CUDD
Colorado University Decision Diagram Package

Software per Sistemi Embedded

Corso di Laurea in Informatica

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Outline

CUDD

- CUDD is the Colorado University Decision Diagram Package.
- It is a C/C++ library for creating different types of decision diagrams:
  - binary decision diagrams (BDD);
  - zero-suppressed BDDs (ZDD);
  - algebraic decision diagrams (ADD)
- This lesson is only on the BDD functionality of CUDD

Getting CUDD

- You can download CUDD by FTP with anonymous login from `vlsi.colorado.edu`
- The latest version is 3.0.0
- How to install it on the Lab Computers (and any Linux distribution):

```
export CUDD_INSTALL_DIRECTORY=$HOME/<install_dir>
mkdir CUDD_INSTALL_DIRECTORY
wget ftp://vlsi.colorado.edu/pub/cudd-3.0.0.tar.gz
tar xzfv cudd-3.0.0.tar.gz
cd cudd-3.0.0
mkdir objdir && cd objdir
../configure --prefix=$CUDD_INSTALL_DIRECTORY
make && make install
```

Including and linking the CUDD library

- The CUDD library has two main header files:
  - `#include<cudd.h>` for the C library
  - `#include<cuddObj.h>` for the C++ library
- We will use the C library
- The package is split into many different libraries:

  `libcudd.a`, `libutil.a`, ...
- To compile and link a C program that use CUDD:

```
gcc -o main main.c -lcudd -lutil -lepd -lmtr -lst -lm
```

Outline

Garbage Collection

- CUDD has a built-in garbage collection system.
- When a BDD is not used anymore, its memory can be reclaimed.
- To facilitate the garbage collector, we need to "reference" and "dereference" each node in our BDD:
  - `Cudd_Ref(DdNode*)` to reference a node
  - `Cudd_RecursiveDeref(DdNode*)` to dereference a node and all its descendants.

Complemented arcs

- Each node of a BDD can be:
  - a variable with two children
  - a leaf with a constant value
- The two children of a node are referred to as the “then” child and the “else” child
- To assign a value to a BDD, we follow “then” and “else” children until we reach a leaf:
  - the value of our assignment is the value of the leaf we reach
- However: “else” children can be complemented:
  - when an “else” child is complemented, then we take the complement of the value of the leaf:
    - i.e., if the value of the leaf is 1 and we have traversed an odd number of complemented arcs, the value of our assignment is 0.

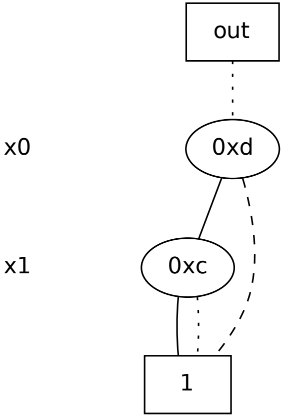
Complemented arcs: example

- $out = x_0 \overline{x}_1$

- "then" arcs are solid
- normal "else" arcs are dashed
- complemented "else" arcs are dotted

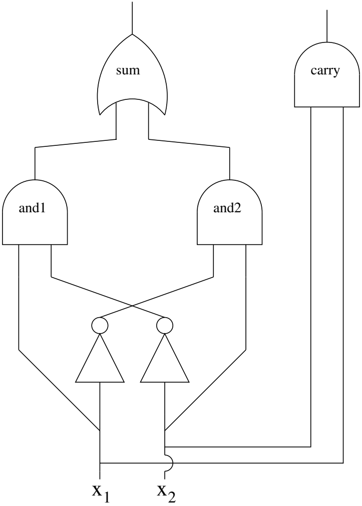
- the `out` arc is complemented:

$$\begin{aligned} \overline{out} &= \overline{x}_0 + x_1 \\ &= \overline{x}_0 + x_0 x_1 \end{aligned}$$



Outline

The half-adder circuit



This is a half adder circuit that we will compile into an OBDD.

It has the following truth table:

x_1	x_2	sum	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

The `DdManager`

The `DdManager` is the central data structure of CUDD:

- It must be created before calling any other CUDD function.
- It needs to be passed to almost every CUDD function.

