Block Diagrams

EECS 20

Lecture 4 (January 24, 2001)

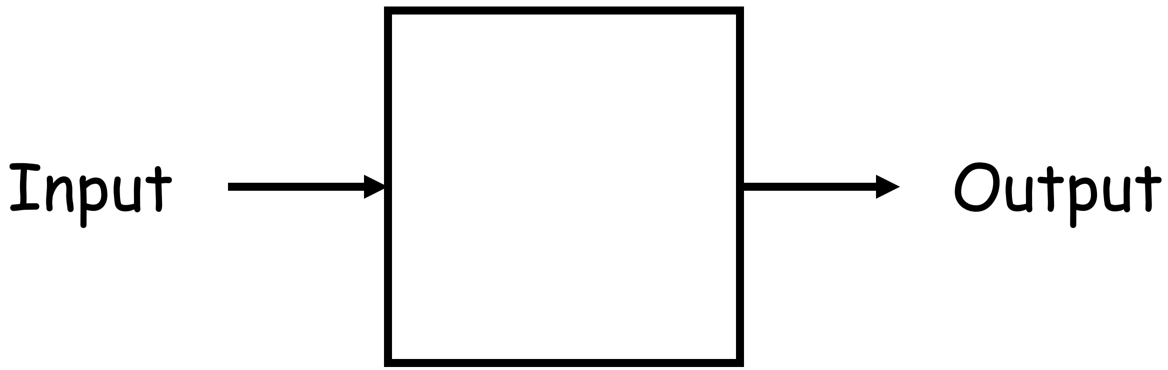
Tom Henzinger

Revised by Matteo Zavatteri and Tiziano Villa, Fall 2020

1 Systems are functions

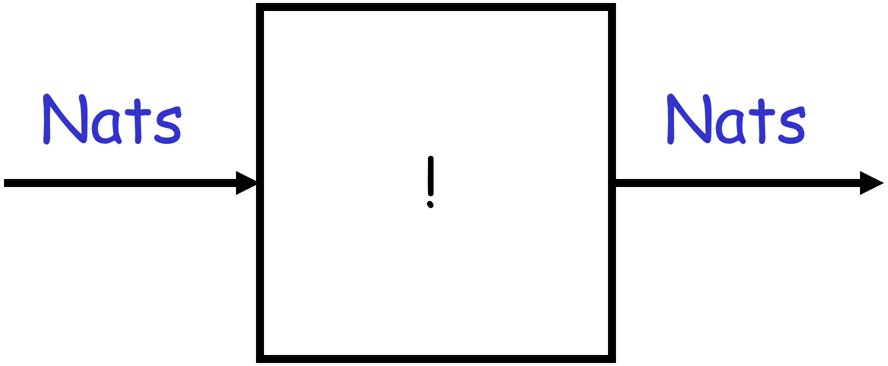
2 Signals are functions

Systems as Functions



Domain:	set of possible inputs
Range:	set of possible outputs
Graph:	set of pairs (input, output)

Factorial System



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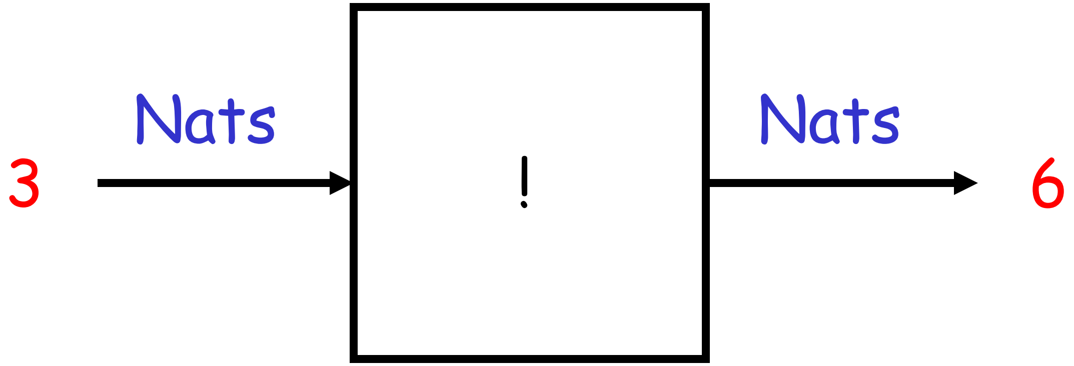

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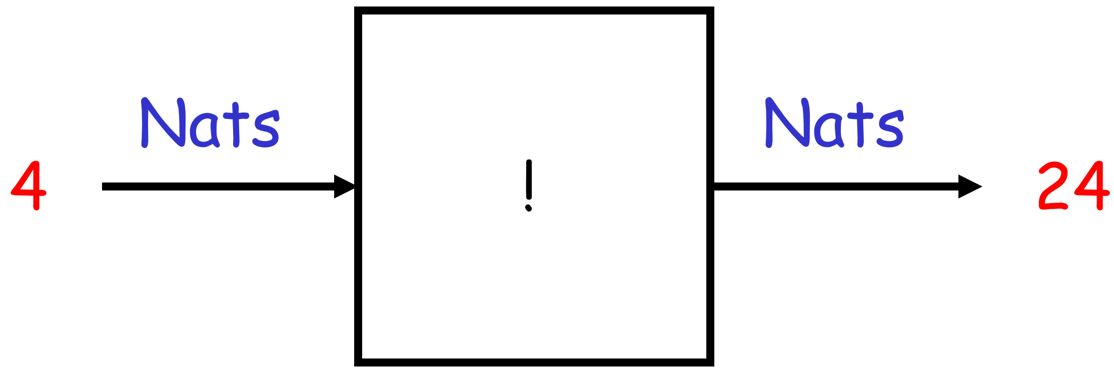
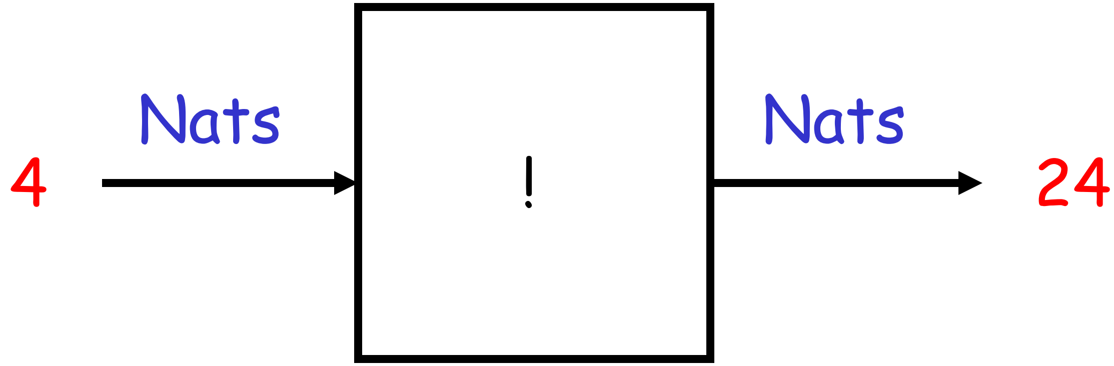


Factorial System

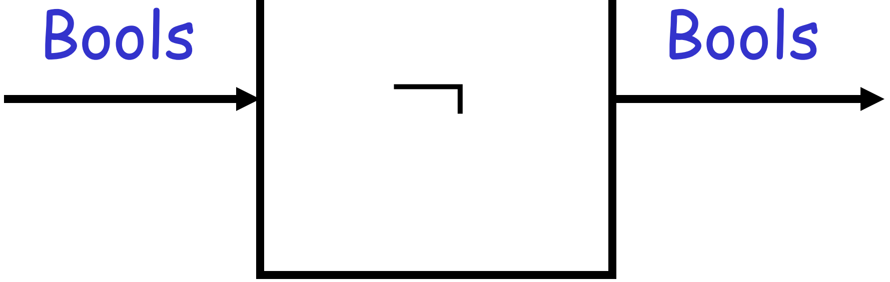


Domain: Nats

Range: Nats

Graph: { (1, 1), (2, 2), (3, 6), (4, 24), ... }

Inverter System



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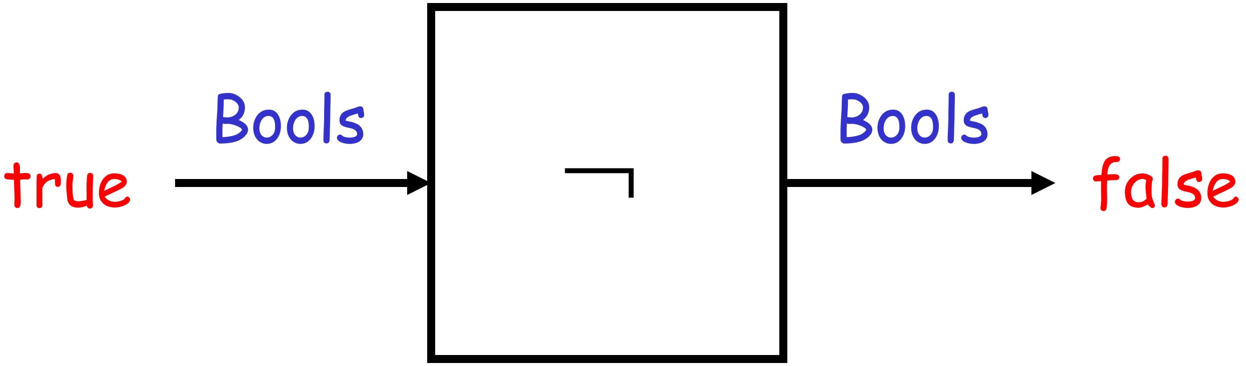

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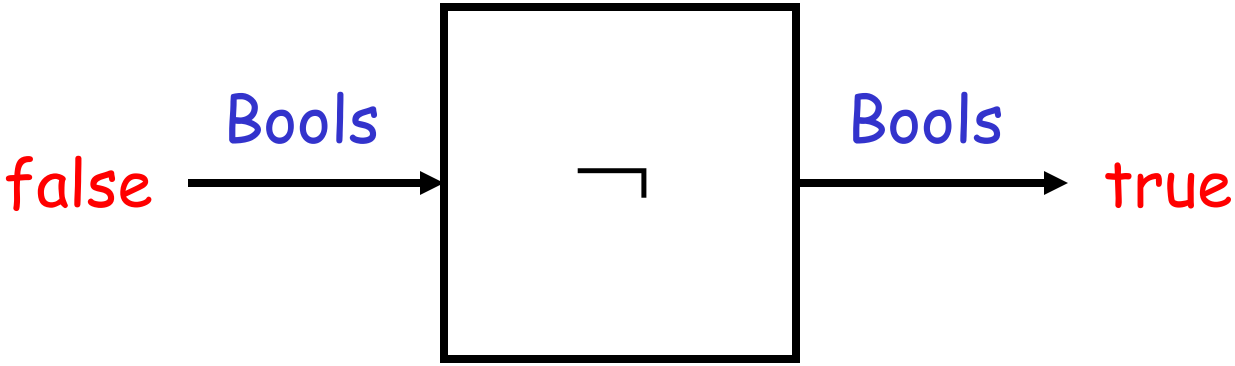
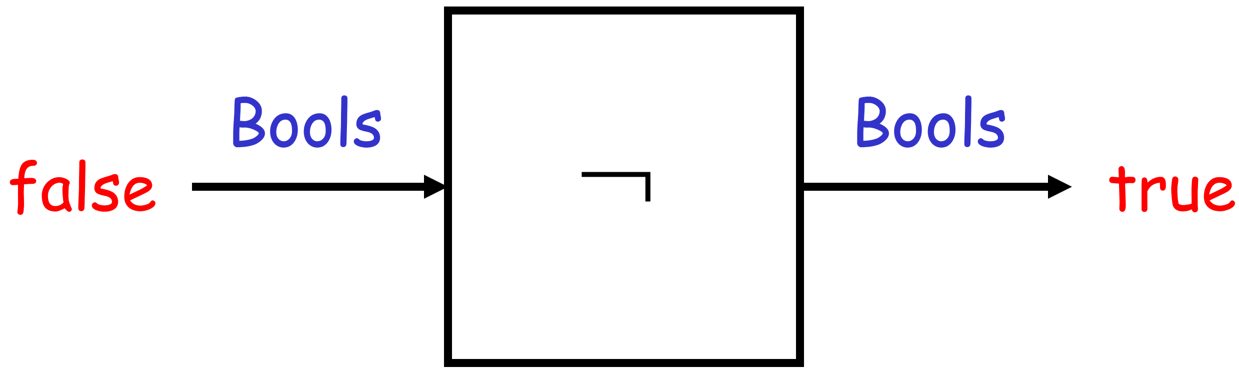


