$R \subseteq A \times A$ RIFL / TRANS / SIMM $\sim \quad \simeq \quad \cong \quad \equiv$

$f : A \to B \qquad \sim_f \subseteq A \times A$

$$\boxed{x \sim_f y \iff f(x) = f(y)}$$

RIFL $\quad \forall a \quad a \sim_f a \quad f(a) = f(a)$ V

TRANS $\quad \forall a, b, c \quad \underbrace{a \sim_f b}_{f(a)=f(b)} \;\&\; \underbrace{b \sim_f c}_{f(b)=f(c)} \Rightarrow a \sim_f c$

$$\underbrace{f(a)=f(b) \qquad f(b)=f(c)}_{f(a)=f(c)} \longrightarrow a \sim_f c$$

SIM $\quad \forall a, b \quad \underbrace{a \sim_f b} \Rightarrow b \sim_f a$

$f(a) = f(b) \Rightarrow f(b) = f(a) \Rightarrow b \sim_f a$

CLASSE DI EQUIVALENZA

$X = Y$
$\iff$
$X \subseteq Y$
&
$Y \subseteq X$

$\sim \subseteq A \times A$

$a \in A \qquad [a]_\sim = \{ x \mid x \sim a \}$

$\forall a \in A \quad [a]_\sim \neq \emptyset \quad a \sim a \qquad a \in [a]$

$[a]_\sim = \{ y \mid a \sim y \} = B$

$[a]_\sim \subseteq B \quad \forall x \quad x \in [a]_\sim \Rightarrow x \in B \qquad x \in [a]_\sim \iff x \sim a \Rightarrow$
$\iff a \sim x \iff x \in B$

$B \subseteq [a]_\sim \quad \forall x \ x \in B \Rightarrow x \in [a]_\sim$

$A \qquad \sim \subseteq A \times A$

$$\underbrace{A/_{\sim}}_{\Downarrow} = \{ [a]_{\sim} \mid a \in A \}$$

INSIEME QUOZIENTE RISPETTO A $\sim$

$\pi : A \to A/_{\sim} \qquad \pi(a) = [a]_{\sim}$

$A, \sim$

$[x]_\sim$

$\uparrow$

RAPPRESENT.

$$[a]_\sim = [b]_\sim \iff a \sim b$$

$$\Rightarrow$$

$$a \in [a]_\sim = [b]_\sim \Rightarrow a \in [b]_\sim \Rightarrow a \sim b$$

$$\Leftarrow$$

$$\underbrace{[a]_\sim \subseteq [b]_\sim} \quad \& \quad \underbrace{[b]_\sim \subseteq [a]_\sim}$$

$$x \in [a]_\sim \qquad x \sim a \quad \Rightarrow x \sim b \in [b]_\sim$$

$$y \in [b] \Rightarrow y \sim b \Rightarrow b \sim y \Rightarrow a \sim y \Rightarrow$$

$$\Rightarrow y \sim a \Rightarrow$$

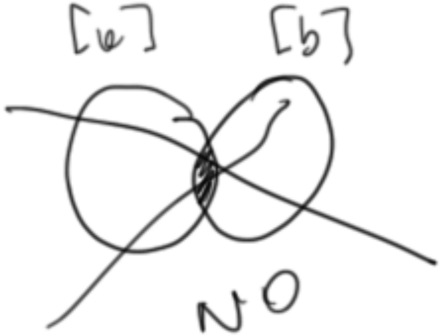
$$\Rightarrow y \in [a]_\sim$$

$$[a]_\sim = [b]_\sim \iff a \sim b$$

$$[a]_\sim = [b] \iff [a]_\sim \cap [b] \neq \emptyset$$

$$\Rightarrow$$

$$[a]_\sim \neq \emptyset \qquad a \in [a]_\sim$$



$$\Leftarrow$$

$$x \in [a]_\sim \quad \& \quad x \in [b]_\sim$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$x \sim a \qquad\qquad x \sim b$$

$$\Updownarrow$$

$$a \sim x$$

$$a \sim b \Rightarrow$$

$$[a]_\sim = [b]_\sim$$

$\mathbb{N}^* \quad x \sim y \iff \exists p \in \mathbb{N} \quad x \cdot p = y$

é di equiv. ?

