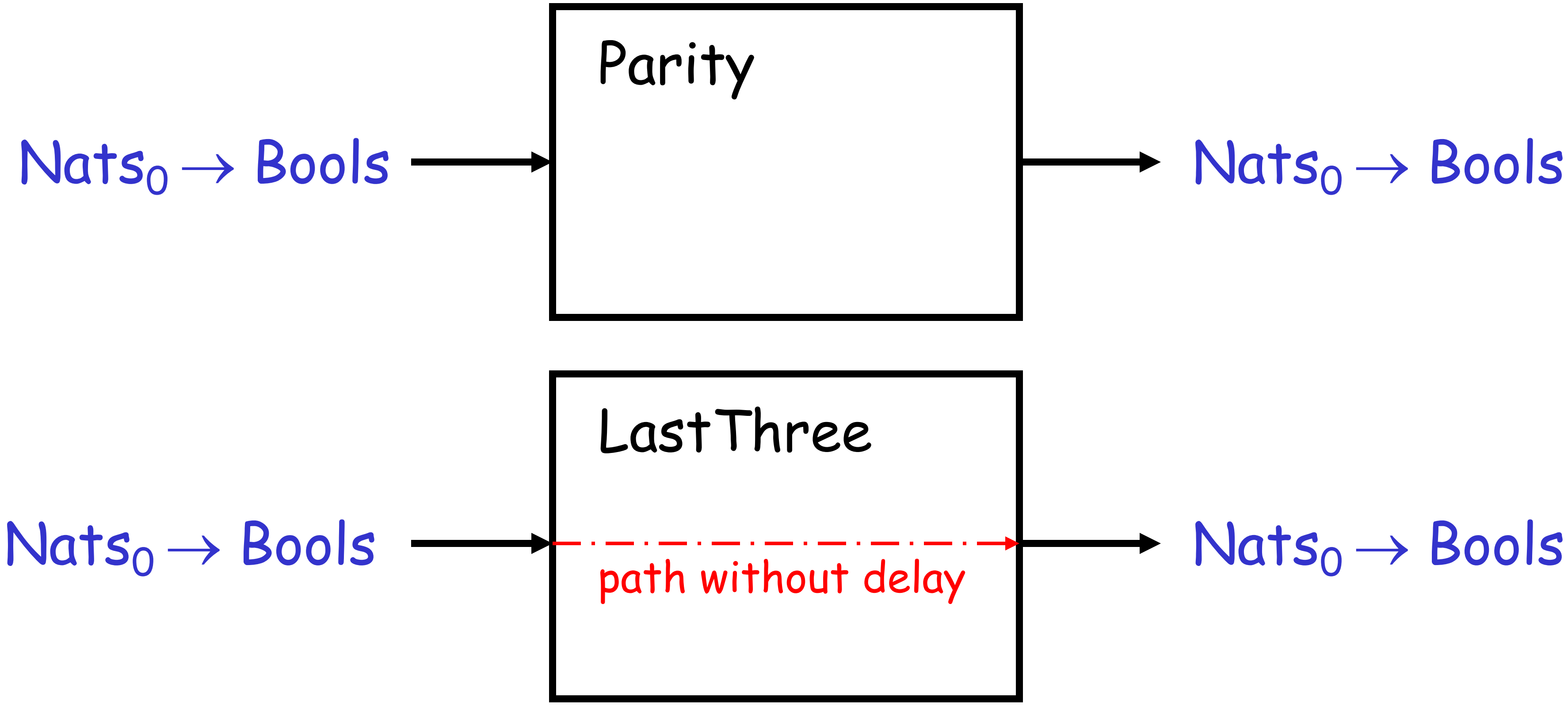
Feedback

EECS 20

Lecture 11 (February 9, 2001)

Tom Henzinger



Parity
LastThree
path without delay
$Nats_0 \rightarrow Bools$
$Nats_0 \rightarrow Bools$
$Nats_0 \rightarrow Bools$
$Nats_0 \rightarrow Bools$

The Parity System :

States [Parity] = { true, false }

initialState [Parity] = true

nextState [Parity] (q,x) = $(q \neq x)$

output [Parity] (q,x) = q

The LastThree System :