Inverter System

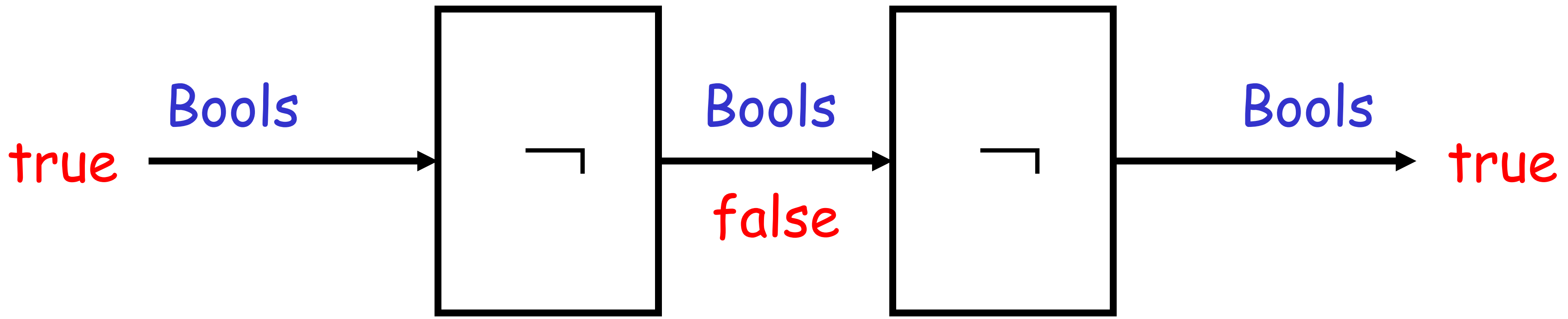


Domain: Bools

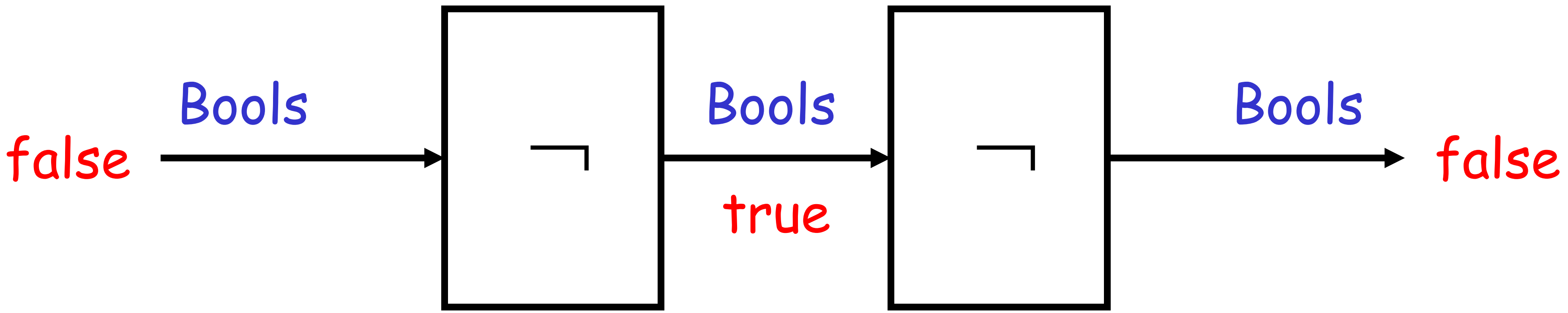
Range: Bools

Graph: { (true, false), (false, true) }

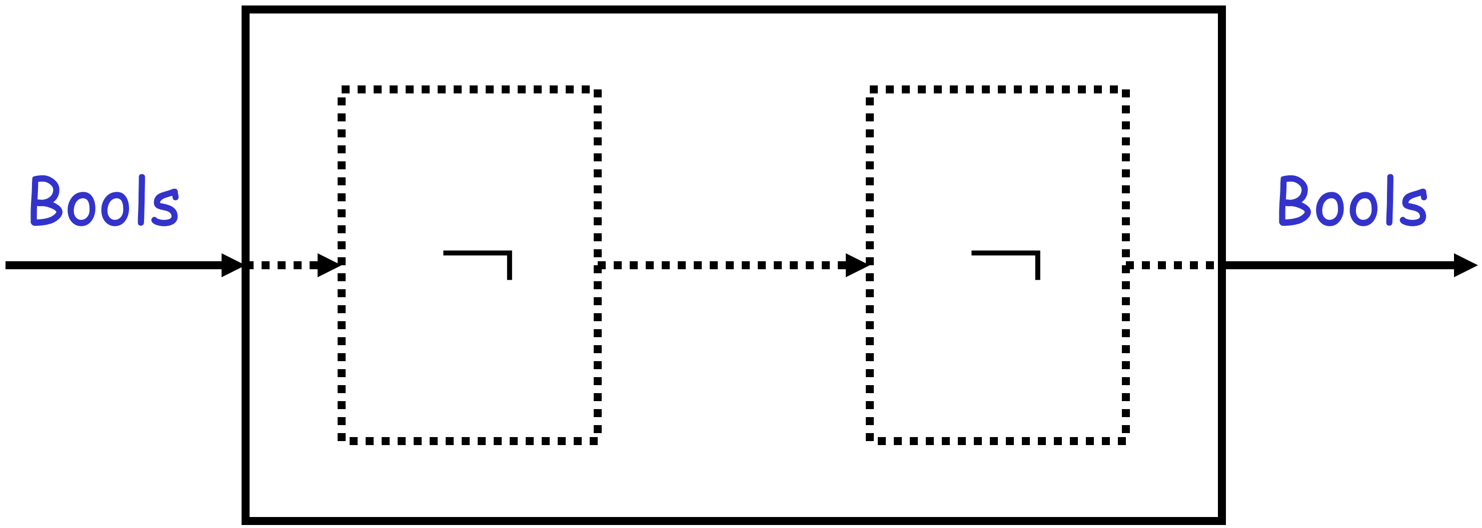
Composition of Systems



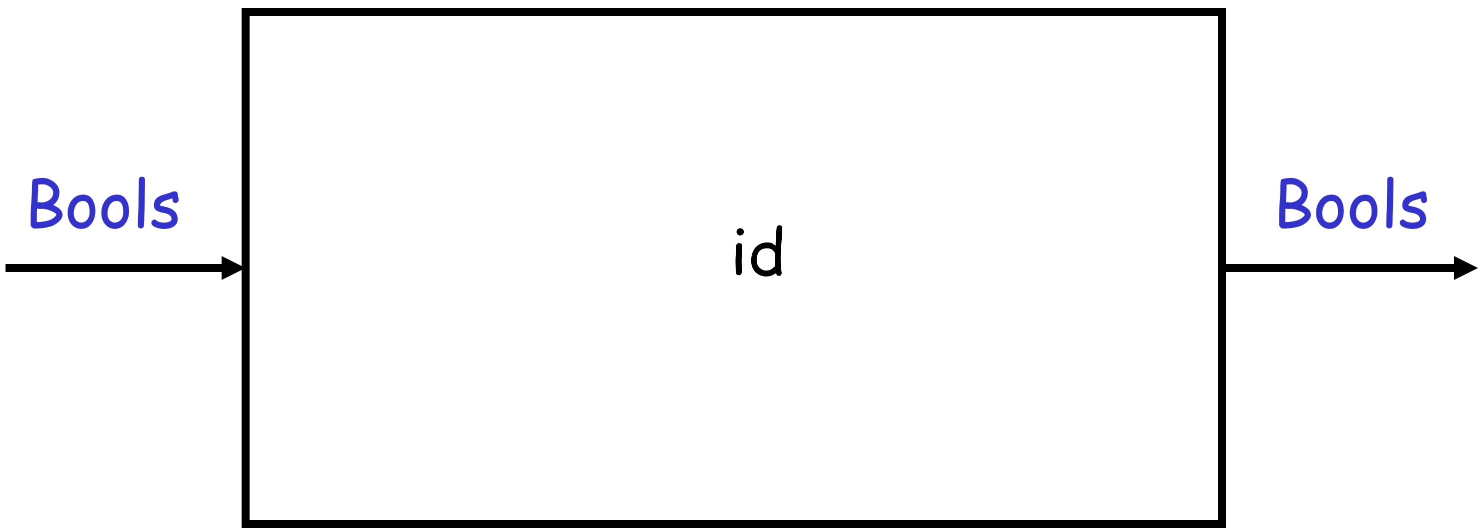
Composition of Systems



This is again a system !



The Identity System



Domain: Bools

Range: Bools

Graph: $\{ (x, y) \in \text{Bools}^2 \mid x = y \}$

System Composition is Function Composition

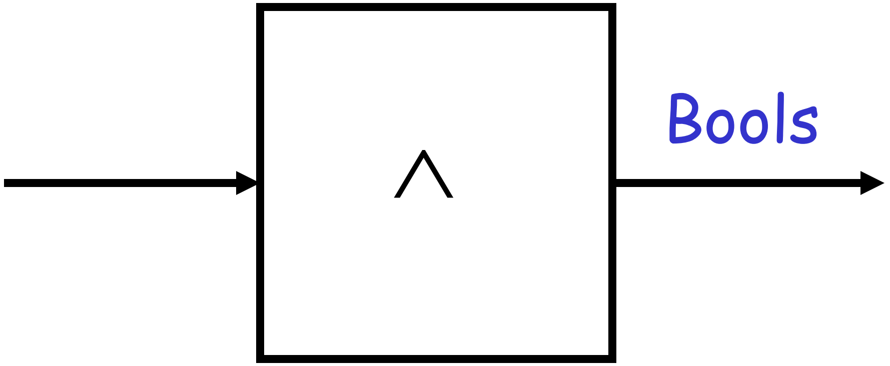
$$\neg \circ \neg \;=\; id$$

because $\quad domain(\neg \circ \neg) \;=\; domain(id)$

$range(\neg \circ \neg) \;=\; range(id)$

$\forall\, x \in Bools,\; (\neg \neg x) \;=\; id(x)$

And System



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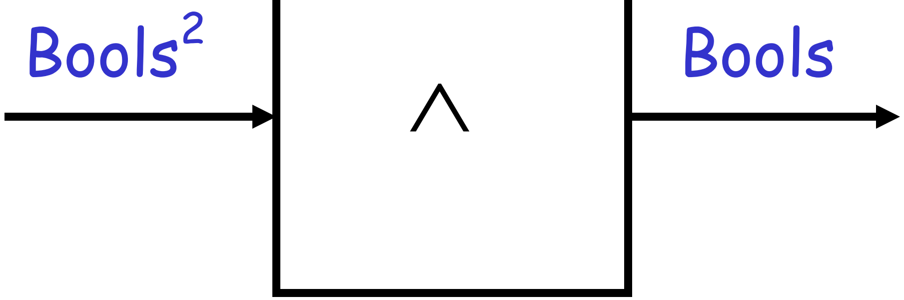

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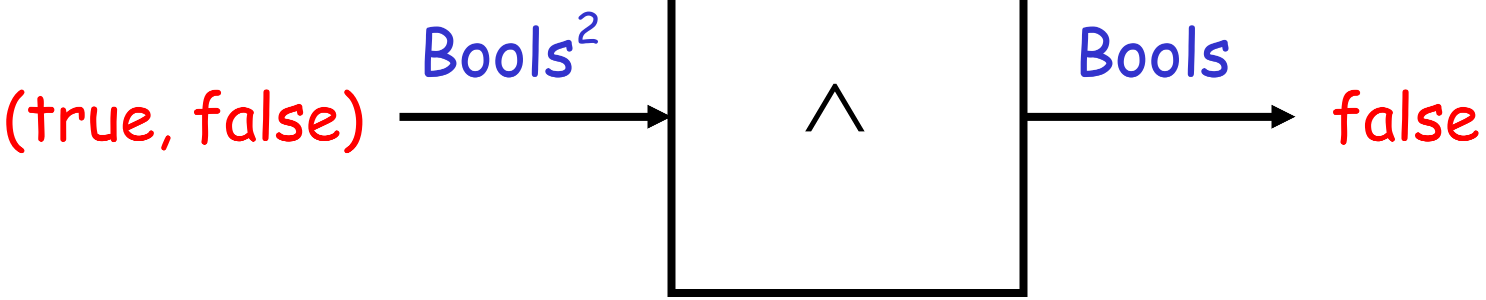

