DIAGRAMMI DI BODE

Andrea Roberti
andrea.roberti@univr.it

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Esercizio tema d'esame 2013

$$H(s) = \frac{s+1}{s^4 + 2s^3 + 100s^2}$$

$$H(j\omega) = \frac{1}{100} \frac{j\omega + 1}{(j\omega)^2(1 + 2j\frac{\omega}{100} - \frac{\omega^2}{100})}$$

MATLAB CODE :

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h˙s = tf([1 1],[1 2 100 0 0])
bode(h˙s)
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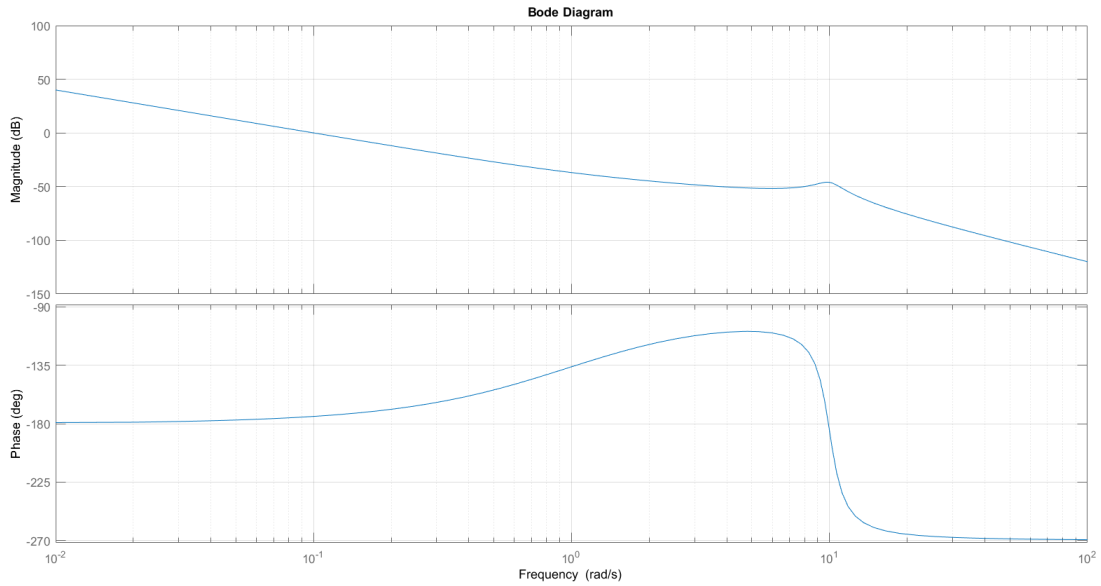


IMPORTANTE

- costante K_b
- monomio $\frac{1}{(j\omega)^\nu}$: proviene da uno zero (se a numeratore) o da un polo (se a denominatore) in s = 0
- binomio $(1 + j\omega\tau)^\mu$ proviene da uno zero (se a numeratore) o da un polo (se a denominatore) *reale* in $-\frac{1}{\tau}$
- trinomio $(1 + j2\zeta\frac{\omega}{\omega_n} - \frac{\omega^2}{\omega_n^2})^\mu$ proviene da una coppia di zeri (se a numeratore) o di poli (se a denominatore) *complessi coniugati* in $a \pm b$

Esercizio 1.1

Si consideri la seguente Funzione di Trasferimento H(s)

$$H(s) = \frac{5}{s^2}$$

Calcolare il diagramma di Bode.