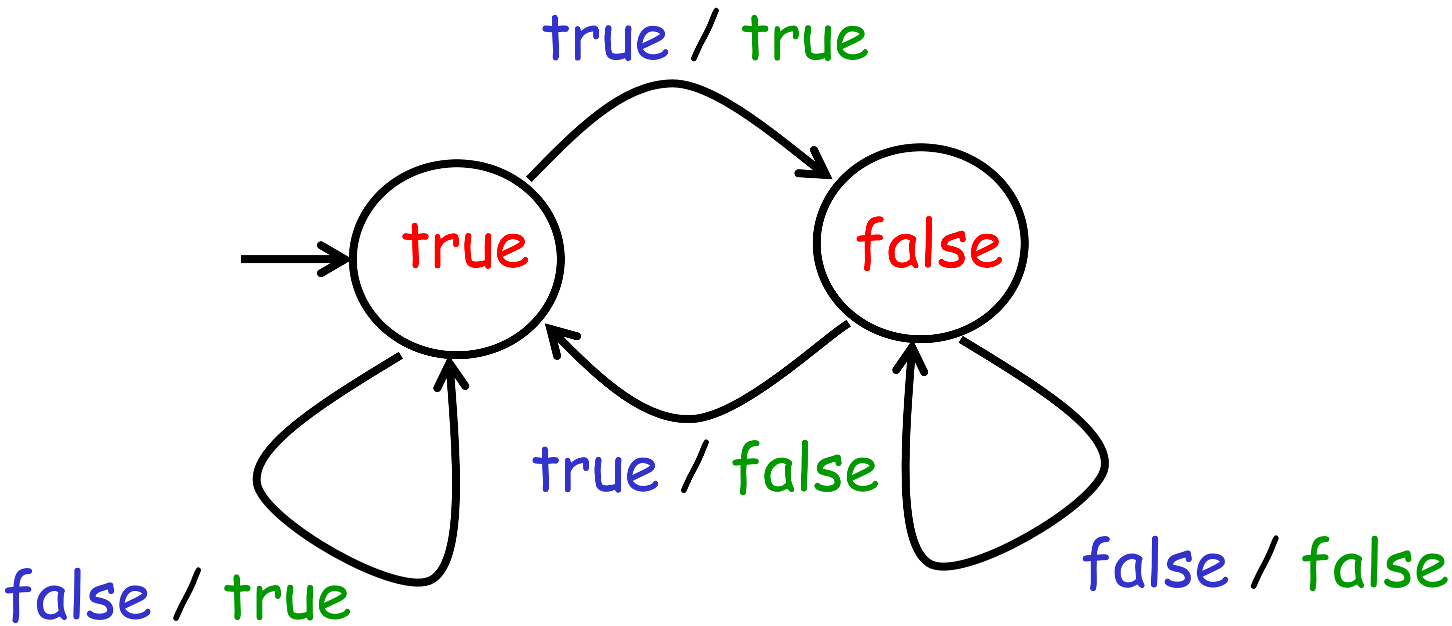
States [LastThree] = { 0, 1, 2 }

initialState [LastThree] = 0

$$\text{nextState [LastThree] }(q,x) = \begin{cases} 0 & \text{if } \neg x \\ \min(q+1, 2) & \text{if } x \end{cases}$$

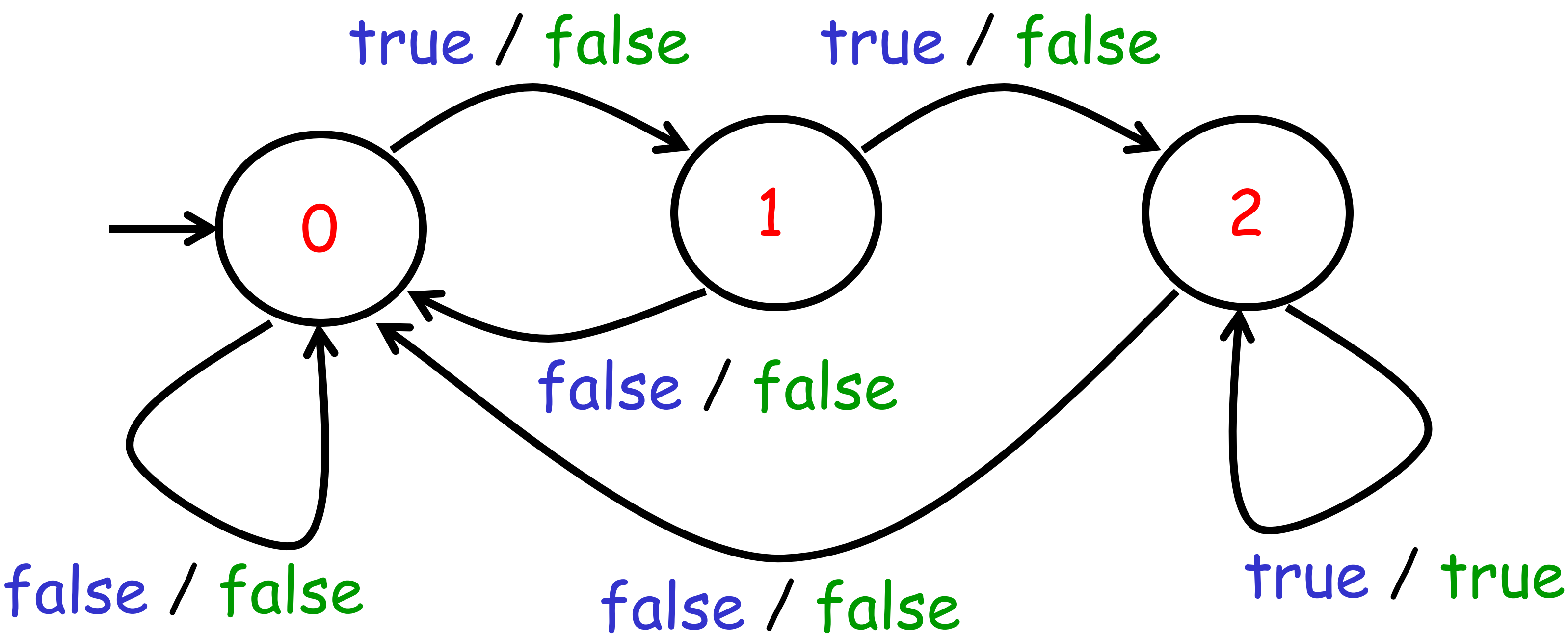
output [LastThree] (q,x) = $((q = 2) \wedge x)$