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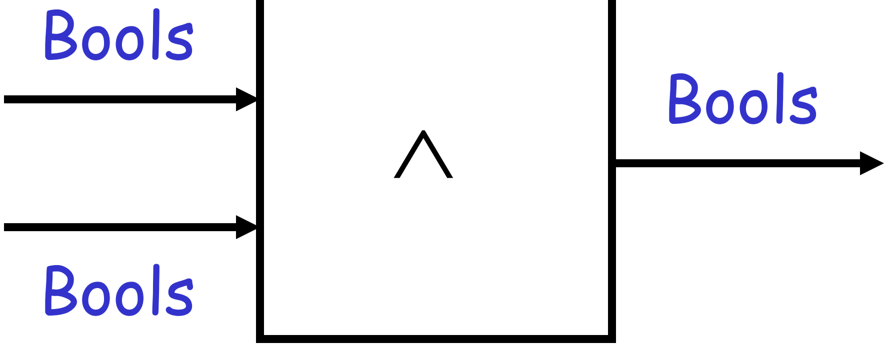

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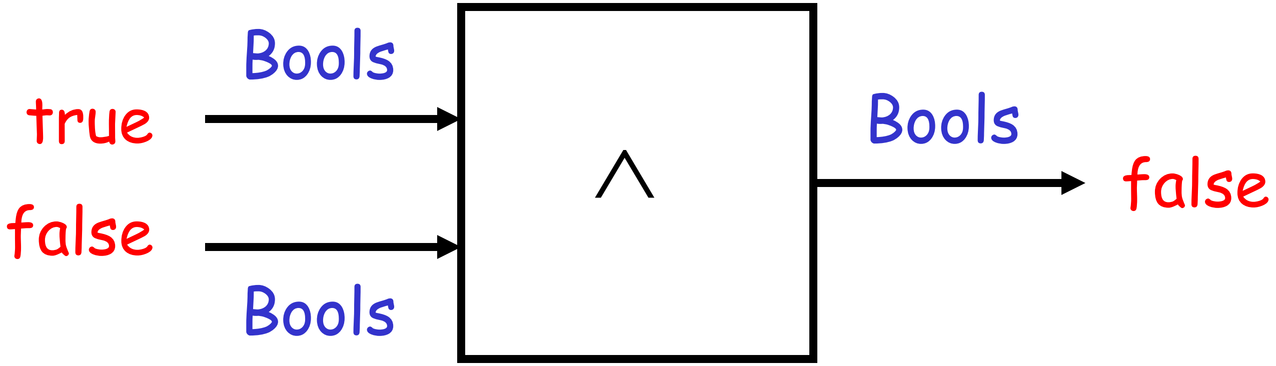
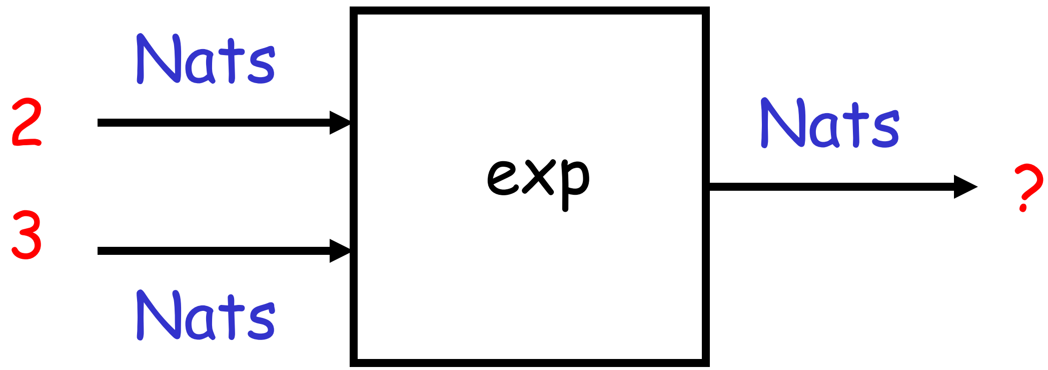
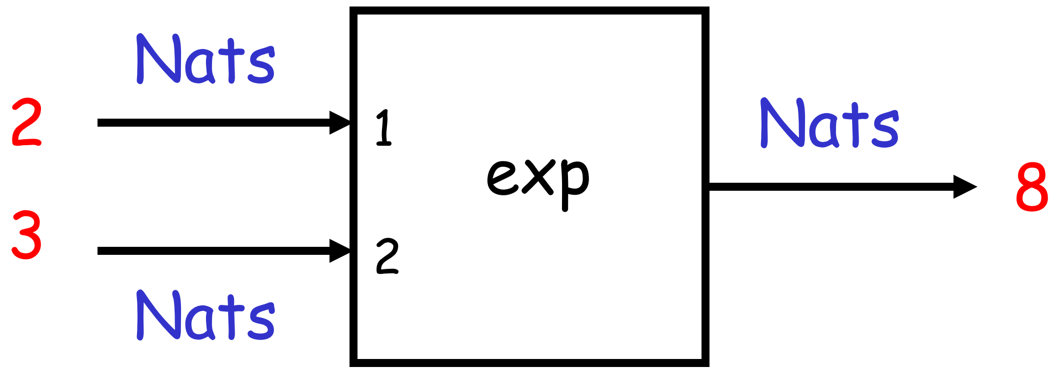


Exponentiation System



$$\text{graph}(\exp) = \{ ((x,y), z) \in \text{Nats}^2 \times \text{Nats} \mid z = x^y \}$$

Exponentiation System



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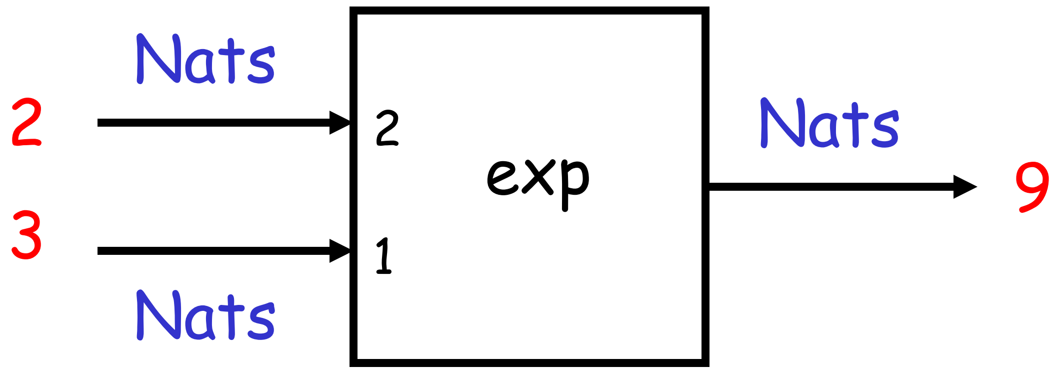

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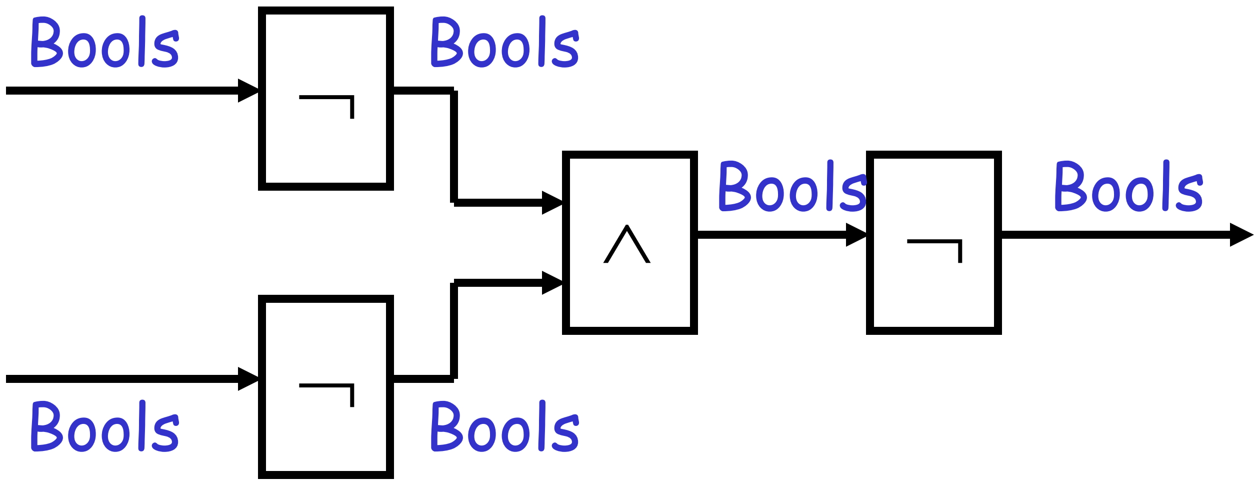
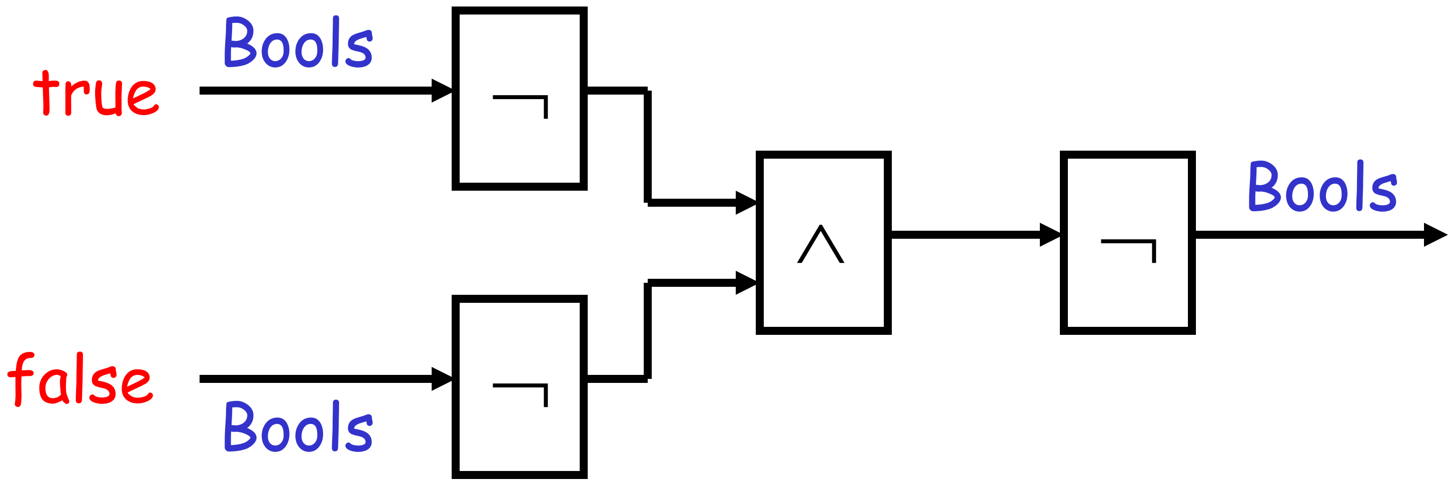
Block Diagram



This cannot be written easily using [illegible] .