To initialize the `DdManager`, we use the following function:

```
DdManager * Cudd_Init(
  unsigned int numVars,     // initial number of BDD variables (i.e., subtables)
  unsigned int numVarsZ,    // initial number of ZDD variables (i.e., subtables)
  unsigned int numSlots,    // initial size of the unique tables
  unsigned int cacheSize,   // initial size of the cache
  unsigned long maxMemory   // target maximum memory occupation.(0 means unlimited)
);
```

Initializing the DdManager

```
#include<stdio.h>
#include<cudd.h>

int main() {
  DdManager* manager=Cudd_Init(0, 0,
      CUDD_UNIQUE_SLOTS,  CUDD_CACHE_SLOTS, 0);
  if(manager == NULL) {
    printf("Error when initializing CUDD.\n");
    return 1;
  }

  ...

  return 0;
}
```

The DdNode

The DdNode is the core building block of BDDs:

```
struct DdNode {
    DdHalfWord index;        // Index of the variable reprented by this node
    DdHalfWord ref;          // reference count
    DdNode *next;            // next pointer for unique table
    union {
    CUDD_VALUE_TYPE value;   // for constant nodes
    DdChildren kids;         // for internal nodes
    } type;
};
```

- index is a unique index for the variable represented by this node.
  - It is permanent: if we reorder variables, the index remains the same
- ref stores the reference count for this node.
  - It is incremented by Cudd_Ref and decremented by Cudd_Recursive_Deref

Create the BDD for **sum**

```
DdNode* x1 = Cudd_bddIthVar(manager, 0);
DdNode* x2 = Cudd_bddIthVar(manager, 1);

DdNode* and1;
and1 = Cudd_bddAnd(manager, x1, Cudd_Not(x2));
Cudd_Ref(and1);

DdNode* and2;
and2 = Cudd_bddAnd(manager, Cudd_Not(x1), x2);
Cudd_Ref(and2);

DdNode* sum;
sum = Cudd_bddOr(manager, and1, and2);
Cudd_Ref(sum);

Cudd_RecursiveDeref(manager, and1);
Cudd_RecursiveDeref(manager, and2);
```

Exercise: write the code for `carry`

Restricting the BDD

- *Restricting* a BDD means assigning a truth value to *some of the variables*

```
DdNode * Cudd_bddRestrict(
  DdManager * manager,   // DD manager
  DdNode * BDD,          // The BDD to restrict
  DdNode * restrictBy)   // The BDD to restrict by.
```

- `BDD` is the original BDD to restrict
- `restrictBy` is the truth assignment of the variables:
  - AND of variables and complemented variables
- the function returns the restricted BDD

Print the truth table of the Half-adder

```
DdNode *restrictBy;
restrictBy = Cudd_bddAnd(manager, x1, Cudd_Not(x2));
Cudd_Ref(restrictBy);

DdNode *testSum;
testSum = Cudd_bddRestrict(manager, sum, restrictBy);
Cudd_Ref(testSum);
DdNode *testCarry;
testCarry = Cudd_bddRestrict(manager, carry, restrictBy);
Cudd_Ref(testCarry);

printf("x1 = 1, x2 = 0: sum = %d, carry = %d\n",
       1 - Cudd_IsComplement(testSum),
       1 - Cudd_IsComplement(testCarry));

Cudd_RecursiveDeref(manager, restrictBy);
Cudd_RecursiveDeref(manager, testSum);
Cudd_RecursiveDeref(manager, testCarry);
```

Exercise:
write the code for the complete truth table

Print the BDD: graphviz

- The function `Cudd_DumpDot` dumps the BDD to a file in GraphViz format
- The `.dot` file can be converted to a PDF by the command `dot`:

```
dot -O -Tpdf half_adder.dot
```

Print the BDD: C code

```
char* inputNames[2];
inputNames[0] = "x1";
inputNames[1] = "x2";
char* outputNames[2];
outputNames[0] = "sum";
outputNames[1] = "carry";

DdNode* outputs[2];
outputs[0] = sum;
Cudd_Ref(outputs[0]);
outputs[1] = carry;
Cudd_Ref(outputs[1]);

FILE* f = fopen("half_adder.dot", "w");

Cudd_DumpDot(manager, 2, outputs, inputNames, outputNames, f);

Cudd_RecursiveDeref(manager, outputs[0]);
Cudd_RecursiveDeref(manager, outputs[1]);
fclose(f);
```

Variable reordering

- The order of variables can have a tremendous effects on the size of BDDs
- CUDD provides a rich set of tools for reordering BDDs:
  - Automatic reordering (using heuristics) when the number of nodes in the BDD passes a certain threshold
  - Manual reordering using different heuristics
  - Manual reordering with a user-specified variable order

The function `Cudd_ShuffleHeap` is used to define the variable order:

```
int Cudd_ShuffleHeap(
  DdManager * manager,   // DD manager
  int * permutation      // required variable permutation
)
```

Exercise: play with the variable order!

- Create the BDD for the function $x_1x_2 + x_3x_4 + x_5x_6$
- Try the following variable orders and compare the results:
  - $x_1 < x_2 < x_3 < x_4 < x_5 < x_6$
  - $x_1 < x_3 < x_5 < x_2 < x_4 < x_6$

HINTS

- `int Cudd_ReadPerm(manager, x2->index)` returns the position of variable `x2` in the order
- `int Cudd_ReadNodeCount(manager)` returns the number of nodes in the BDD