Parity System



States = Bools
Inputs = Bools
Outputs = Bools

LastThree System



States = { 0, 1, 2 }
Inputs = Bools
Outputs = Bools

A Run of the LastThree System

Time	0	1	2	3	4
Input	t	t	t	t	f
Output					
State					

A Run of the LastThree System

Time		0	1	2	3	4
Input		t	t	t	t	f
Output						
State	0					

A Run of the LastThree System

Time		0		1		2		3		4
Input		t		t		t		t		f
Output		f								
State	0		1							

A Run of the LastThree System

Time		0		1		2		3		4
Input		t		t		t		t		f
Output		f		f						
State	0		1		2					

A Run of the LastThree System

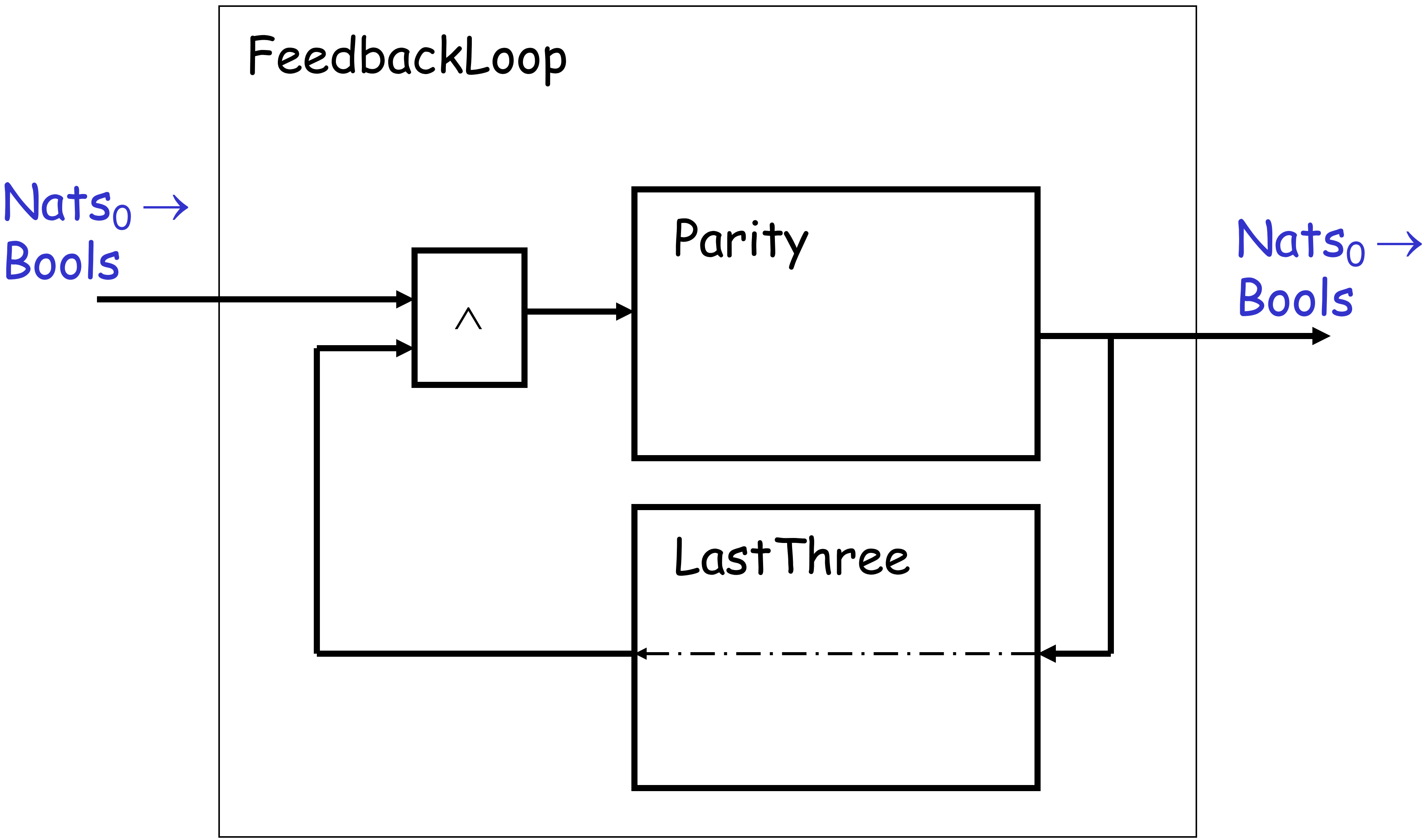
Time		0		1		2		3	4
Input		t		t		t		t	f
Output		f		f		t			
State	0		1		2		2		

A Run of the LastThree System

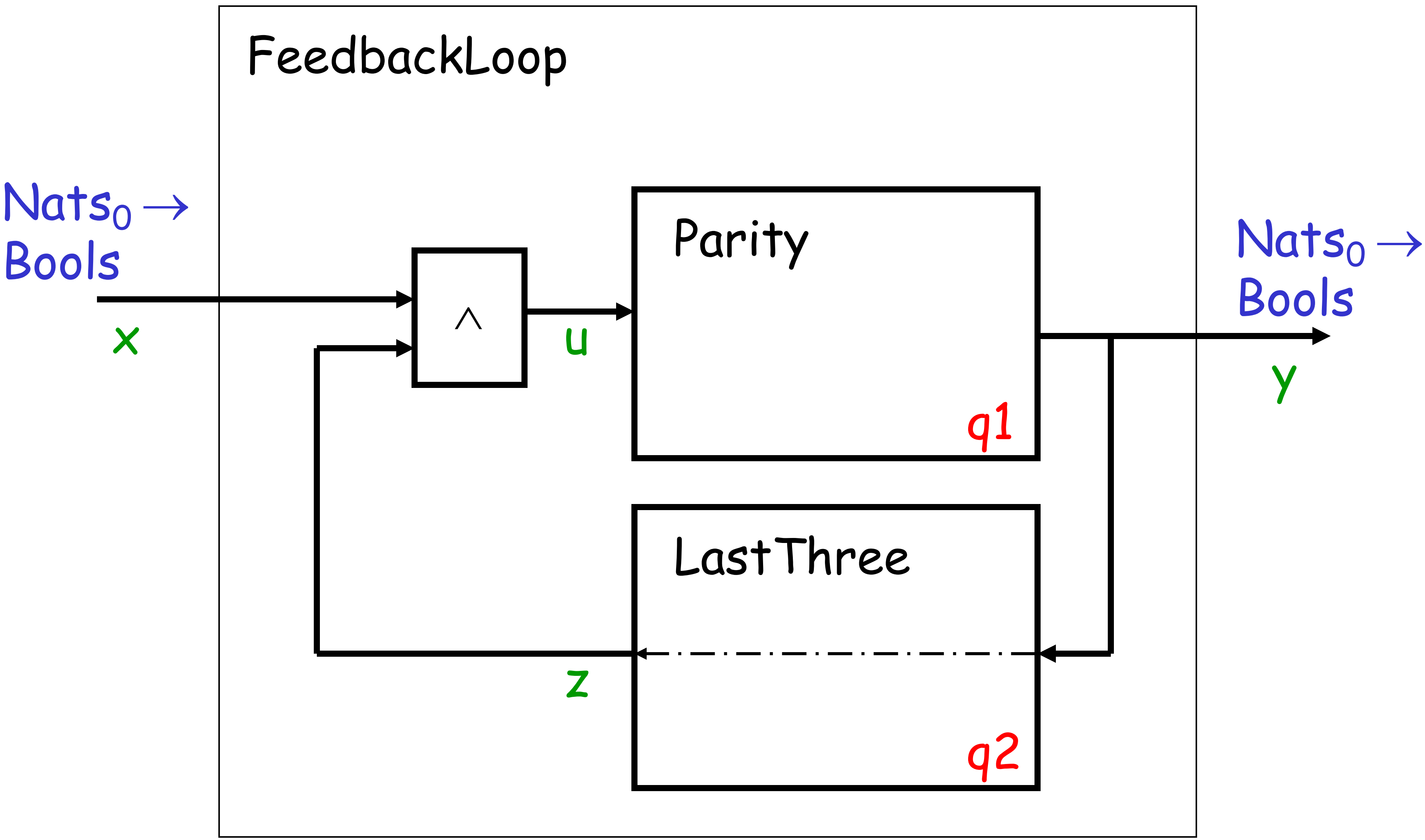
Time		0		1		2		3		4
Input		t		t		t		t		f
Output		f		f		t		t		
State	0		1		2		2		2	