Block Diagram



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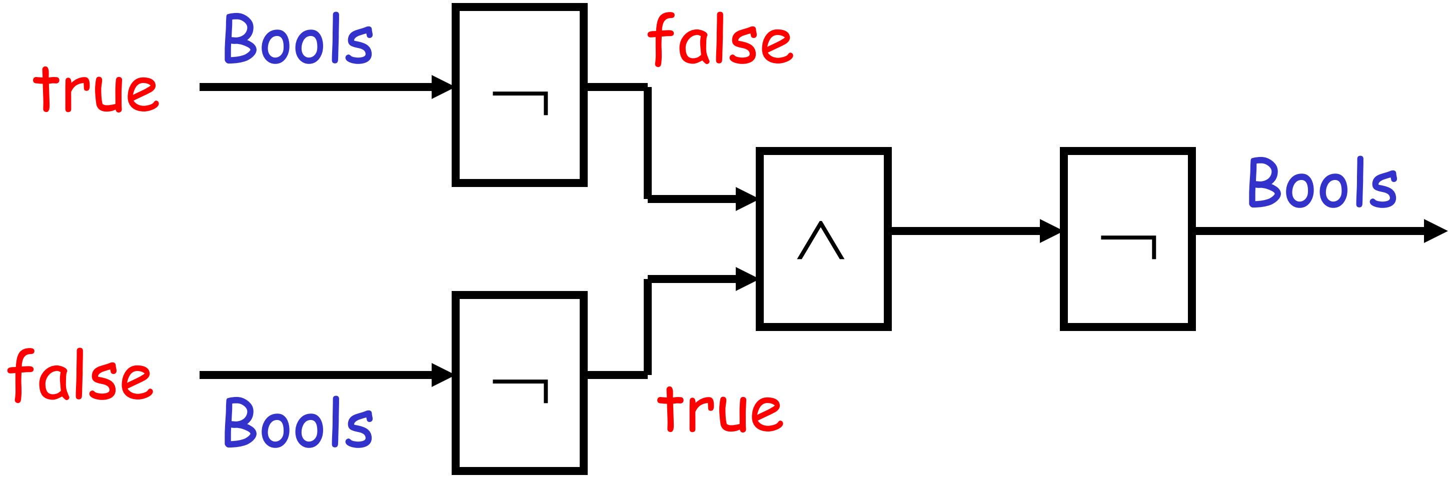

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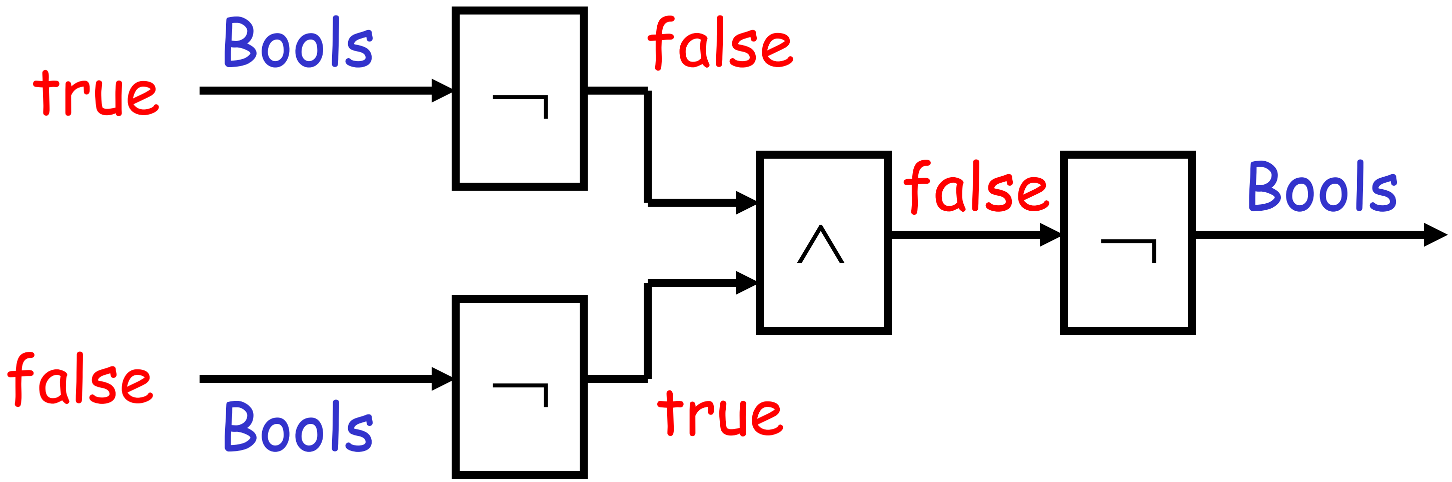

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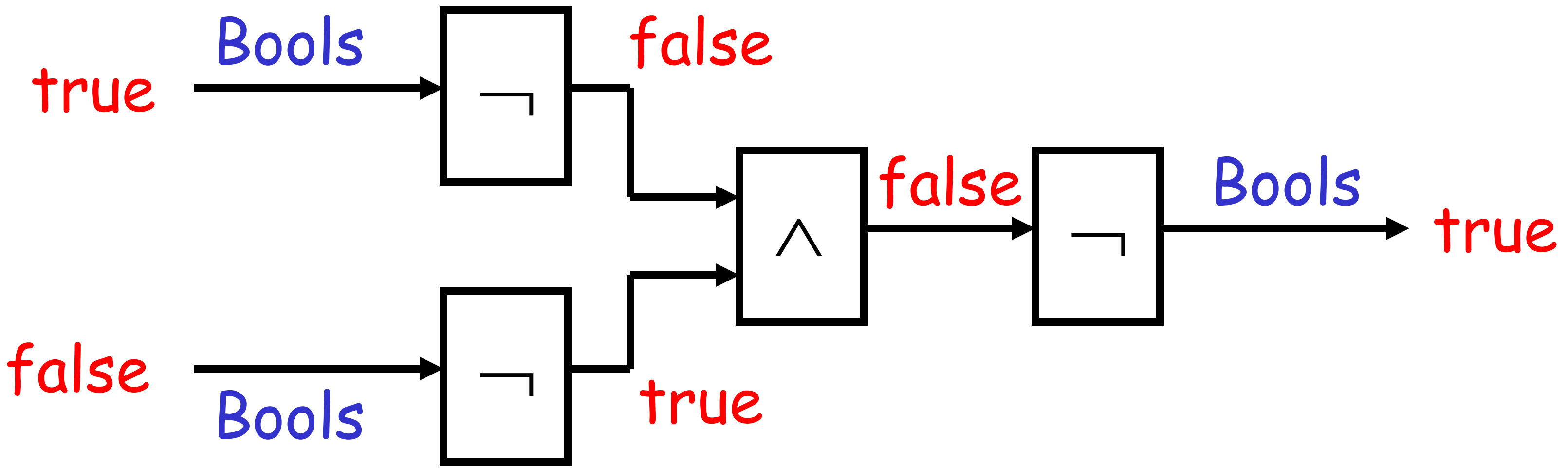
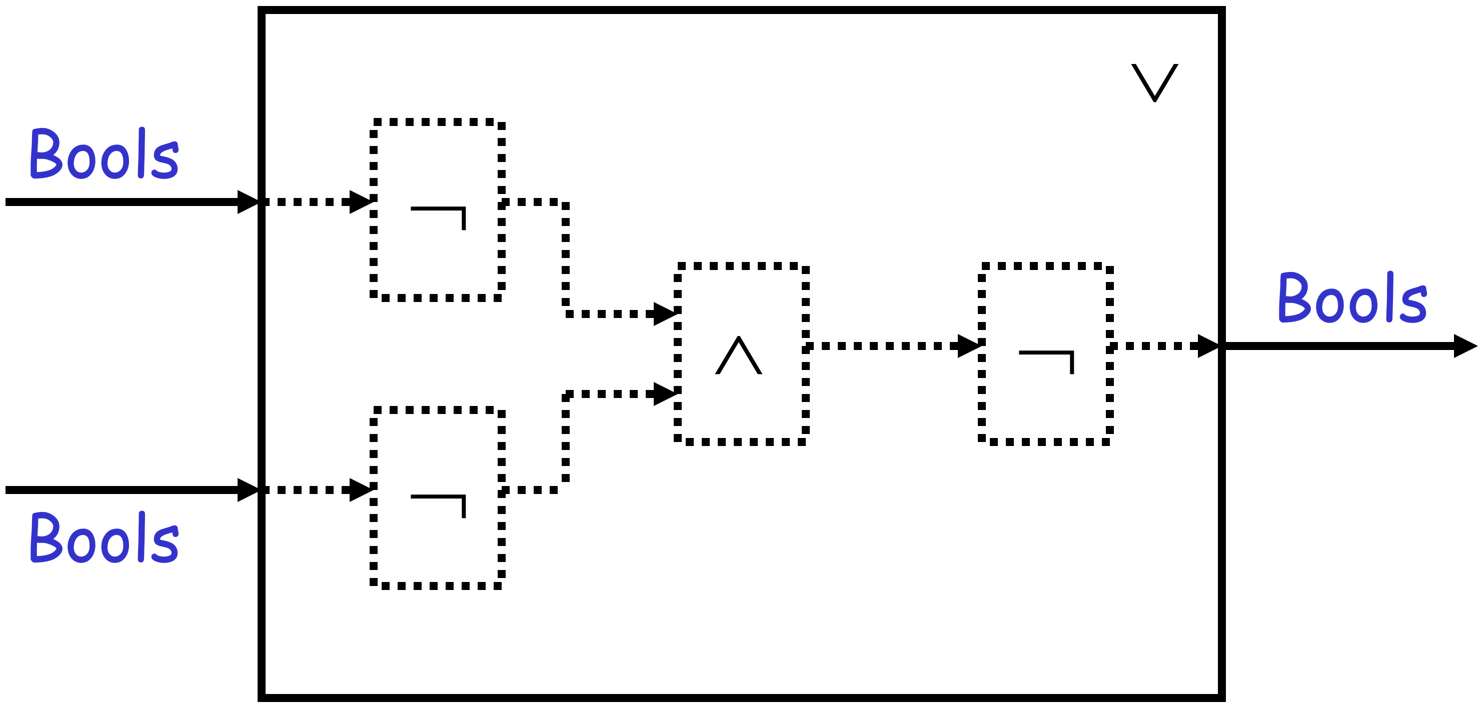


Or System

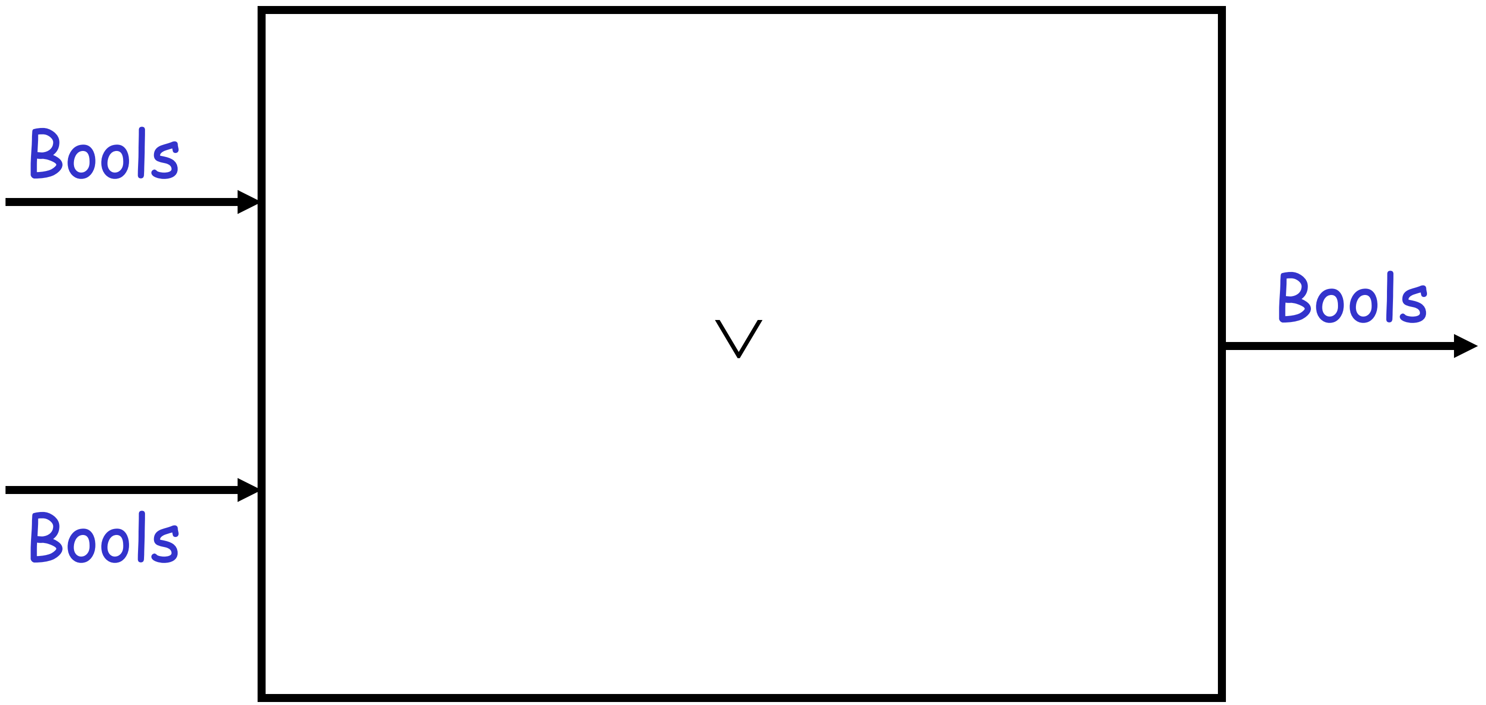


$$\text{domain}(\vee) = \text{Bools}^2$$

$$\text{range}(\vee) = \text{Bools}$$

$$\text{graph}(\vee) = \{ ((x,y), z) \in \text{Bools}^2 \times \text{Bools} \mid z = x \vee y \}$$