$H(s) = 5\frac{1}{(j\omega)^2}$

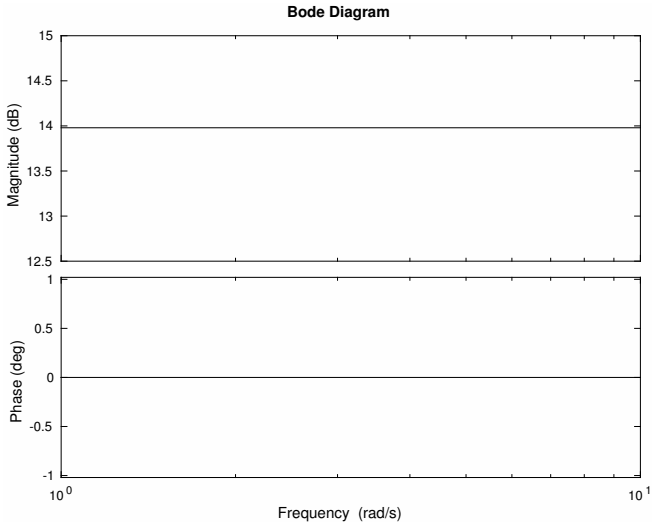
1) $K_{\mathrm{b}} = 5$

Modulo o Ampiezza

$|H(j\omega)|_{\mathrm{dB}} = 20log_{10}|K_{\mathrm{b}}| = 20log_{10}5 = 13.9dB$

Fase

$K_{\mathrm{b}} > 0,$ $SI.Quindi\ 0^{\mathrm{o}}$



2) $|H(j\omega)| = \frac{1}{(j\omega)^2}$

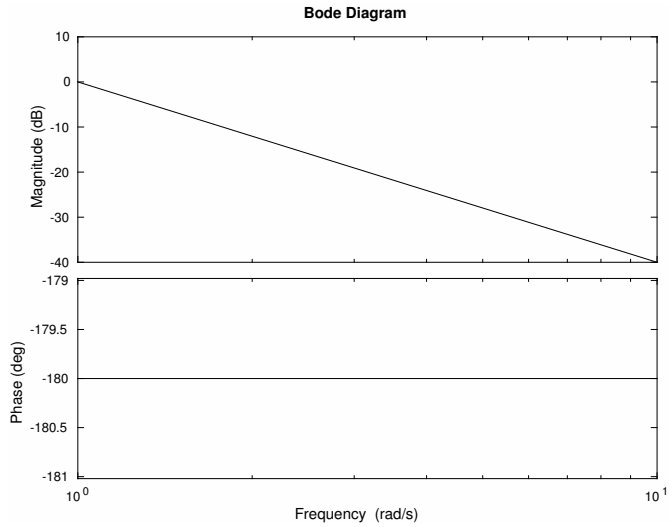
$\nu = 2$

Modulo o Ampiezza

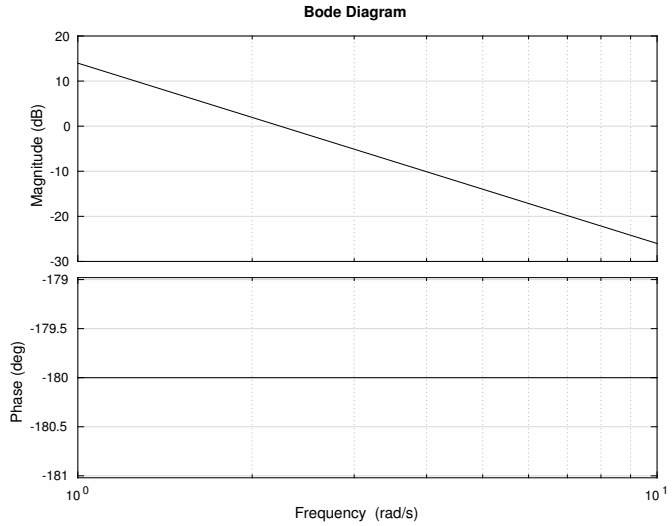
$|H(j\omega)|_{\text{dB}} = 20log_{10}\frac{1}{(j\omega)^2} = -40log_{10}\omega = -40\frac{db}{Decade}$

Fase

$arg(\frac{1}{(j\omega)^2}) = -\nu * 90^o = -180^o$



3) Somma di tutte le forme di bode



MATLAB CODE :

H_s = tf(5,[1 0 0])	Creo la Tf, col 5 al numeratore e s^2 al denominatore
bode(H_s)	Visualizzo il diagramma di Bode

Esercizio 1.2

Si consideri la seguente Funzione di Trasferimento H(s)

$$H(s) = -3s^5$$

Calcolare il diagramma di Bode.