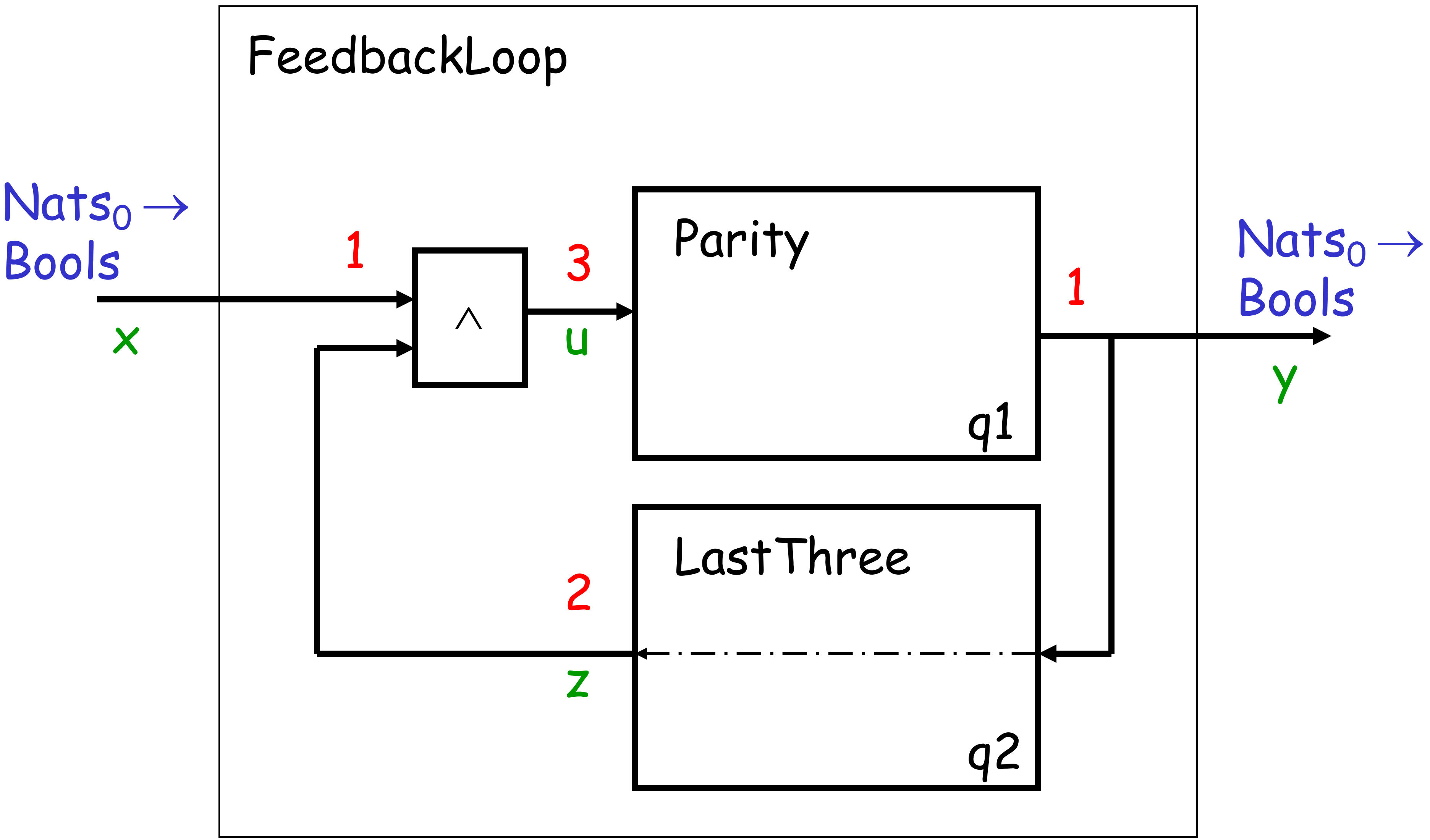
A Run of the LastThree System

Time		0		1		2		3		4	
Input		t		t		t		t		f	
Output		f		f		t		t		f	
State	0		1		2		2		2		0



This block diagram is ok, because every cycle contains a delay.


FeedbackLoop
$Nats_0 \to Bools$
x
∧
u
Parity
q1
$Nats_0 \to Bools$
y
LastThree
z
q2


FeedbackLoop
Nats0 → Bools
1
3
x
^
u
Parity
q1
1
Nats0 → Bools
y
LastThree
2
z
q2

A Run of the FeedbackLoop System

		0	1	2	3	4
	Time	0	1	2	3	4
	Input x	t	t	f	t	f
1	Output y					
2	Aux z					
3	Aux u					
	State q1					
	State q2					

A Run of the FeedbackLoop System

	Time		0	1	2	3	4
	Input x		t	t	f	t	f
1	Output y						
2	Aux z						
3	Aux u						
	State q1	t					
	State q2	0					

A Run of the FeedbackLoop System

	Time		0	1	2	3	4
	Input x		t	t	f	t	f
1	Output y		t				
2	Aux z						
3	Aux u						
	State q1	t					
	State q2	0					

A Run of the FeedbackLoop System

	Time		0	1	2	3	4
	Input x		t	t	f	t	f
1	Output y		t				
2	Aux z		f				
3	Aux u						
	State q1	t					
	State q2	0					

A Run of the FeedbackLoop System

	Time		0	1	2	3	4
	Input x		t	t	f	t	f
1	Output y		t				
2	Aux z		f				
3	Aux u		f				
	State q1	t					
	State q2	0					

A Run of the FeedbackLoop System

	Time		0		1	2	3	4
	Input x		t		t	f	t	f
1	Output y		t					
2	Aux z		f					
3	Aux u		f					
	State q1	t		t				
	State q2	0		1				