Or System



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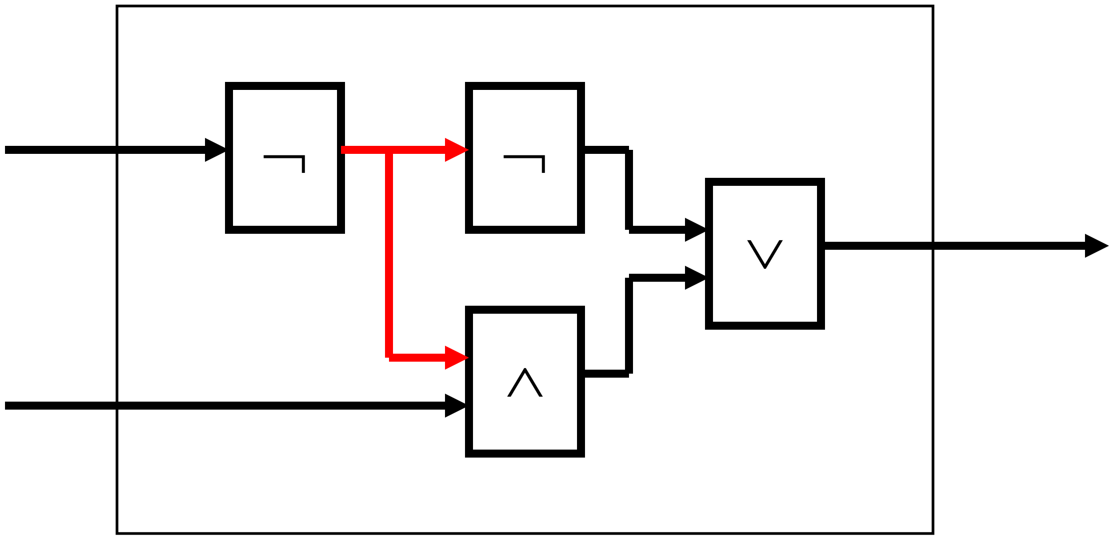
Block Diagrams with Forks



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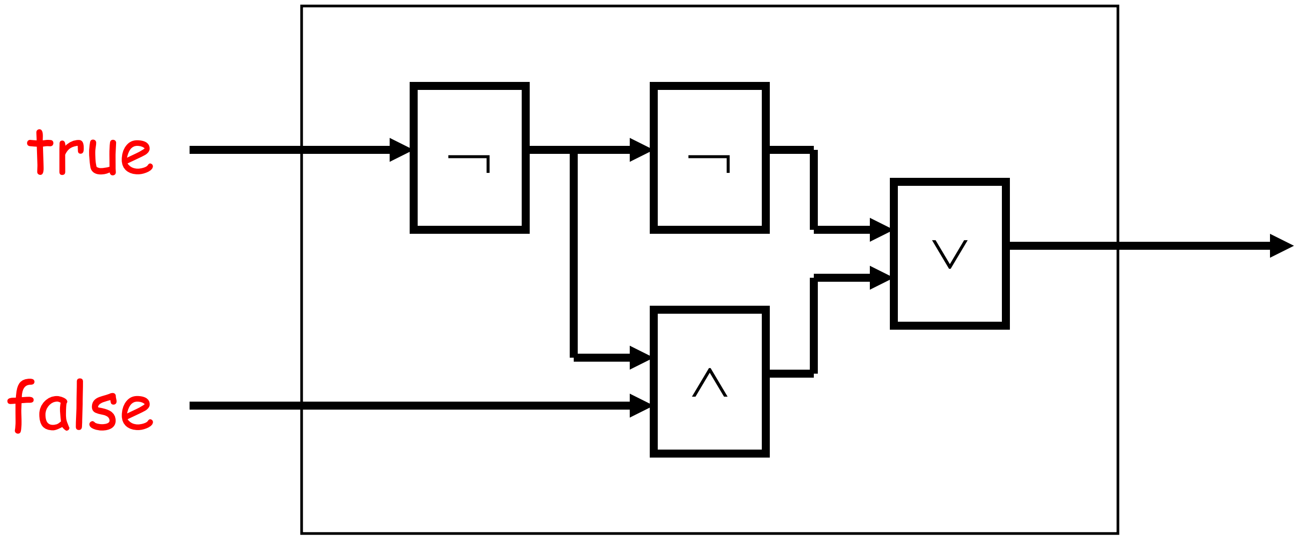

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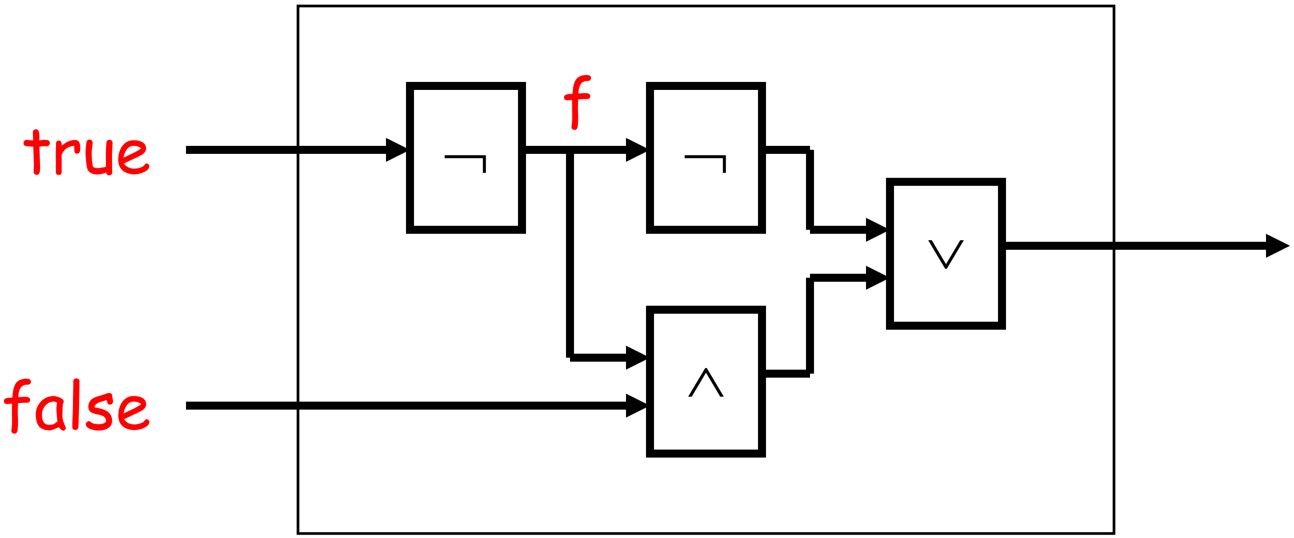

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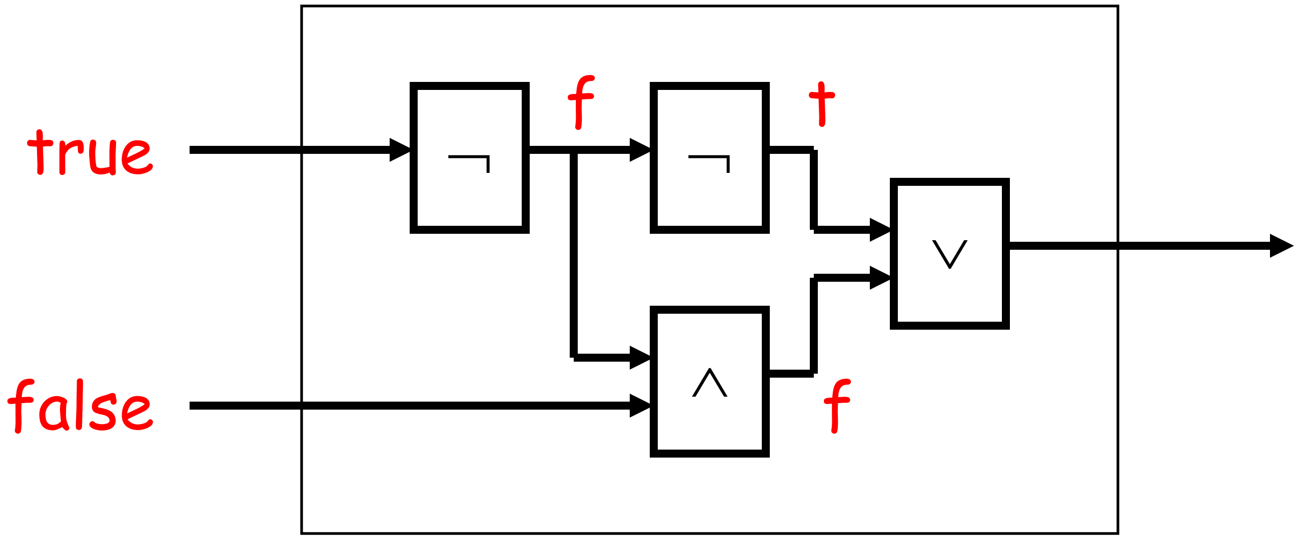

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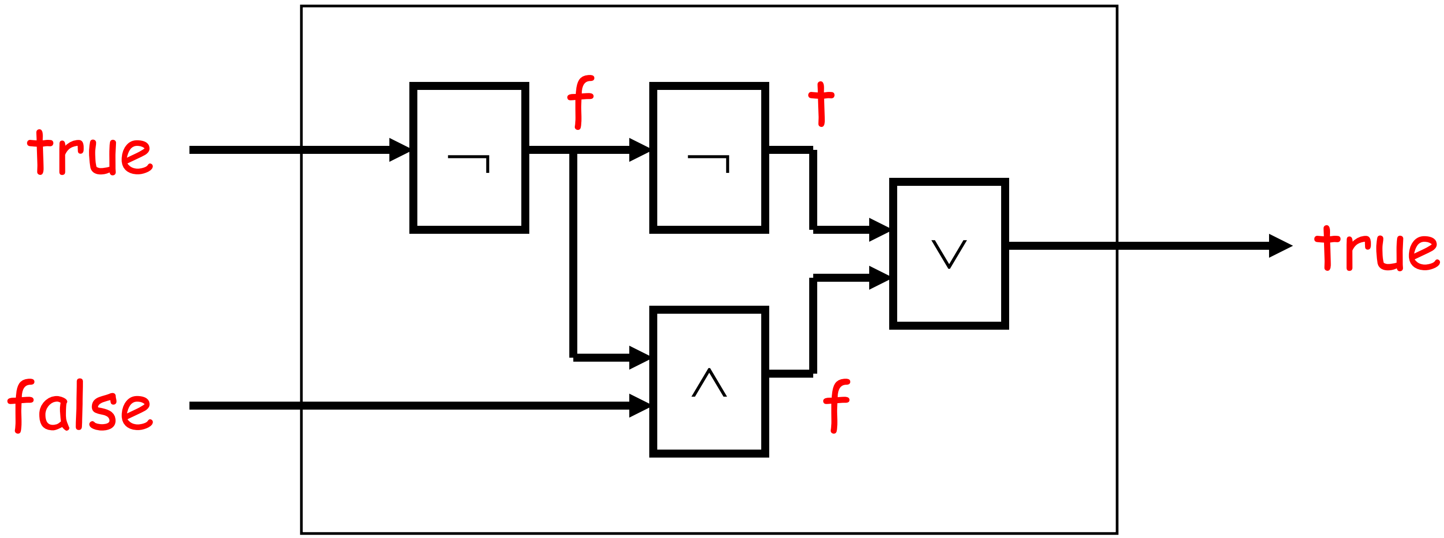
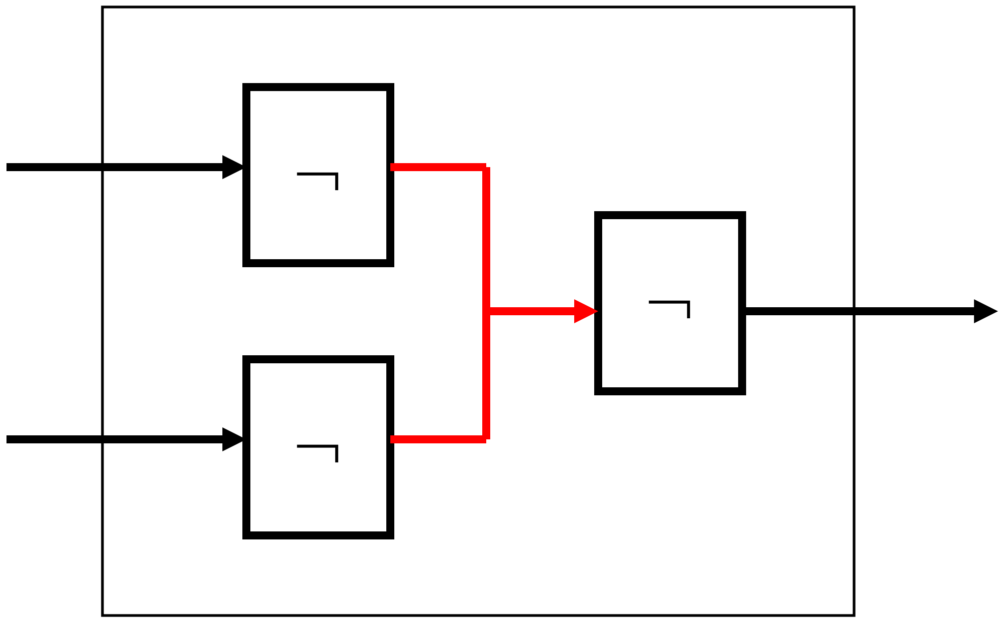