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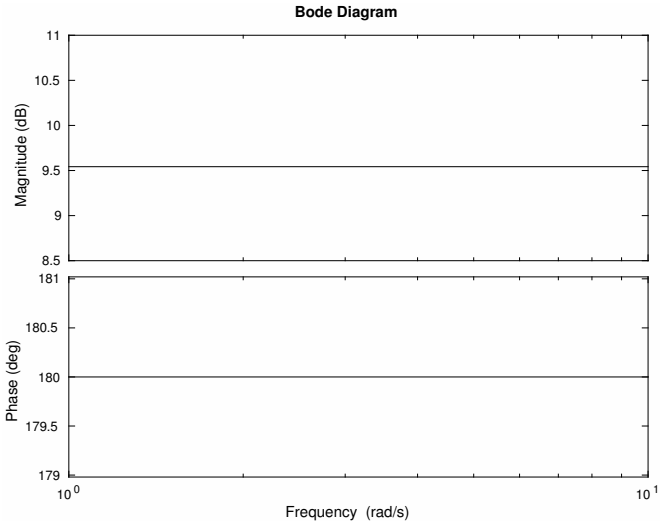
1) **-3**

$|H(j\omega)|_{\text{dB}} = 20log_{10}|K_{\text{b}}| = 20log_{10}|3| = 9.54dB$

$arg(K_{\text{b}}) = 180^o$



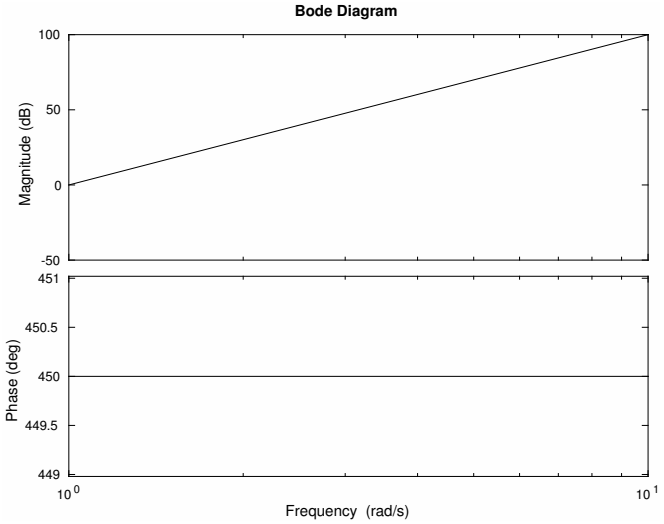
2) $\mathbf{s^5}$

$|H(j\omega)| = \frac{1}{(j\omega)^{-5}}$

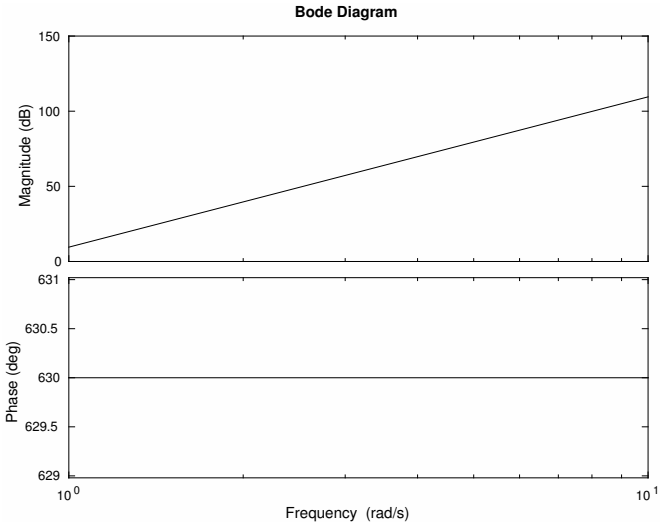
$\nu = -5$ Negativo, quindi il grafico del modulo é crescente