A Run of the FeedbackLoop System

	Time		0		1	2	3	4
	Input x		t		t	f	t	f
1	Output y		t		t			
2	Aux z		f					
3	Aux u		f					
	State q1	t		t				
	State q2	0		1				

A Run of the FeedbackLoop System

	Time		0		1	2	3	4
	Input x		t		t	f	t	f
1	Output y		t		t			
2	Aux z		f		f			
3	Aux u		f					
	State q1	t		t				
	State q2	0		1				

A Run of the FeedbackLoop System

	Time		0		1	2	3	4
	Input x		t		t	f	t	f
1	Output y		t		t			
2	Aux z		f		f			
3	Aux u		f		f			
	State q1	t		t				
	State q2	0		1				

A Run of the FeedbackLoop System

	Time		0		1		2	3	4
	Input x		t		t		f	t	f
1	Output y		t		t				
2	Aux z		f		f				
3	Aux u		f		f				
	State q1	t		t		t			
	State q2	0		1		2			

A Run of the FeedbackLoop System

Time		0		1		2		3		4	
Input x		t		t		f		t		f	
Output y		t		t		t		t		f	
Aux z		f		f		t		t		f	
Aux u		f		f		f		t		f	
State q1	t		t		t		t		f		f
State q2	0		1		2		2		2		0

The FeedbackLoop System

Inputs [FeedbackLoop] = Bools

Outputs [FeedbackLoop] = Bools

States [FeedbackLoop]

= States [Parity] $\times$ States [LastThree]

= { true, false } $\times$ { 0, 1, 2 }

initialState [FeedbackLoop]

= (initialState [Parity], initialState [LastThree])

= (true, 0)

The FeedbackLoop System, continued

output [FeedbackLoop] ((q1, q2) , x)

= output [Parity] (q1, any)

The value of this does not matter !

The FeedbackLoop System, continued

output [FeedbackLoop] ((q1, q2) , x)

= output [Parity] (q1, any)

Let this be y .

nextState [FeedbackLoop] ((q1, q2) , x)$_2$

= nextState [LastThree] (q2, y)

The FeedbackLoop System, continued

output [FeedbackLoop] ((q1, q2) , x)

= output [Parity] (q1, any)

Let this be y .

nextState [FeedbackLoop] ((q1, q2) , x) $_2$

= nextState [LastThree] (q2, y)

Let z = output [LastThree] (q2, y) .

The FeedbackLoop System, continued

output [FeedbackLoop] ((q1, q2) , x)

= output [Parity] (q1, any)

Let this be y .

nextState [FeedbackLoop] ((q1, q2) , x) $_2$

= nextState [LastThree] (q2, y)

Let z = output [LastThree] (q2, y) .

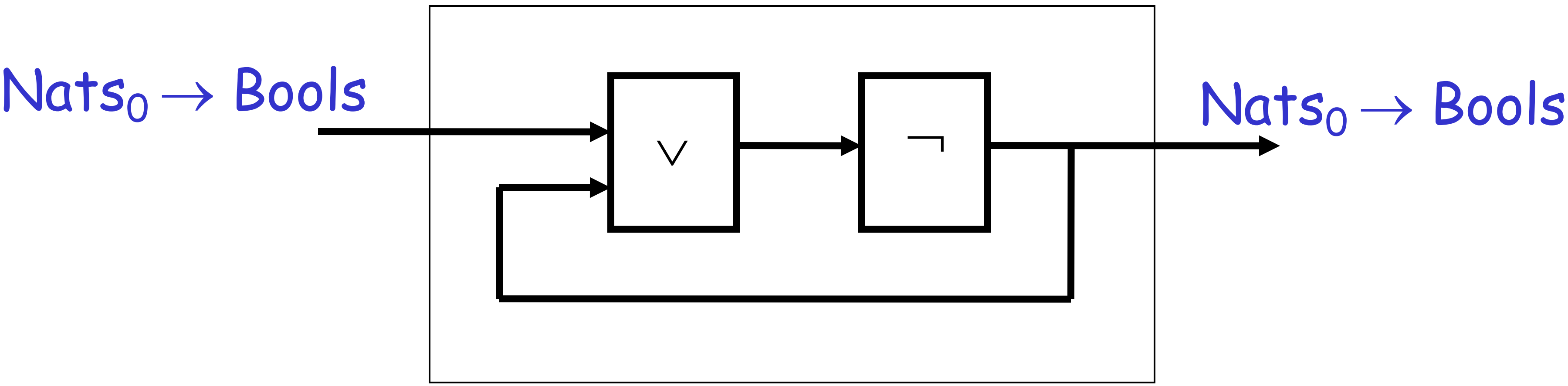
nextState [FeedbackLoop] ((q1, q2) , x) $_1$

= nextState [Parity] (q1, x $\wedge$ z)

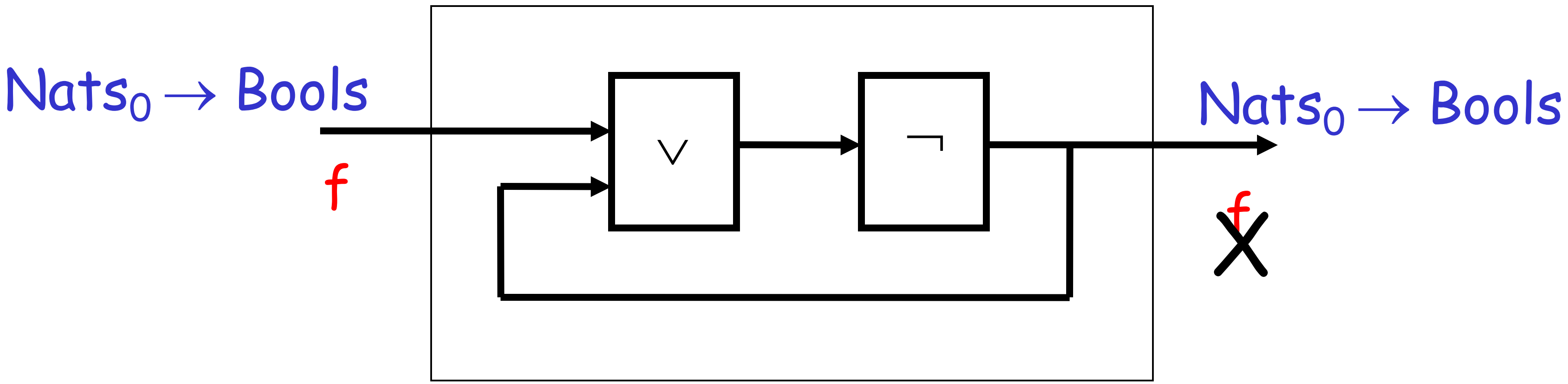
If every cycle contains a delay, then all values can be computed.