Joins are illegal



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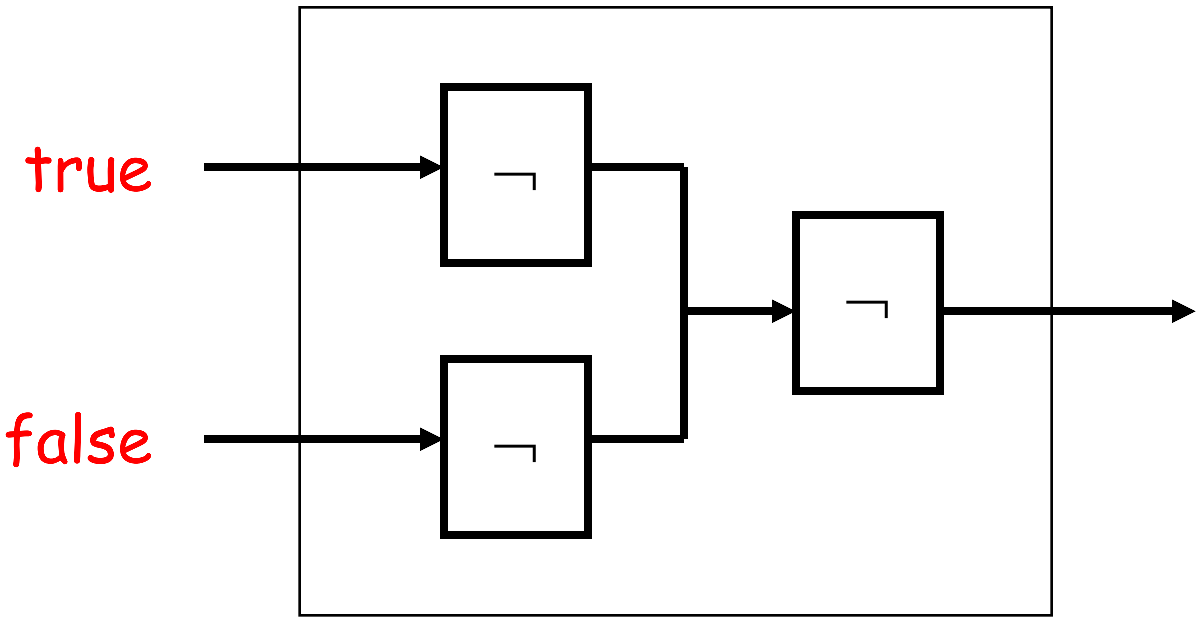

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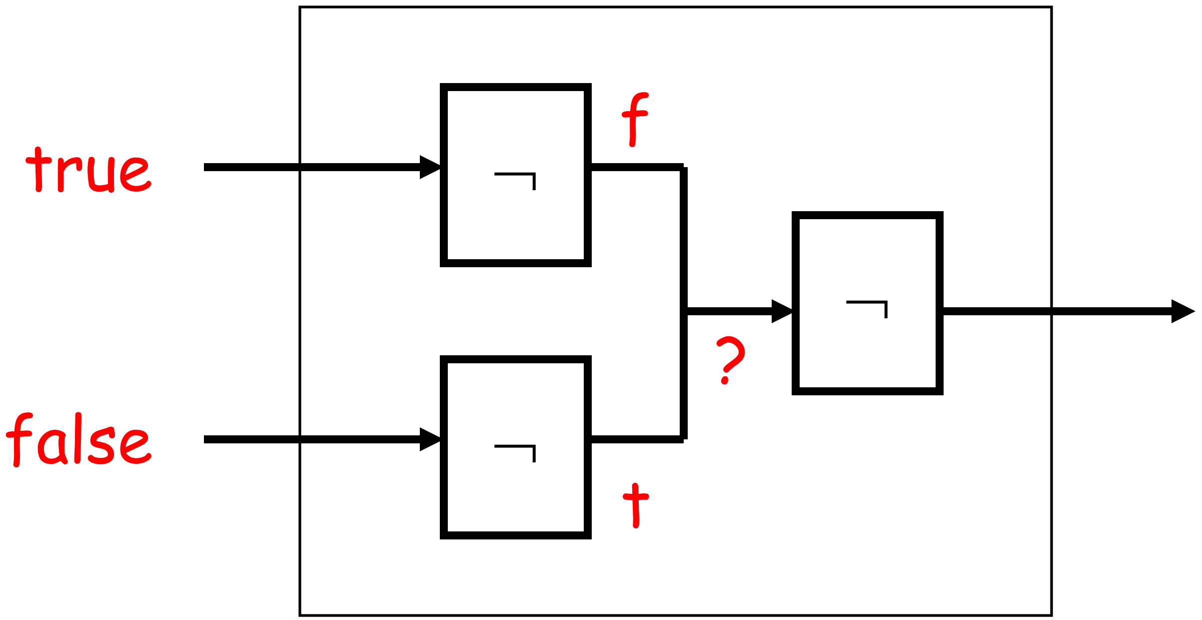
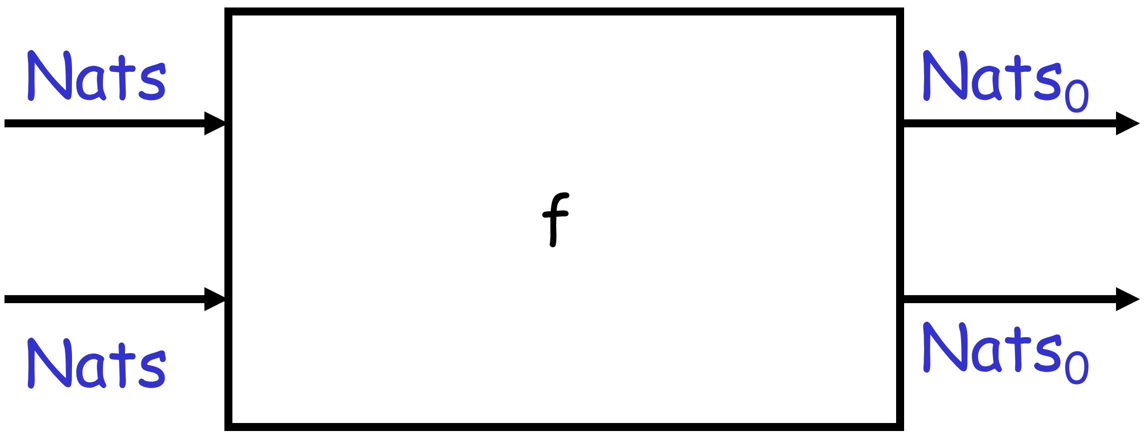


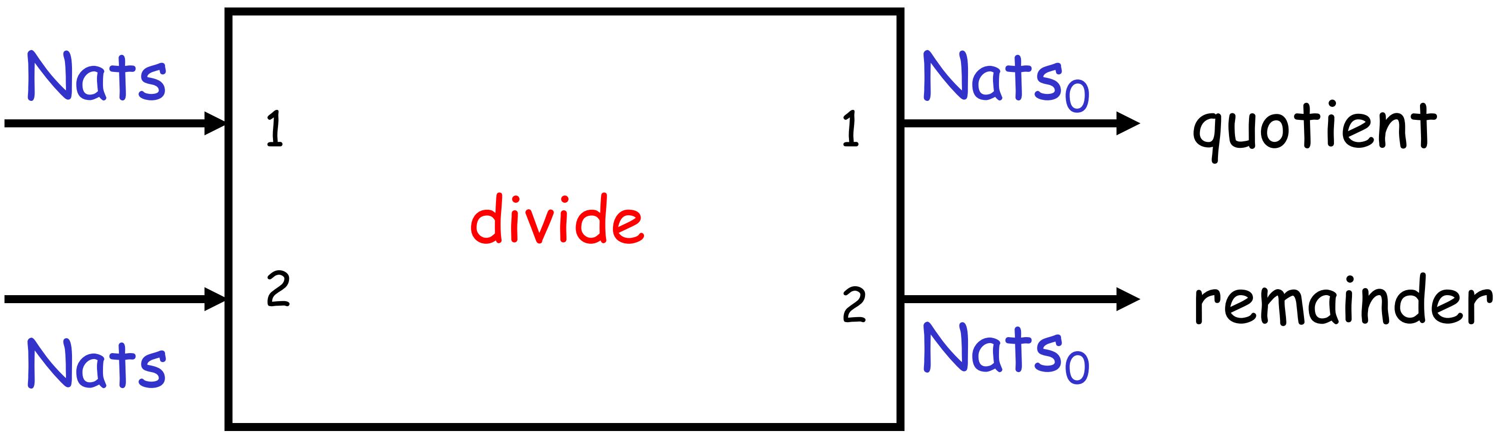
Multiple Outputs



f: $Nats^2 \to Nats_0^2$ such that

$\forall x,y \in Nats,\; f(x,y) = \{ (q,r) \in Nats_0^2 \mid x = q \cdot y + r \wedge r < y \}$

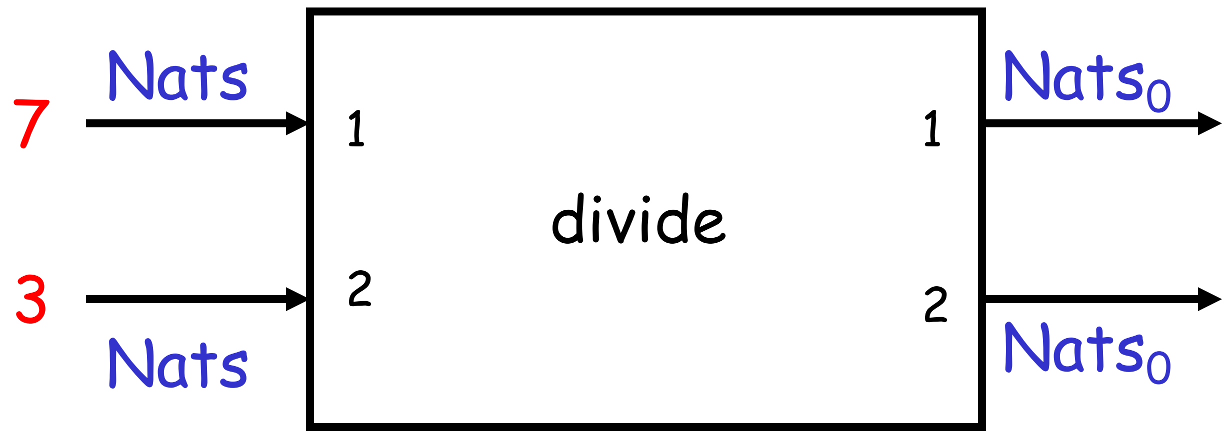
Division System



divide: $Nats^2 \rightarrow Nats_0{}^2$ such that

$\forall x,y \in Nats,\ \ divide\,(x,y) \ = \ \{\, (q,r) \in Nats_0{}^2 \mid x = q \cdot y + r \wedge r < y \,\}$

Division System



divide: $Nats^2 \rightarrow Nats_0^2$ such that

$\forall x,y \in Nats$, divide $(x,y) = \{ (q,r) \in Nats_0^2 \mid x = q \cdot y + r \wedge r < y \}$

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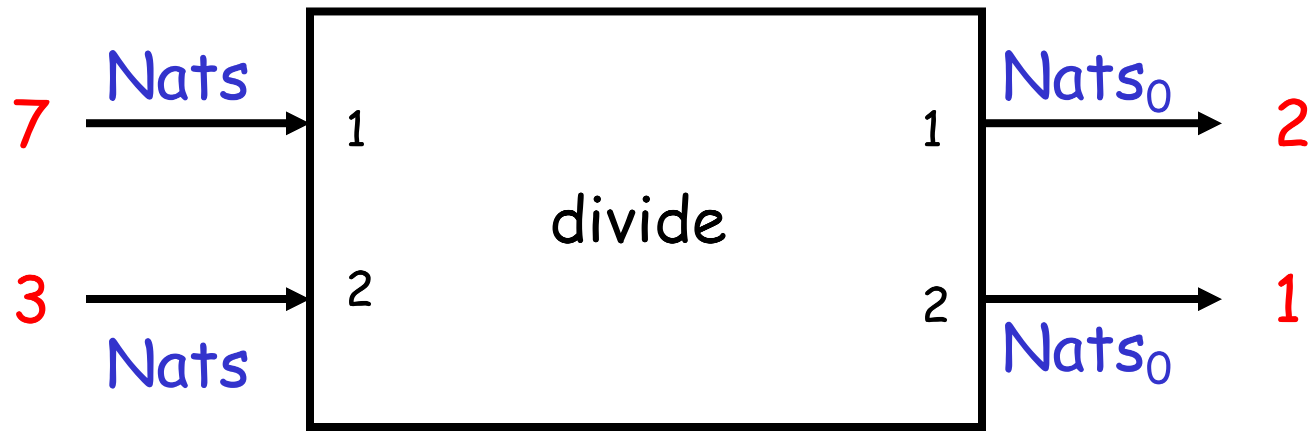

