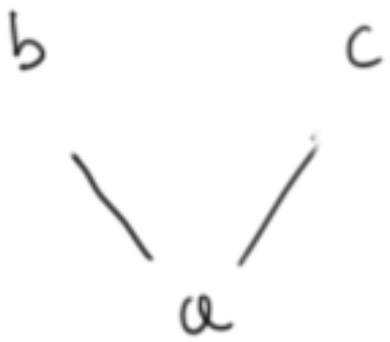


$b \leq b \qquad c \leq c$

$a \leq a$

$a \leq b \qquad a \leq c$

UN MINIMO a

UN MINIMALE a

NON HA MASSIMO

CI SONO <u>DUE</u> MASSIMALI b e c

PARTIZIONE

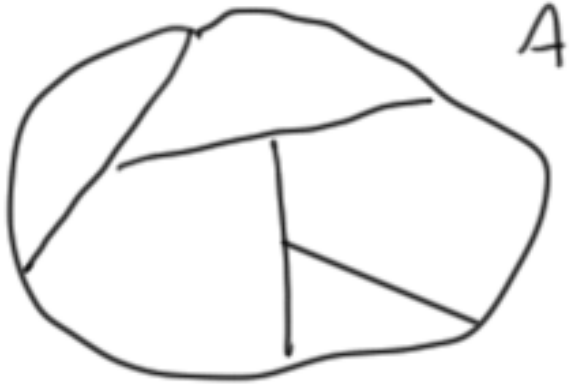
$A \quad \mathcal{Y} \subseteq \mathcal{P}(A)$

È DETTO PARTIZIONE SSE

1) $\forall X \in \mathcal{Y} \quad X \neq \emptyset$

2) $\forall X, Y \in \mathcal{Y} \quad X \neq Y \Rightarrow X \cap Y = \emptyset$

3) $\bigcup_{X \in \mathcal{Y}} = A$



1) SIA $\sim \subseteq A \times A$ UNA REL. DI EQ.

$A/_\sim$ È UNA PARTIZIONE DI A

DIM.

$\Rightarrow$ 1) $[a]_\sim \in A/_\sim \Rightarrow [a]_\sim \neq \emptyset$

2) $[a]_\sim \neq [b]_\sim \Rightarrow [a]_\sim \cap [b] = \emptyset$

$\left([a]_\sim \cap [b]_\sim \neq \emptyset \Rightarrow [a]_\sim = [b]_\sim \right)$

3) $\displaystyle\bigcup_{[a]_\sim \in A/_\sim} [a]_\sim = A$

ESERCIZIO

1) $[a]_\sim \in A/_\sim \Rightarrow [a]_\sim \neq \emptyset$

$a \sim a \Rightarrow a \in [a]_\sim$

2) $[a]_\sim \cap [b]_\sim \neq \emptyset \Rightarrow [a]_\sim = [b]_\sim$

$[x]_\sim = \{ y \mid x \sim y \}$

$[a]_\sim \subseteq [b]$

i) $a \sim b$

$[a]_\sim \cap [b]_\sim \neq 0 \Rightarrow \exists x \quad x \in [a]_\sim \ \& \ x \in [b]$

$$\begin{array}{ccc} x \in [a]_\sim & & x \in [b] \\ \Updownarrow & & \Updownarrow \\ a \sim x & & b \sim x \\ & & \Updownarrow \\ & & x \sim b \end{array} \Rightarrow a \sim b$$

ii) $\underbrace{[a]_\sim \subseteq [b]_\sim} \ \& \ [b]_\sim \subseteq [a]$

$z \in [a]_\sim \Rightarrow a \sim z \Rightarrow \underline{z \sim a}$

poiché $\underline{a \sim b} \Rightarrow z \sim b \Rightarrow b \sim z \Rightarrow z \in [b]_\sim$

$\mathcal{Y}$ UNA PARTIZIONE DI A

$a \approx b \iff \exists X \in \mathcal{Y} \quad a, b \in X$

ESERCIZIO VERIFICARE CHE $\approx$ È UNA RELAZ. DI EQUIV.

1) RIFLESSIVITÀ $a \approx a$?

$\forall a \in A \quad \exists X \in \mathcal{Y}$ t.c. $a \in X \quad \left(\bigcup_{X \in \mathcal{Y}} X = A \right)$

2) $a \approx b \Rightarrow \exists X \in \mathcal{Y} \; a, b \in X \Rightarrow b \approx a$

3) $a \approx b \;\&\; b \approx c \Rightarrow a \approx c$??

$a \approx b \;\&\; b \approx c \Rightarrow \exists X_1, X_2 \in \mathcal{Y}$

$a, b \in X_1$ e $b, c \in X_2$

$b \in X_1 \cap X_2 \Rightarrow X_1 = X_2 \Rightarrow$

$\exists X \in \mathcal{Y} \quad a, b, c \in X \Rightarrow a \approx c$

$$\mathcal{G} \subseteq \mathcal{P}(A)$$

ASSIOMA DI SCELTA

$$\exists\, \psi : \mathcal{G} \to A \quad \text{t.c.} \quad \forall X \in \mathcal{G}$$

$$\psi(X) \in X$$

ORDINAMENTI

A , $R \subseteq A \times A$

R È DETTA REL. D'ORDINE (PARZIALE)

SSE

1) $\forall a \in A \quad aRa$

2) $\forall a, b, c \in A \ (aRb \ \& \ bRc) \Rightarrow aRc$

3) ANTISIMMETRIA

$\forall a, b \in A \ (aRb \ \& \ bRa) \Rightarrow a = b$

$\mathbb{N}$ $\qquad a \leq b \Leftrightarrow \underline{\exists z \in \mathbb{N} \quad a+z=b}$

$\leq$ è RIFL / TRANS / ANTISIMM.

1) RIFL. $\quad a \leq a$ sí poiche $a+0=a$

2) TRANS. $\quad \underbrace{a \leq b}_{a+x=b} \ \& \ \underbrace{b \leq c}_{b+z=c} \Rightarrow a \leq c$

$$\underbrace{a+x=b \qquad b+z=c}$$

$\Rightarrow a+x=c-z \Rightarrow a+\underbrace{(x+z)}_{\in \mathbb{N}}=c \Rightarrow a \leq c$

3) ANTISIMM.

$\boxed{a \leq b \ \& \ b \leq a} \Rightarrow a=b$

$\Rightarrow \exists \ x,y \in \mathbb{N} \ \begin{cases} a+x=b \\ b+y=a \end{cases}$

$\cancel{a}+\cancel{b}+x+y=\cancel{b}+\cancel{a}$

$x+y=0 \Rightarrow$

$x=y=0 \quad a+0=b \Rightarrow a=b$

1) $\left[\begin{array}{ll} \{a, b\} & R = \{(a,a), (b,b), (a,b)\} \\ x R y & (x,y) \in R \end{array}\right.$

2) $\{a, b, c\}$ $\quad R = \{(a,a), (b,b), (c,c), (a,b), (a,c)\}$

3) $\mathbb{N}$ $\quad \sqsubseteq \quad x \sqsubseteq y \Leftrightarrow \begin{cases} x = y \\ \text{oppure} \\ x = 0 \end{cases}$

4) $\mathbb{N} \cup \{\infty\}$ $\quad x \sqsubseteq y \Leftrightarrow \begin{cases} x, y \in \mathbb{N} \ \& \ x \leq y \\ y = \infty \end{cases}$