$|H(j\omega)|_{\text{dB}} = 20log_{10}\frac{1}{(j\omega)^{-5}} = -20\nu log_{10}\omega = 100\frac{db}{Decade}$

$arg(\frac{1}{(j\omega)^{-5}}) = -\nu * 90^o = +450^o$



3) **Grafico totale :**



Esercizio 1.3

A partire dalla seguente equazione differenziale trovare la Funzione di Trasferimento H(s)

$$\ddot{v}(t) - \dot{v}(t) - 2v(t) = \ddot{u}(t) + 2\dot{u}(t) + u(t)$$

Calcolare il diagramma di Bode.

Esercizio 1.4

Si consideri la seguente Funzione di Trasferimento H(s)

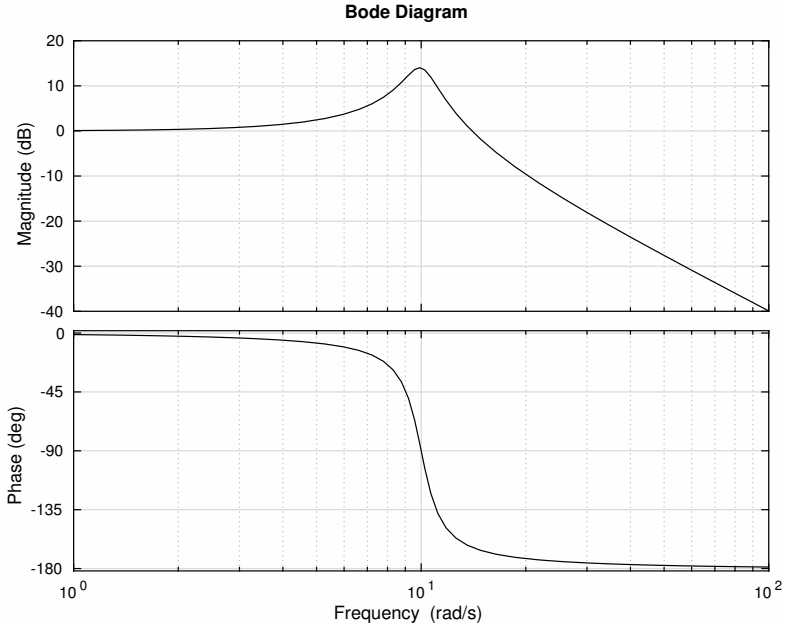
$$H(s) = \frac{1}{\frac{s^2}{100} + \frac{s}{50} + 1}$$

Calcolare il diagramma di Bode.

$\omega_{\mathrm{n}} = 10; \qquad \mu = -1 \qquad \zeta = \frac{1}{10} = 0.1 \qquad \omega_{\mathrm{r}} = 9.9$

$Fase = (\frac{1}{5^{|\zeta|}}\omega_{\mathrm{n}}; 0^o) \qquad (5^{|\zeta|}\omega_{\mathrm{n}}; \mu sign(\zeta)180^o)$

$Fase = (8.5; 0^o) \qquad (11.8; -180^o)$



Esercizio 1.5
Esame del 30/11/2011
Si consideri la seguente Funzione di Trasferimento H(s)

$$H(s) = \frac{(s^2 - 0.01)(s - 0.1)}{(s^2 + 16s + 100)(s + 100)}$$

Calcolare il diagramma di Bode.

Prima cosa , scomporre la funzione di trasferimento in forma di Bode.

$NUMERATORE : (s^2 - 0.01)(s - 0.1) = (s - 0.1)(s + 0.1)(s - 0.1)$
Ora devo portare il termine noto = 1 .