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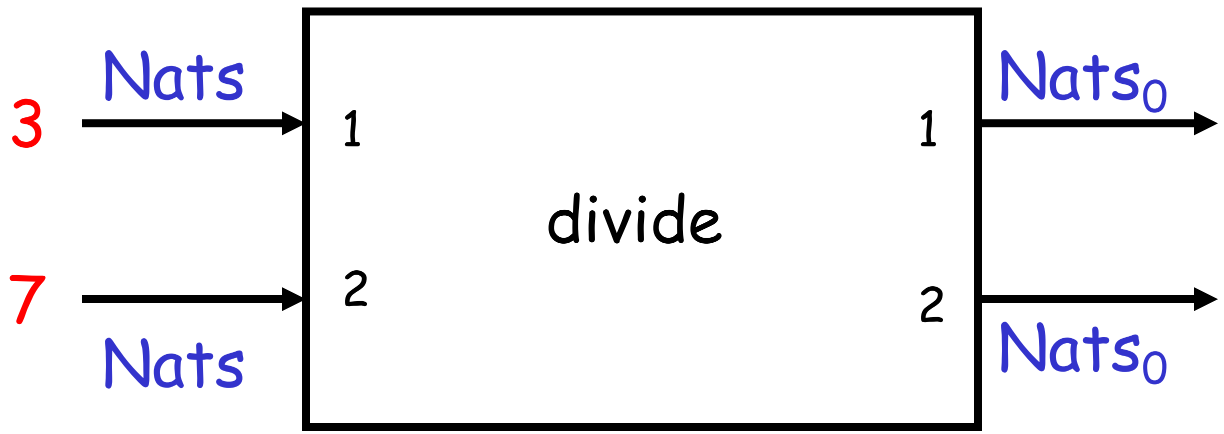

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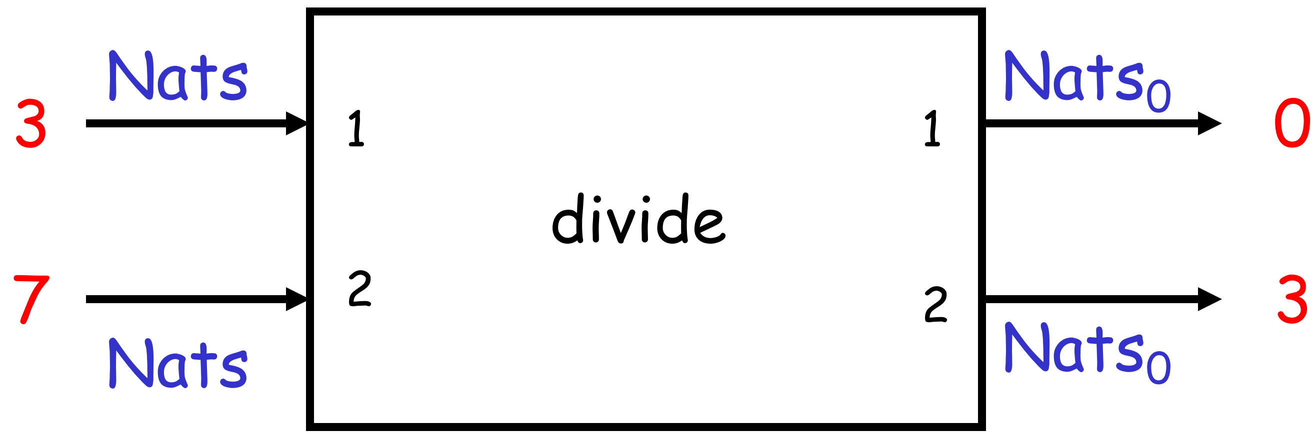

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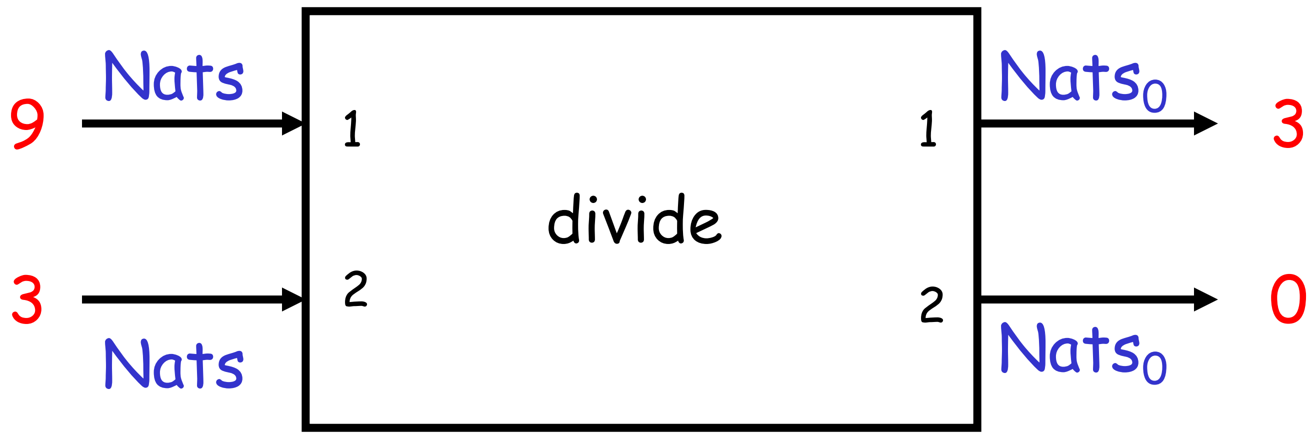

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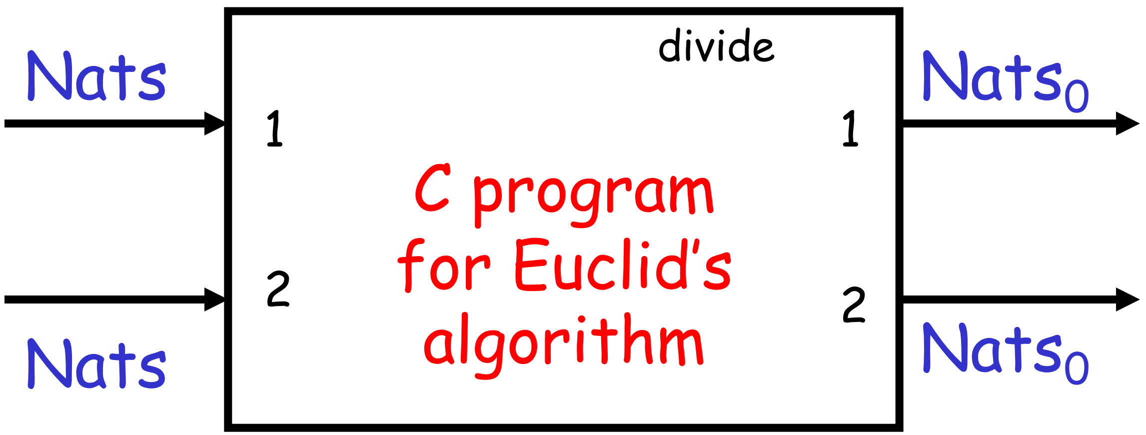
Many possible implementations



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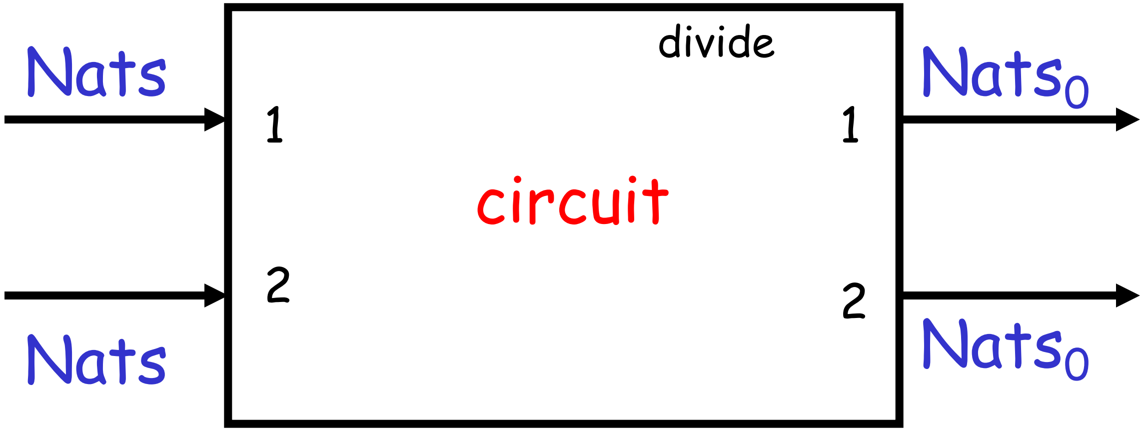
Many possible implementations



$\text{divide}: Nats^2 \to Nats_0^2$ such that

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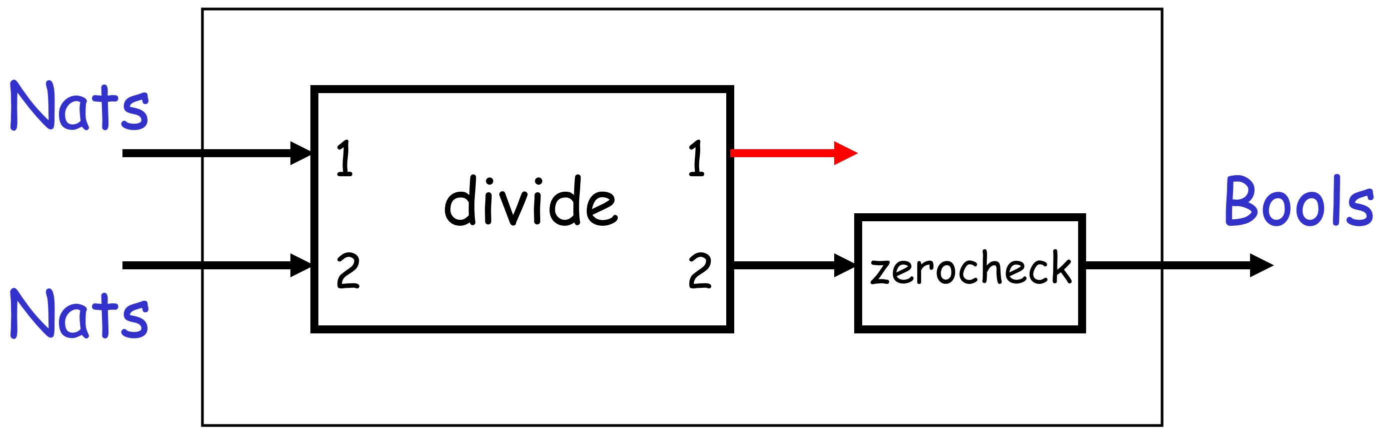
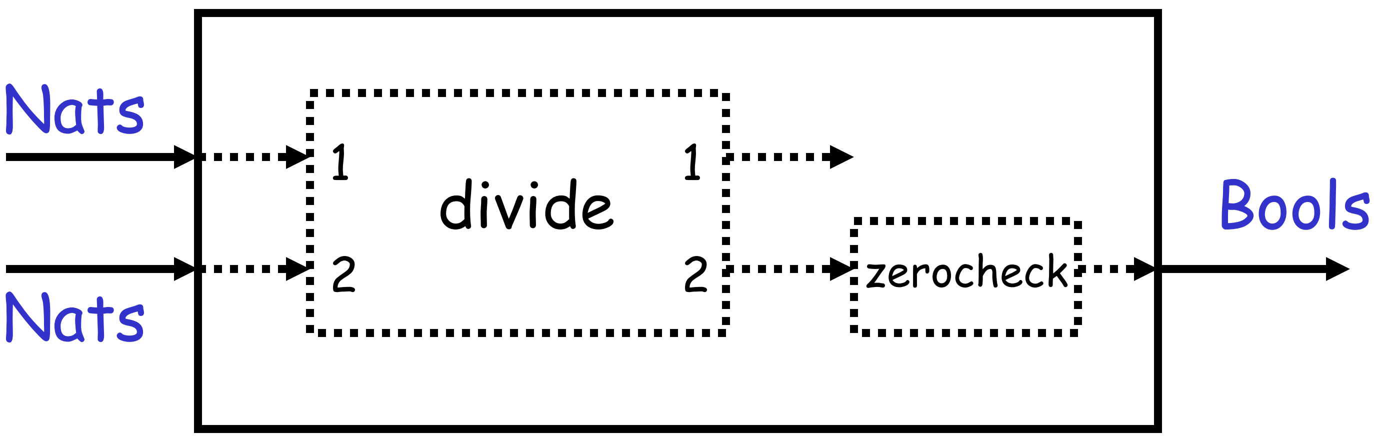
Block diagrams can hide outputs



zerocheck: $\text{Nats}_0 \to \text{Bools}$ such that

$\forall x \in \text{Nats},\ \text{zerocheck}(x) \Leftrightarrow x = 0$.

