ECOLE NATIONALE SUPERIEURE
DES TRAVAUX PUBLICS

NATIONAL ADVANCED SCHOOL
OF PUBLIC WORKS

MASTERS IN ENGINEERING (BAC + 5 ans)

ANNEE ACADEMIQUE/*ACADEMIC YEAR* :

DEPARTEMENT/DEPARTMENT :   *CLASSE/CLASS* :

COMPOSITION DE FIN DE SEMESTRE/*END OF SEMESTER EXAMINATION* : SIMULATION #1

EPREUVE/*COURSE TITLE* :   *CODE/CODE* :

DATE/*DATE* : 17 AUGUST 2013   *DUREE/DURATION* : 2 HOURS

EXAMINATEUR/*EXAMINER* :

INSTRUCTIONS/*INSTRUCTIONS* :

Multiple choice questions (2 points each, 50 points maximum)
Read carefully the text of each question and mark the box with the best answer.

1. How many floating point numbers has $\mathbb{F}(2, 3, -1, 1)$?

   ☐ 21 ☐ 24 ☐ 25 ☐ 27

2. The machine precision for $\mathbb{F}(10, 2, -3, 4)$ is

   ☐ 0.001 ☐ 0.01 ☐ 0.05 ☐ 0.1

3. The maximum number in $\mathbb{F}(10, 2, -3, 4)$ is

   ☐ 9.9 ☐ 99 ☐ 990 ☐ 9900

4. Let $x = 0.5$, $y = 1$ and $z = 10$ be floating point numbers of $\mathbb{F}(10, 1, -1, 2)$. Which is the result of $(x \oplus y) \otimes z$?

   ☐ 10 ☐ 15 ☐ 20 ☐ overflow

5. The problem of finding the solution of a non singular linear system $A\mathbf{x} = \mathbf{b}$ is always a well conditioned problem.

   ☐ True ☐ False

6. The absolute value of the error in the bisection method is always non increasing, i.e., $|e_{k+1}| \leq |e_k|$ for each $k \geq 0$

   ☐ True ☐ False

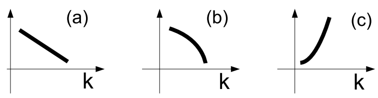
7. How many fixed points has the function $f(x) = x^3$?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3

8. The fixed point iterations $x_{k+1} = x_k^2$ with $x_0 = 0.5$ goes toward the fixed point α equals to

   ☐ 0 ☐ 0.5 ☐ 1 ☐ $+\infty$

9. Consider the method $x_{k+1} = \phi(x_k)$ with fixed point $\alpha = 1$. Assume that x_k goes toward α and $\phi'(\alpha) = 0$. Then, the plot of $\log_{10}(|e_k|)$, choosing from the figure below, may be



   ☐ (a) ☐ (b) ☐ (c) ☐ (a) or (c)

10. The Newton method has the absolute value of the error $|e_5| = 10^{-6}$ for the computation of the root of $e^x + x = 0$. Assuming a unitary asymptotic error constant, we expect $|e_6|$ equals to about

    ☐ 10^{-6} ☐ 10^{-8} ☐ 10^{-10} ☐ 10^{-12}

11. The Newton method for the solution ξ of equation $f(x) = 0$ gives $|x_6 - x_5| = 10^{-3}$ and $|x_7 - x_6| = 2 \cdot 10^{-6}$. The estimation of the absolute value of the error $|e_7|$ is

☐ 10^{-12} ☐ $4 \cdot 10^{-12}$ ☐ $8 \cdot 10^{-12}$

12. The number of iterations required to the Newton method to compute the root of $3x - 2 = 0$ with an error not greater then 10^{-6} and starting at $x_0 = 2.0$ are

☐ 1 ☐ 14 ☐ 35 ☐ 68

13. The equation $f(x) = 0$ has a unique root ξ in $(0, 1)$. Assume that $f'(x) > 0$ and $f''(x) < 0$ in $[0, 1]$. Starting from $x_0 = 0$, the sequence x_k produced by the Newton method fulfills

☐ $x_k < x_{k+1}$ ☐ $x_k > x_{k+1}$ ☐ $x_k > \xi$

14. The spectral radius of the matrix

$$\begin{pmatrix} 2 & 2 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -4 \end{pmatrix}$$

is

☐ -4 ☐ 2 ☐ 4 ☐ 24

15. The number of arithmetic operations required to the backward substitution algorithm to solve an upper triangular linear system is about