$NUMERATORE : \frac{1}{10}(10s - 1)\frac{1}{10}(10s + 1)\frac{1}{10}(10s - 1)$

NUMERATORE : $\frac{1}{1000}(10s - 1)^2(10s + 1)$

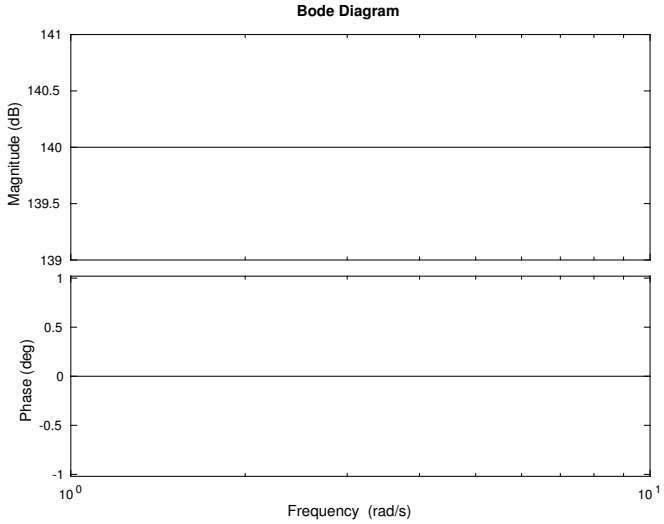
$DENOMINATORE : 100(\frac{s^2}{100} + \frac{16s}{100} + 1)100(\frac{s}{100} + 1)$

DENOMINATORE : $10000(\frac{s^2}{100} + \frac{16s}{100} + 1)(\frac{s}{100} + 1)$

$H(s) = 10000000\frac{(1 - 10s)^2(10s + 1)}{(\frac{s^2}{100} + \frac{16s}{100} + 1)(\frac{s}{100} + 1)}$

Ora posso studiare le singole parti della forma di Bode

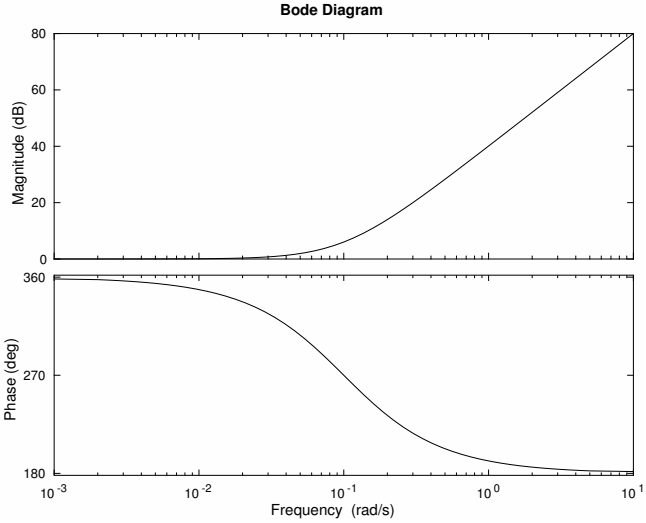
1) **10000000**



2) $\mathbf{(1\ -10s)^2}$

$\tau = -10 \qquad \mu = 2 \qquad \omega_{\mathrm{f}} = \frac{1}{|\tau|} = 0.1$

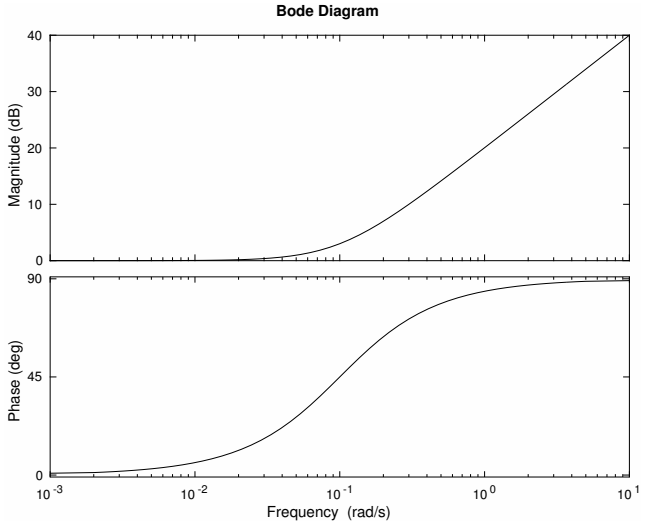
FASE : $(\frac{1}{50}; 0^o) \qquad (\frac{1}{2}; -180^o)$