If there is a cycle without delay, then some values may not be well-defined.

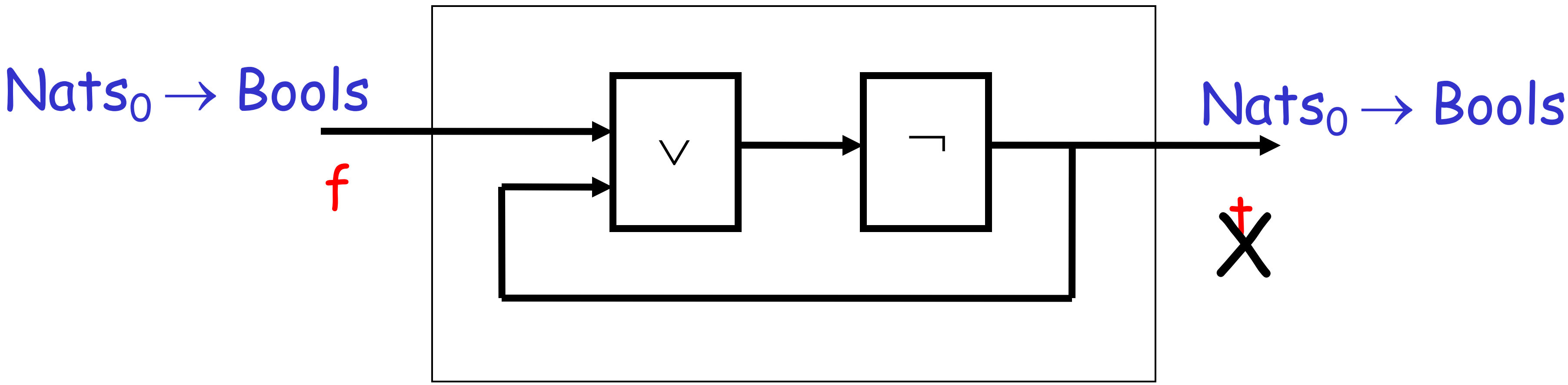
An Ill-Defined System



An Ill-Defined System

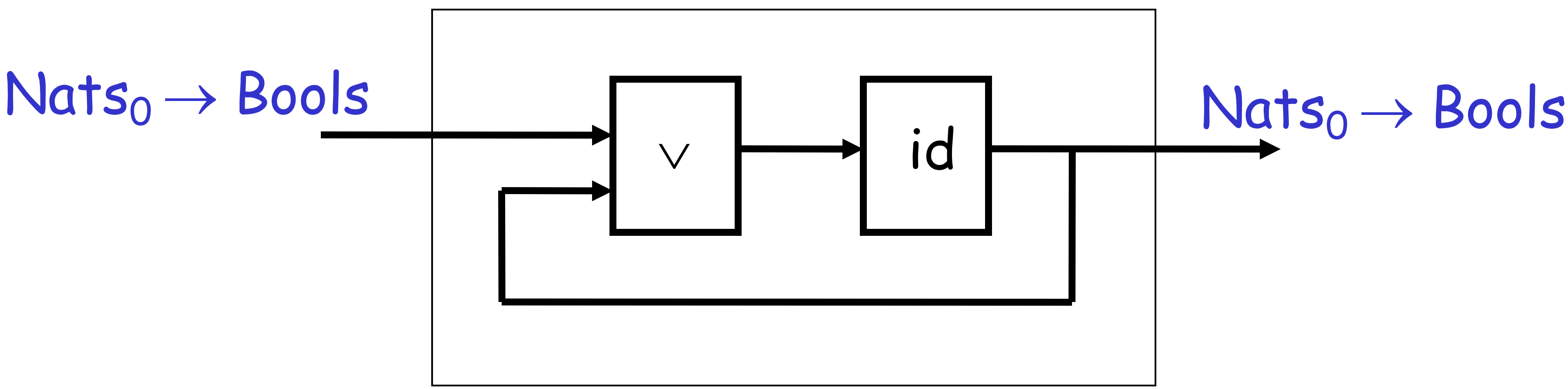


An Ill-Defined System

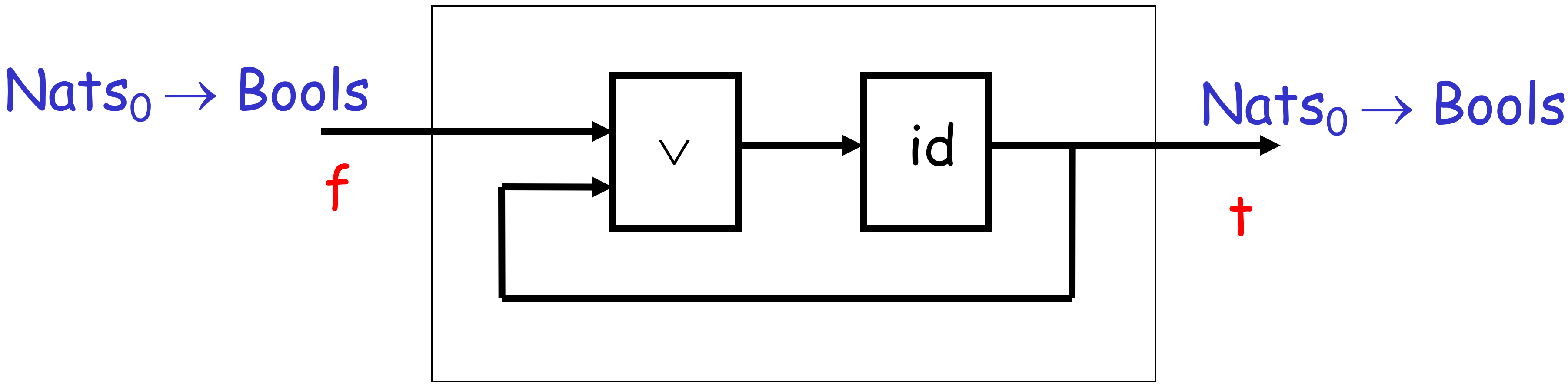


No legal output !