$3 \sim 9 \qquad \cancel{9 \sim 3}$

$\mathbb{N}^* \quad x \approx y \iff x + y$ ē pari

1) SIMM $\quad x \approx y \Rightarrow y \approx x \qquad x+y$ ē pari $\Rightarrow$
$y+x$ ē pari

2) RIFL $\quad x \approx x \qquad x+x$ ē pari

3) TRANS $\quad \underbrace{x+y}$ ē pari & $y+z$ ē pari $\Rightarrow x+z$ ē pari ?

$\mathbb{N}^* \quad x \approx y \iff x + y$ è pari

$$\mathbb{N}^*/_{\approx}$$

$$f : \mathbb{N}^*/_{\approx} \to \{0,1\}$$

$$f([a]_{\approx}) = \begin{cases} 1 & \text{se } a \text{ è pari} \\ 0 & \text{se } a \text{ è dispari} \end{cases}$$

$$[a]_{\sim} = [b]_{\sim} \Rightarrow f([a]_{\sim}) = f([b]_{\sim})$$

$\mathbb{N}^*$ $\quad x \sim y \iff x = y$ OPPURE x & y sono multipli di 3

$$\sim = \{ (m,n) \mid m = n \text{ oppure } \exists k_1, k_2 \text{ t.c. } 3 \cdot k_1 = m \ \& \ 3 \cdot k_2 = n \}$$

$$\cancel{f : \mathbb{N}^*/_\sim \to \{0,1\} \qquad f([a]_\sim) = \begin{cases} 1 & \text{se } a \text{ è pari} \\ 0 & \text{se } a \text{ è dispari} \end{cases}}$$

OBBROBRIO! NON È UNA FUNZIONE !!!

$$\forall x, y \in \mathbb{N}^* \quad [x]_\sim = [y]_\sim \Rightarrow f([x]_\sim) = f([y])$$

$$\boxed{\begin{array}{l} f([9]_\sim) = 0 \\ f([12]_\sim) = 1 \\ [9]_\sim = [12]_\sim \end{array}}$$

$\mathbb{Z}$ numeri interi

$S = \{+, -\}$

$\mathbb{Z} = S \times \mathbb{N}$

(s, n)

$\{1, 0\}$

$\{\star, \square\}$

$(+, 5) \quad +5$

$(-, 7) \quad -7$

PESANTE!!

BRUTTO!

$(0, 5)$

$(1, 6)$

$(-, 8) + (+, 3)$

$(+, 0)$

$(-, 0)$

$\mathbb{Z} = S \times \mathbb{N}^* \cup \{(+, 0)\}$

$S_1 = \{+, -\}$

$S_2 = \{☆, □\}$

$(m, n) \in \mathbb{N} \times \mathbb{N}$

"$m - n$"

$(3, 5) \sim (8, 10)$

$3 - 5$ $\quad$ $8 - 10$

-2 $\quad$ -2

$$\sim \ \subseteq (\mathbb{N}\times\mathbb{N}) \times (\mathbb{N}\times\mathbb{N})$$

$$(m, n) \sim (m', n') \iff \boxed{m + n' = n + m'}$$

$$\iff$$

$$(m - n) = (m' - n')$$

$$\iff$$

$$m + n' = m' + n$$

$$\sim \subseteq (\mathbb{N}\times\mathbb{N}) \times (\mathbb{N}\times\mathbb{N})$$

$$(m,n) \sim (m',n') \iff \boxed{m+n' = n+m'}$$

1) RIFL $\quad (m,n)\sim(m,n) \quad m+n = n+m$

2) $(m,n)\sim(p,q) \Rightarrow (p,q)\sim(m,n)$

$$m+q = n+p \Rightarrow p+n = q+m$$

3) $(m_1,n_1)\sim(m_2,n_2)$ & $(m_2,n_2)\sim(m_3,n_3) \Rightarrow (m_1,n_1)\sim(m_3,n_3)$

$$m_1+n_2 = n_1+m_2 \quad \& \quad m_2+n_3 = n_2+m_3 \Rightarrow m_1+n_3 = n_1+m_3$$

$$\mathbb{Z} = \mathbb{N}^2/\sim$$

$(m$

$- : \mathbb{Z} \to \mathbb{Z}$

$- : \mathbb{N}^2/_{\sim} \to \mathbb{N}^2/_{\sim}$

0

$-(m, n) = (n, m)$

$+ : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$

$\times : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$

0