ECOLE NATIONALE SUPERIEURE DES TRAVAUX PUBLICS

NATIONAL ADVANCED SCHOOL OF PUBLIC WORKS

MASTERS IN ENGINEERING (BAC + 5 ans)

ANNEE ACADEMIQUE/*ACADEMIC YEAR* :------------------------------

DEPARTEMENT/DEPARTMENT :-- *CLASSE/CLASS* :------------------------

COMPOSITION DE FIN DE SEMESTRE/*END OF SEMESTER EXAMINATION* : SIMULATION #2

EPREUVE/*COURSE TITLE* :-- *CODE/CODE* :--------------------------

DATE/*DATE* : 24 AUGUST 2013 ---------------------------- *DUREE/DURATION* : 2 HOURS

EXAMINATEUR/*EXAMINER* :--

INSTRUCTIONS/*INSTRUCTIONS* :---

Multiple choice questions

Read <u>carefully</u> the text of each question and mark the box with the best answer.

1. The maximum floating point number x_{max} and the machine precision eps of the floating point system $\mathbb{F}(10, 2, -1, 1)$ are

   ☐ $x_{max} = 9.9$ eps $= 0.05$
   ☐ $x_{max} = 9.0$ eps $= 0.10$
   ☐ $x_{max} = 9.9$ eps $= 0.05$
   ☐ $x_{max} = 9.0$ eps $= 0.10$

2. Consider the fixed point iterations given by $x_{k+1} = x_k/2 + 1$. Let α be the unique fixed point. Starting at $x_0 = 1$, the absolute value of the error $e_2 = \alpha - x_2$ of the second iteration x_2 is

   ☐ 0.25 ☐ 0.50 ☐ 1.0 ☐ 1.5

3. The order of convergence of the fixed point method $x_{k+1} = 2 - 2x_k + x_k^2$ when $x_0 = 0.5$ is

   ☐ 1 ☐ 2 ☐ 3 ☐ 4

4. The order of convergence of the Newton method for the solution of the non linear equation $e^x = 2 - x$ is

   ☐ 0.5 ☐ 1 ☐ 2 ☐ more then 2

5. The order of convergence of the Newton method is always less or equal to 2

   ☐ True ☐ False

6. The Hilbert matrices are an example of well conditioned matrices

   ☐ True ☐ False

7. Let
   $$A = \begin{pmatrix} 10 & 0 \\ 0 & 0.01 \end{pmatrix}$$
   The condition number $K_2(A)$ of the matrix A is

   ☐ 0.01 ☐ 10 ☐ 100 ☐ 1000

8. The LU factorization of the matrix A gives $|U| = 4$. The determinant of A^{-2} is

   ☐ 16 ☐ $\dfrac{1}{16}$ ☐ $\dfrac{1}{4}$ ☐ 4

9. The L matrix of the LU-factorization of the matrix A given by

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & 4 \end{pmatrix}$$

is

☐ $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$ ☐ $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -2 & 1 \end{pmatrix}$

☐ $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -2 & 1 \end{pmatrix}$ ☐ $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{pmatrix}$

10. Given the splitting $A = D - E - F$ where E is strictly lower triangular, F is strictly upper triangular and D is diagonal, the iteration matrix for the Jacobi method is

☐ $D^{-1}(E + F)$ ☐ $D(E + F)$

☐ $-D^{-1}(E + F)$ ☐ $-D(E + F)$

11. Starting from $\mathbf{x}_0 = (0, 0)^T$, the norm of the residual $\mathbf{r}_1 = \mathbf{b} - A\mathbf{x}_1$ after the first Gauss-Seidel iteration for the linear system $A\mathbf{x} = \mathbf{b}$ given by

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

is

☐ $\dfrac{\sqrt{130}}{6}$ ☐ $\dfrac{7}{6}$ 0 $\left[-\dfrac{7}{6}\,,\, 0 \right]^T$

12. The Lagrange polynomials depends only on the nodes of the points (x_i, y_i), $i = 0, \ldots, n$

☐ True ☐ False

13. The sum of all the Lagrange polynomials depends on the values of the function f in the interpolating points

☐ True ☐ False

14. If we want to approximate a function in an interval $[a, b]$ using equally spaced nodes, a higher degree interpolating polynomial always works better then a lower degree one

☐ True ☐ False

15. The regression line for the set of points

x_i	-1	0	1	2
y_i	0	2	3	3

is

☐ $y = x + 1.5$ ☐ $y = 1.5x + 1$

☐ $y = x + 1$ $y = 1.5x + 1.5$

16. The quadrature formula

$$\int_0^1 \sqrt{x} f(x) \approx \sum_{i=0}^{3} \alpha_i f(x_i)$$

has the maximum degree of precision. Then the number

$$d = \left| \int_0^1 \left(\sqrt{x} + x\sqrt{x} \right) dx \; - \sum_{i=0}^{3} \alpha_i x_i \right|$$

is equal to

☐ 0 ☐ 1/3 ☐ 2/3 ☐ 1

17. Given a positive n, the sum of all Cotes numbers $C_i^{(n)}$, $i = 0, \ldots, n$ is

☐ 1 ☐ n ☐ $\sqrt{n}$ ☐ $\dfrac{n}{2}$

18. The error for the computation of

$$\int_0^{100} \left(x^3 + 13254x \right) dx$$

using the Cavalieri-Simpson formula is

☐ 10^{-3} ☐ 10^{-2} ☐ 10^{-1} ☐ 0

19. The second derivative of f does not change much in the integration interval. Then, using the composite trapezoidal rule we expect that the ratio of the errors E_{2m}/E_m is