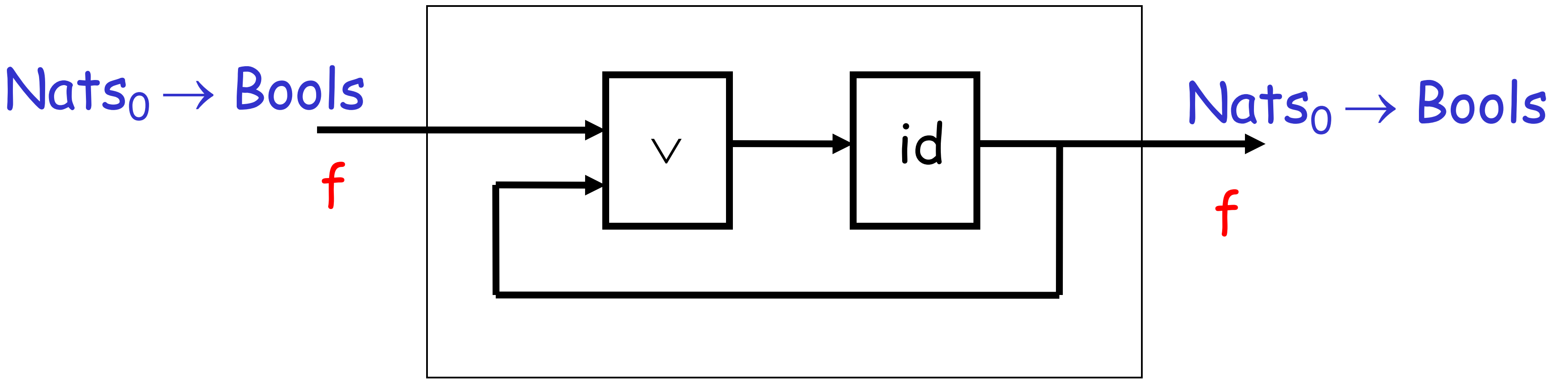
Another Ill-Defined System



Another Ill-Defined System



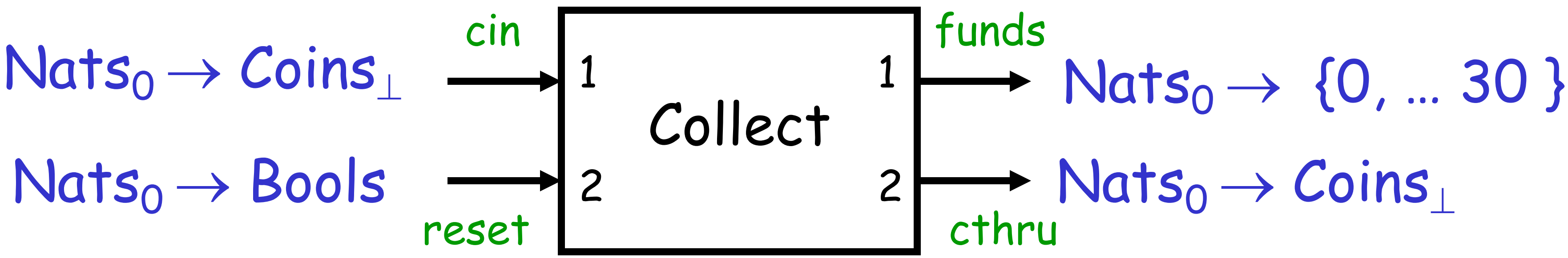
Another Ill-Defined System



Multiple legal outputs !

