

# System Design laboratory

Computer Engineering for Robotics and Smart Industry  
Tiziano Villa, Matteo Zavatteri

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Nome e Cognome:

Matricola:

Posta elettronica:

1. Consider the function  $f(x_1, x_2, x_3, x_4, x_5) = x_5 \oplus ((x_2 \oplus x_3) \cdot ((x_1 + x'_3) + x_4))$ .
  - (a) Build a ROBDD without complemented edges for  $f$  applying the procedure ITE. Use the variable order  $x_5 < x_4 < x_2 < x_3 < x_1$ .
  - (b) Introduce complemented edges following the convention that a) no 1-edge is complemented and b) there is no leaf labeled 0. Apply the following procedure:
    - i. Replace every incoming edge to the 0-leaf with a complemented edge to the 1-leaf.
    - ii. Apply De Morgan transformations to move up the complementation attributes (inversion bubbles) from the 1-edges, going from the lower levels of the BDD upwards, until when no 1-edge is left with inversion bubbles (show snapshots of the process drawing the transformations one level at a time).  
 Notice that by pushing up bubbles, it may happen that a negation is propagated up to the edge pointing to the root node, meaning that the ROBDD represents the negation of the function and so we must point to it by a complemented edge. Since the 1-edges are never complemented, the following check must be true:  $f(1, \dots, 1) = 0$  iff the edge to the ROBDD of the function is complemented (again: in this case the ROBDD represents the negation of the function and so we must point to it by a complemented edge).
    - iii. Cancel even numbers of bubbles on all 0-edges.
    - iv. Simplify the resulting BDD with complemented edges by merging nodes that are roots of isomorphic subtrees, i.e., nodes with the same functionality.
  - (c) Build the ROBDD with complemented edges by applying the CUDD library, and compare the ROBDD that you got by hand and the one produced by CUDD.
  - (d) Report the steps of how the CUDD library builds directly the ROBDD with complemented edges, by tracing the ITE procedure of CUDD and showing snapshots of all the intermediate steps of the ROBDD construction, i.e., each time a new node is added show how the unique table is updated line by line.
  - (e) Compare the construction that you carried on by hand with the construction by the ITE procedure of CUDD.

Compare the ITE procedure without complemented edges that you applied in the first step with the ITE procedure with complemented edges implemented in the CUDD library.

Extract the pseudo-code of the ITE procedure with complemented edges implemented in the CUDD library.

- (f) (Extra credit) Going back to the ROBDD without complemented edges built at the beginning, swap the order of variables  $x_2$  and  $x_3$  and show the transformed ROBDD by exchanging them locally by hand.