Block Diagrams can hide outputs



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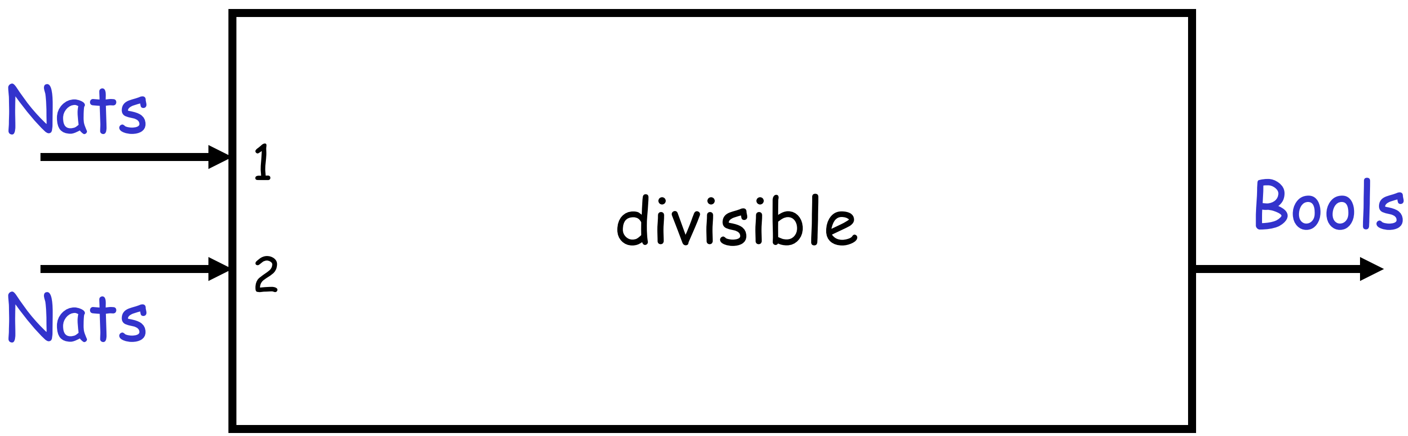
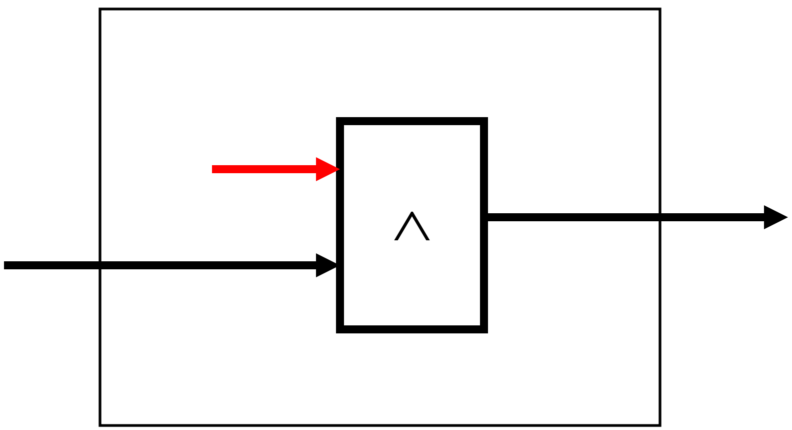


divisible: $Nats^2 \to Bools$ such that

$\forall x,y \in Nats,\ \ divisible\,(x,y) \Leftrightarrow (\ \exists q \in Nats,\ \ x = q \cdot y\ )\ .$

Hidden inputs are illegal, for now



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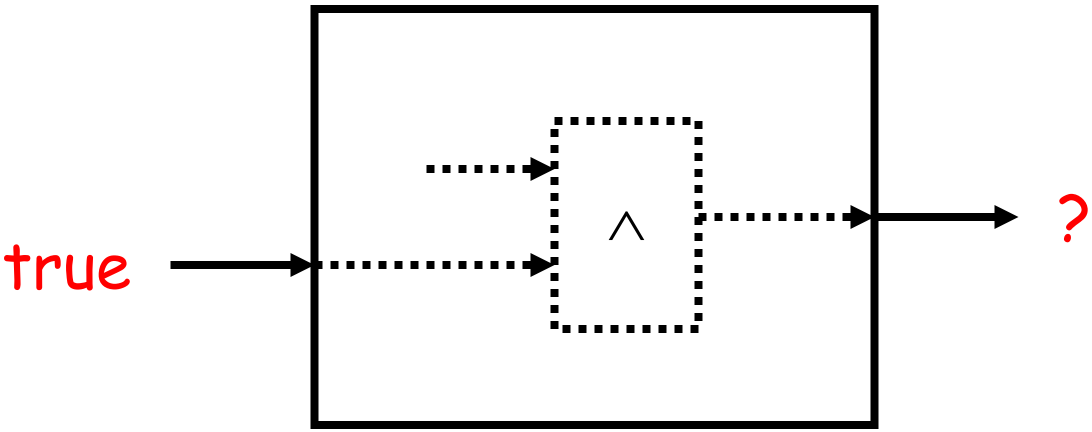
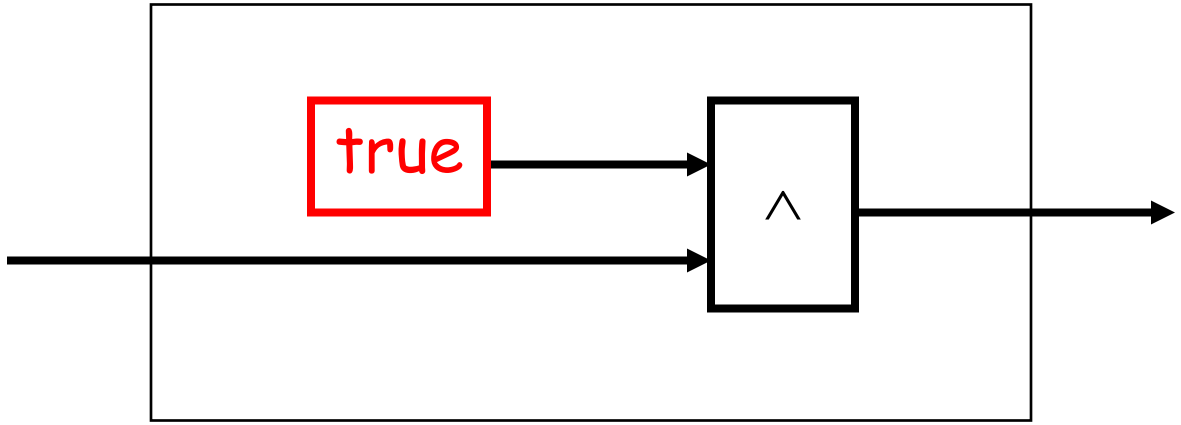
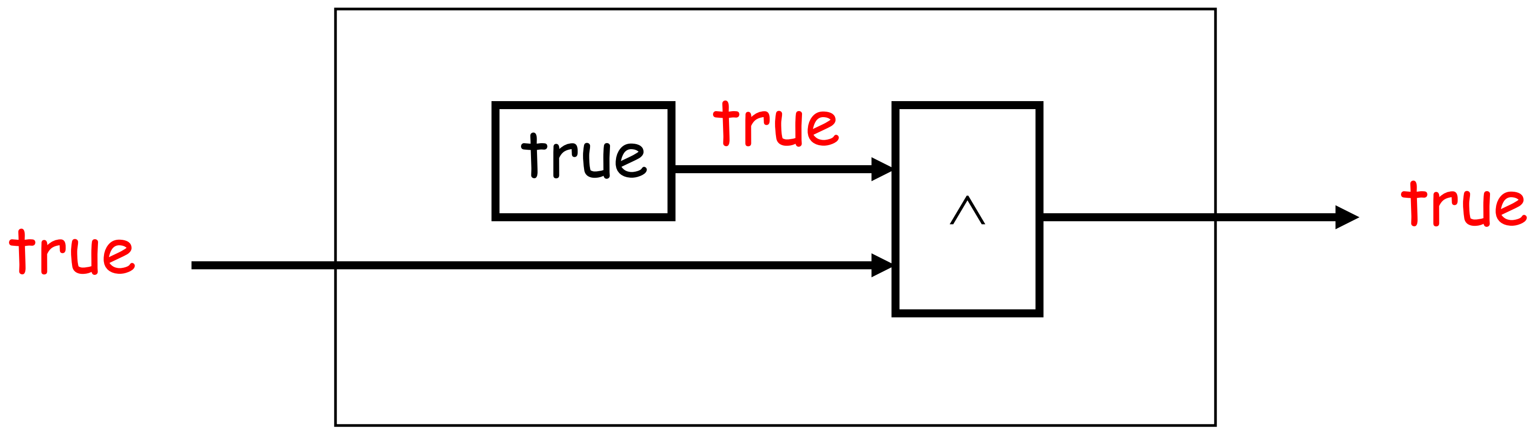


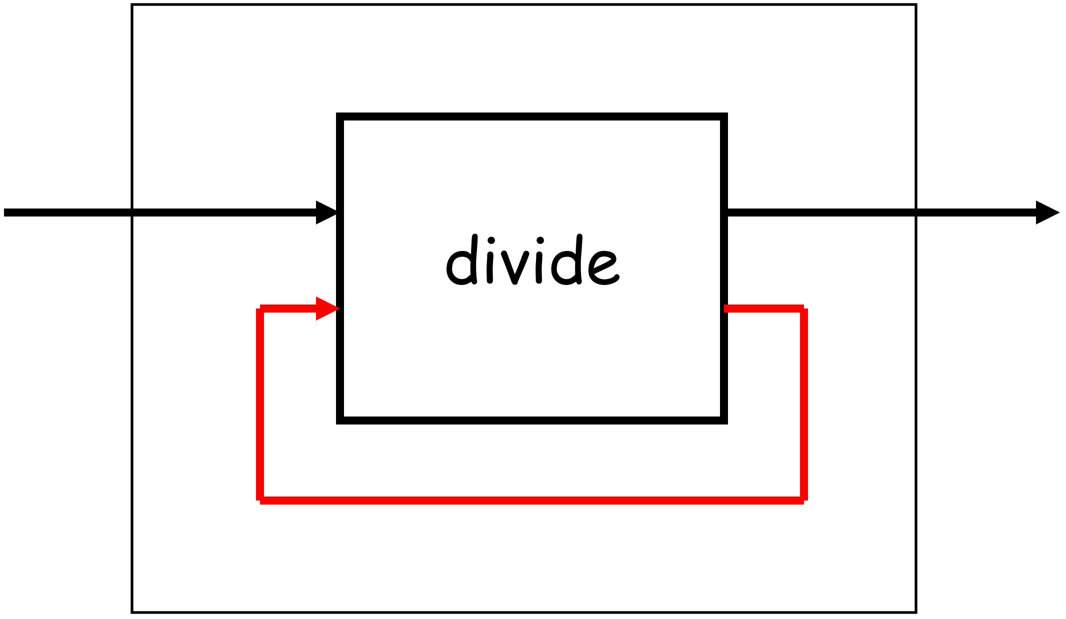
Constant functions have no inputs



Constant function



Cycles are illegal, for now



Block Diagrams

-are nested, directed, acyclic graphs

-allow compositional, hierarchical system description

Quiz

1. $\forall$ set x, $x \subseteq P(x)$
2. $\exists$ function f, $\{ x \in \text{domain}(f) \mid x = f(x) \}$
3. $\forall n \in \text{Nats}$, $n = 2 \Rightarrow (n, n+1) \in \{1, 2, 3\}^2$
4. $\exists f \in [\text{Nats} \to \text{Nats}]$, $f(x) = x^2$