3) $\mathbf{(1\ +10s)}$

$\tau = 10 \qquad \mu = 1 \qquad \omega_{\mathrm{f}} = \frac{1}{|\tau|} = 0.1$

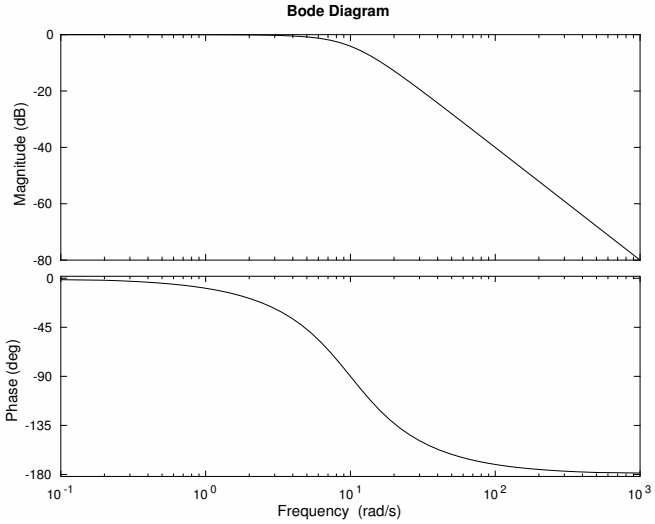
FASE : $(\frac{1}{50}; 0^o) \qquad (\frac{1}{2}; 90^o)$



4) $\mathbf{(s^2/100\ +16s/100\ +1)^{-1}}$

$\omega_{\mathrm{n}} = 10 \qquad \zeta = \frac{1}{20} \qquad \mu = -1$

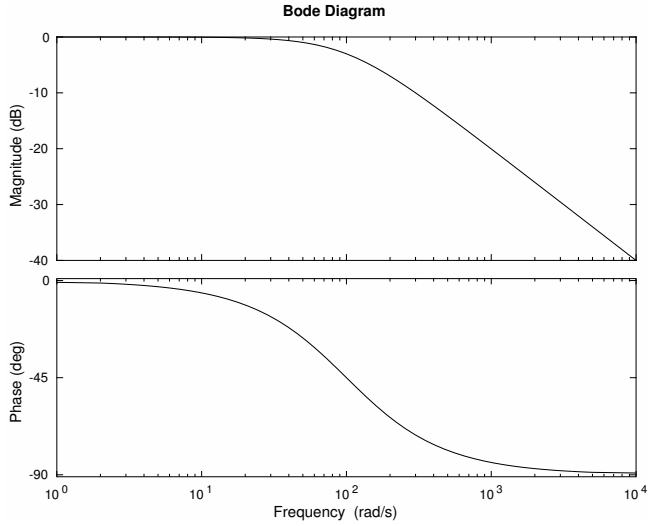
FASE : $(9; 0^o) \qquad (11; -180^o)$



5) $\mathbf{(s/100\ +1)^{-1}}$

$\tau = \frac{1}{100} \qquad \mu = -1 \qquad \omega_{\mathrm{f}} = \frac{1}{|\tau|} = 100$

FASE : $(20; 0^o) \qquad (500; -90^o)$



TO DO :

$H(s) = -2s^{\frac{1}{2}}$

$H(s) = \dfrac{5}{s^{\frac{2}{5}}}$

$H(s) = 6s^2 + 12s + 6$

$H(s) = s^2(3+s)^3$