☐ 4 ☐ 1/4 ☐ near 1/4

20. The Cavalieri-Simpson approximation of the integral

$$\int_0^1 \sqrt{x}\, dx$$

is

☐ $\dfrac{2\sqrt{2} + 1}{6}$ ☐ $\dfrac{2}{3}$ ☐ $\dfrac{1}{6}$ ☐ 1

21. The divided difference $f[x_0, x_1, x_2]$ of the following table

$$\begin{array}{r|l} x_0 = -1 & 2 \\ x_1 = 0 & 3 \\ x_2 = 1 & 6 \end{array}$$

is

☐ 1 ☐ 3 ☐ −1 ☐ 36

22. Let $p(x)$ be the Newton expression of the interpolating polynomial for the points (x_i, y_i), $i = 0, \ldots, n$. If we add a new point (x_{n+1}, y_{n+1}) with $x_0 < x_{n+1} < x_1$ we have to recompute all the divided difference table

☐ True ☐ False

23. The composite trapezoidal formula gives the results of the following table

A_0	A_1	A_2
1	0.875	0.844

The best approximation for the integral is then

☐ 0.844 ☐ 0.833 ☐ 0.906 ☐ 0.875

24. Which is the value of `n` at the end of the Matlab code

```
toll = 1E2;
n = 5;
while( 10^n > toll & n >= 2 )
   n = n - 2;
end
```

☐ 0 ☐ 1 ☐ 2 ☐ 3

25. Consider the following Matlab code

```
S = 5;

for k=1:3
   if k>=3
      S = S*k;
   else
      S = S-k;
   end
end
```

At the end of the loop, the variable `S` is equal to

☐ 1 ☐ 4 ☐ 6 ☐ 9

26. After the execution of the following Matlab code, the variable `r` is equal to

```
A = diag( diag( [1 2; 3 4] ) );
r = eig( A );
```

☐ $[1\ \ 4]^T$ ☐ $[1\ \ 2]^T$ ☐ 4 ☐ 1

27. After the execution of the following Matlab code, the variable `v` is equal to

```
v = [ sum( 3:4:14 )  length( 1:4 )  ];
v = v.^2;
```

☐ [441 16] ☐ 7056 ☐ 4 ☐ [21 4]☐

28. Given the Matlab code

```
v = [ 1 2 3; 4 5 6; 6 7 8 ];
v = v(2,[2 3]);
```

gives

☐ [5 6] ☐ [4 5 6] ☐ $[5\ 8]^T$ ☐ $[2\ 5]^T$☐

29. To plot a function with the command `plot(x,y)`, the vector `x` and `y` must have the same size

☐ True
☐ False
☐ It depends on the function

30. The command `clear all` makes the command Window clear but does not clear the variables in the Workspace

☐ True ☐ False

Open questions

Write clearly all the answers in the exam's booklet.

1. Prove that the condition number $K(A)$ fulfills $K(A) \geq 1$ for each matrix A. Give an example of well conditioned matrix and one of an ill conditioned matrix.

2. Consider the iterative method $\mathbf{x}_{k+1} = B\mathbf{x}_k + \mathbf{f}$ to solve the linear system $A\mathbf{x} = \mathbf{b}$. Prove the relationship $\mathbf{e}_k = B\mathbf{e}_{k-1}$, $k = 1, 2, \ldots$ where $\mathbf{e}_k = \mathbf{x} - \mathbf{x}_k$ is the error at the k-th step. Give necessary and sufficient conditions on the iteration matrix B in order to have a convergent sequence for each starting point $\mathbf{x}_0$. Write the iteration matrix for the Jacobi method.

3. Given the set of points (x_i, y_i), $i = 0, \ldots, 3$ in the following table

x_i	-2	0	1	2
y_i	1	1	2	3

   write the Newton expression of the interpolation polynomial. Compute the minimum value of the function $S(m, q)$
$$S(m, q) = \sum_{k=0}^{3} [\ y_i - mx_i - q\ ]^2$$
   and give the values of m and q for which this minimum is reached.

4. Show the composite trapezoidal rule using m intervals . Recalling that the error for the trapezoidal rule is
$$E = -\frac{h^3}{12} f''(\xi)$$
   where h is the amplitude of the integration interval and ξ is a suitable point inside the integration interval, find the expression for the error in the composite trapezoidal formula.

5. Write a Matlab code for the computation of the sum of elements of the vector `x` using just a for loop.