☐ n ☐ n^2 ☐ n^3 ☐ $\frac{2n^3}{3}$

16. The matrix U of the LU factorization of the matrix A

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

is

☐ $\begin{pmatrix} 2 & 1 \\ 0 & 1/2 \end{pmatrix}$ ☐ $\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$

☐ $\begin{pmatrix} 2 & 1 \\ 0 & 3/2 \end{pmatrix}$ ☐ $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

17. Each square matrix A may be written as $A = L \cdot U$ where L is lower triangular with unitary elements on the main diagonal and U is upper triangular

☐ True ☐ False

18. The matrix A has an LU factorization with the determinant of U equal to $|U| = -2$. The determinant of A^3 is

☐ -8 ☐ $-1/8$ ☐ 3

☐ we cannot compute since we not have A

19. Consider the linear system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

☐ $\|A\|_F = 10$ ☐ GS converges
☐ A is singular ☐ $\rho(A) < 1$

20. The condition number $K_2(A)$ of the matrix A of the previous item is

☐ 1 ☐ 2 ☐ 3 ☐ 4

21. The following Matlab code

```
1.   v = 2:3:10;
2.   A = [ v; v + 1 ];
```

☐ is wrong in line 2 ☐ $A = \begin{pmatrix} 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$

☐ $A = \begin{pmatrix} 2 & 5 & 8 \\ 3 & 5 & 8 \end{pmatrix}$ ☐ $A = \begin{pmatrix} 2 & 5 & 8 \\ 2 & 5 & 9 \end{pmatrix}$

22. Consider the following Matlab code

```
1.   v = linspace(2,10,5);
2.   w = length( size( v' * v ) );
```

The variable `w` is

☐ 2 ☐ 3 ☐ [2 2] ☐ $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

23. Consider the following Matlab code

```
1.   S = 0;
2.   for k=1:4
3.      S = S + k;
4.      S = 2 * S;
5.   end
```

At the end of the loop, the variable `S` is equal to

☐ 0 ☐ 6 ☐ 26 ☐ 52

24. Which is B at the end of the Matlab code

```
A = [1 2 3; 4 5 6];
B = A( :, 1:2 ).^2;
```

☐ $\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$ ☐ $\begin{pmatrix} 1 & 4 \\ 16 & 25 \end{pmatrix}$

☐ $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ ☐ $\begin{pmatrix} 9 & 12 \\ 24 & 33 \end{pmatrix}$

25. Which is the value of n at the end of the Matlab code

```
n = 0;
while( n<=3 )
   n = n + 2;
   if n >= 4
      n = n-1;
   end
end
```

☐ 3 ☐ 4 ☐ 5 ☐ 6

Open questions (60 points maximum)
Write clearly all the answers in the exam's booklet.

1. (**10 points**) Describe the bisection method. Then, apply it to the function $f(x) = x - 1$ with starting interval $[a_0, b_0] = [0,\ 1.9]$. Compute x_0, x_1 and the corresponding errors and justify the obtained result.
2. (**10 points**) Consider the Newton method for the approximation of the root $\xi = 1$ of the function $f(x) = x^2 - 1$. Assume the starting point to be $x_0 = 2$.
   (a) Compute the first two iterations of the Newton method and the corresponding absolute value of the errors.
   (b) Sketch a *qualitative* graph of $\log_{10}(\,|\,e_k\,|\,)$ as a function of the iteration number k. Justify your answer.
   (c) What happens if we choose as new starting point $x_0 = 0$? Justify your answer.
   (d) Write a fixed point method of order $p = 2$ for the computation of the root ξ. Justify your answer.
3. (**20 points**) Consider the upper triangular linear system $U\mathbf{x} = \mathbf{b}$.
   (a) Show, with all the mathematical details, the backward substitution method.
   (b) Show, with all details, the computational cost of the method.
   (c) Write the Matlab code to solve such a linear system using the for loop.
4. (**20 points**) Consider the linear system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}$$

   (a) (**10 points**) Prove that the matrix A is positive definite, compute the Cholesky factorization and solve the linear system. Check out that the obtained solution is correct.
   (b) (**2 points**) Prove that the Jacobi method converges to the solution starting from $\mathbf{x_0} = [0\ \ 0\ \ 0]^T$.
   (c) (**8 points**) Compute the first two iterations of the Gauss-Seidel method starting from $\mathbf{x_0} = [0\ \ 0\ \ 0]^T$.