supplied again.



If Target $i = 1, 2$ is supplied, then Target $(i \bmod 2) + 1$ must be supplied at least once before Target i is supplied again.

Requirement 4

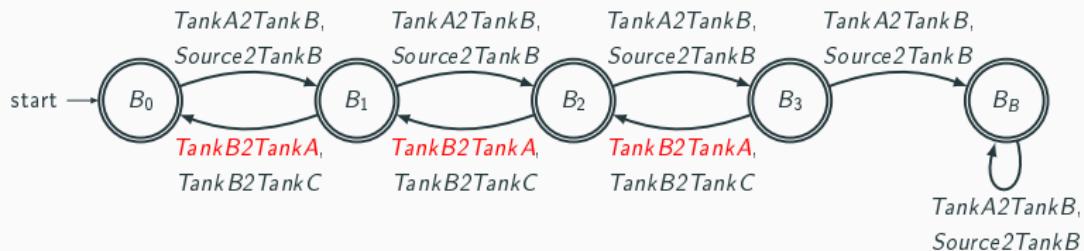
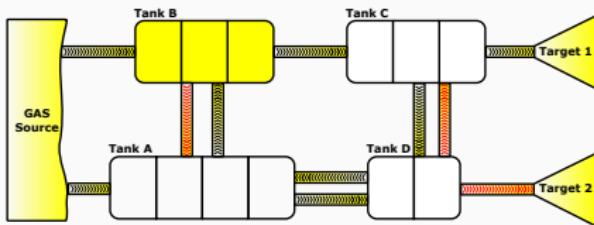
Requirement 4: TankB can supply TankC only if TankB is full



- States?
- Transitions?
- Event controllability?

Requirement 4

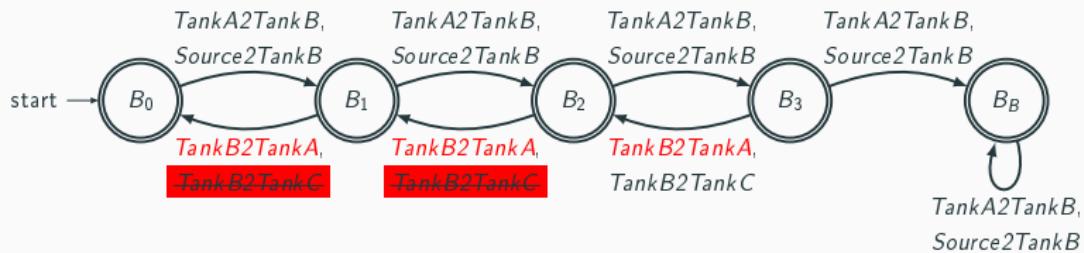
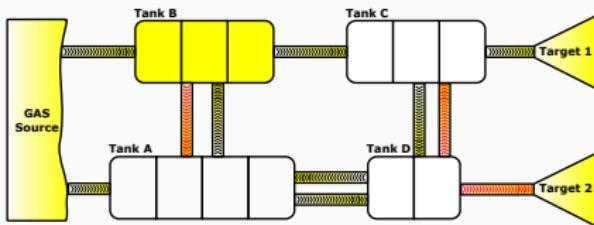
Requirement 4: TankB can supply TankC only if TankB is full



Step 1: Start from a copy of the automaton for Tank B.

Requirement 4

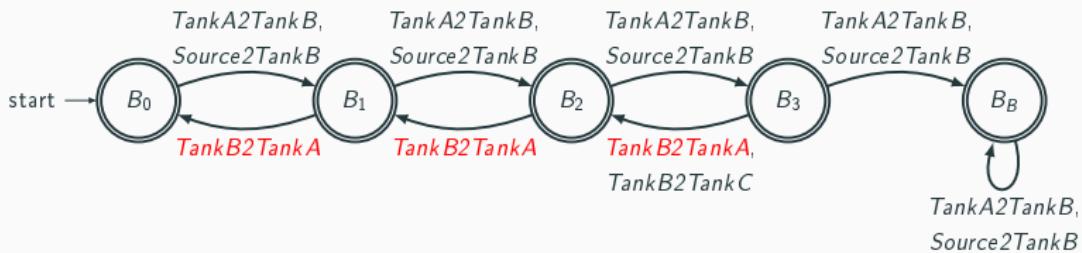
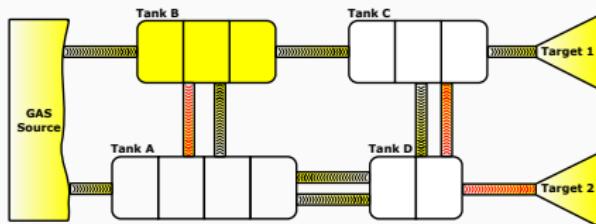
Requirement 4: TankB can supply TankC only if TankB is full



Step 2: Detect and remove the illegal transitions.

Requirement 4

Requirement 4: TankB can supply TankC only if TankB is full



Final requirement.