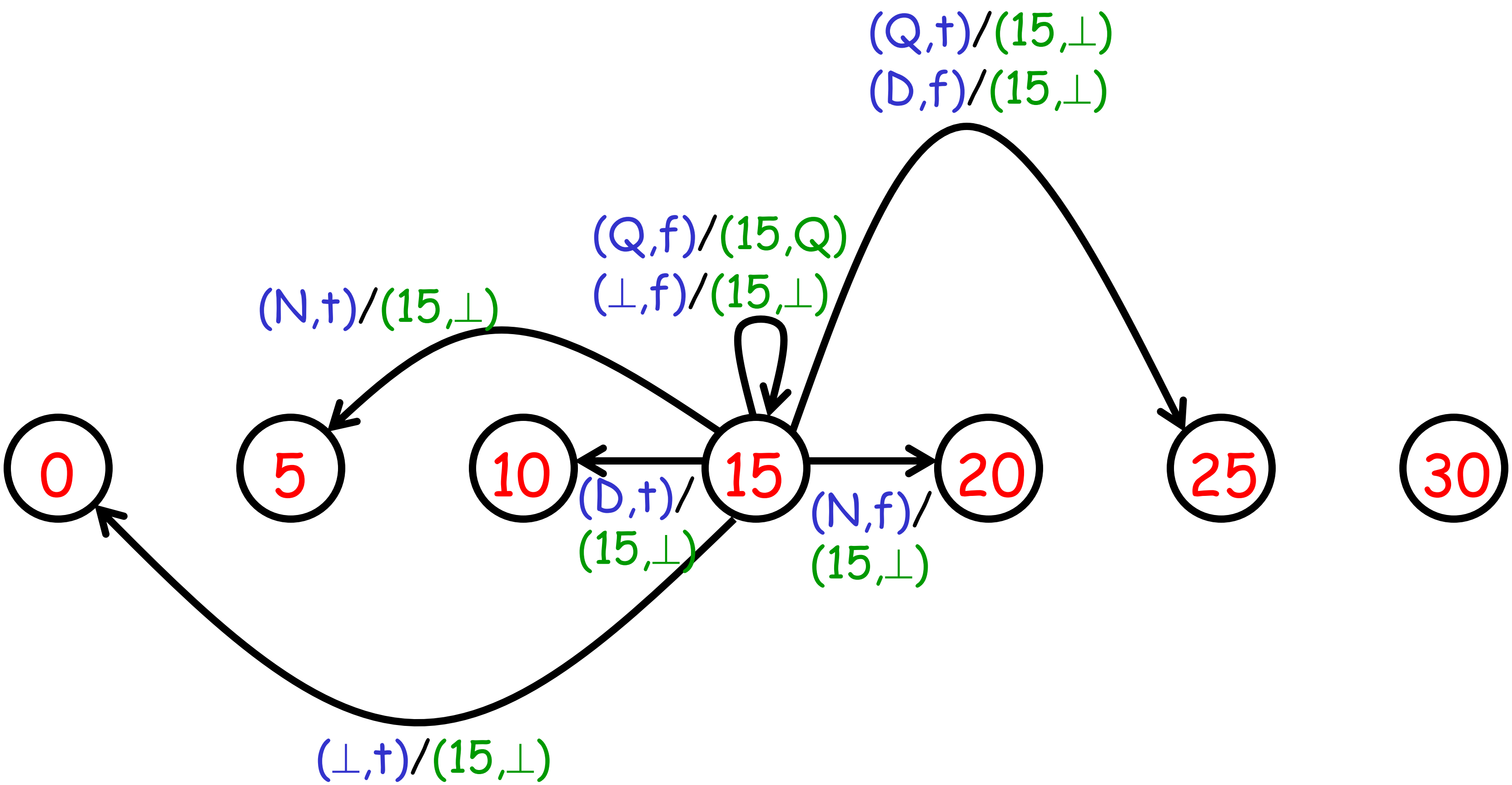
Example: Vending Machine

Coin Collector

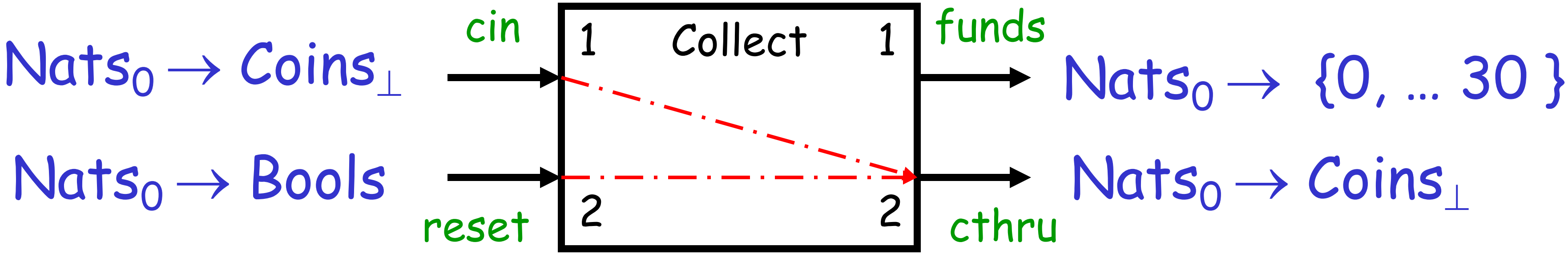


Let Coins = { Nickel, Dime, Quarter } .

Coin Collector



Coin Collector

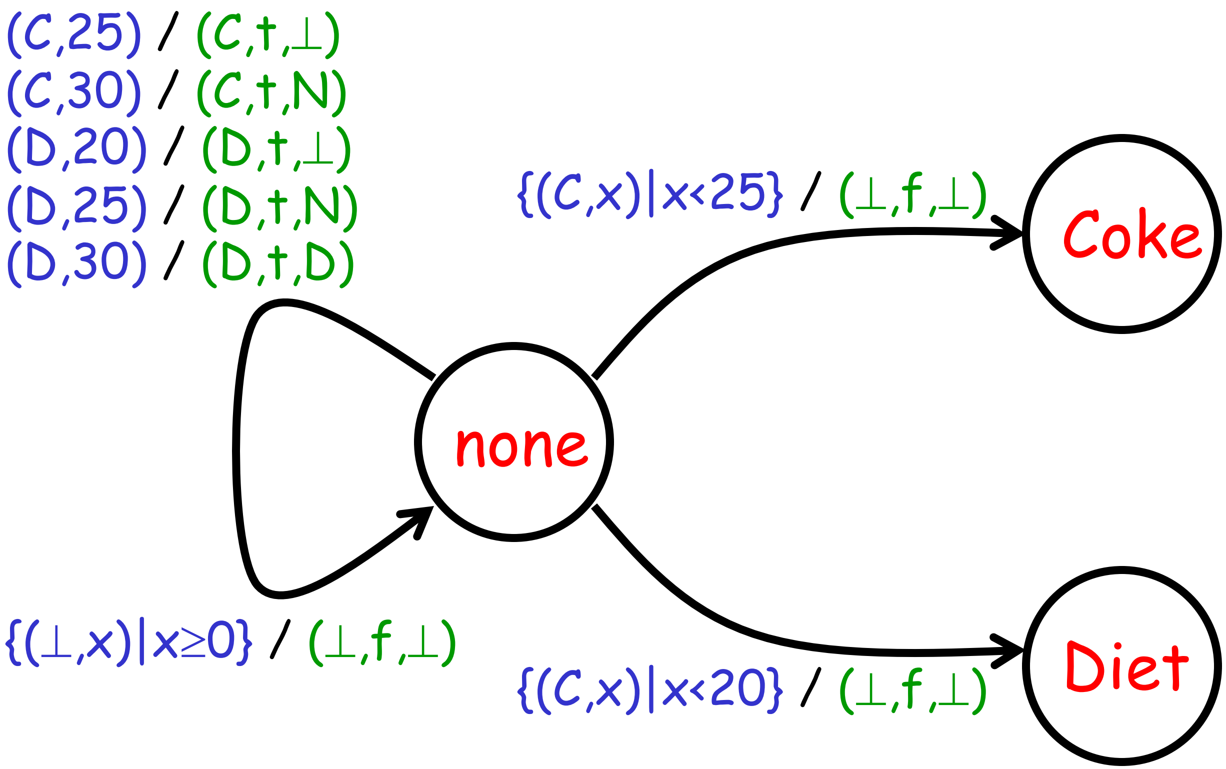


Soda Dispenser

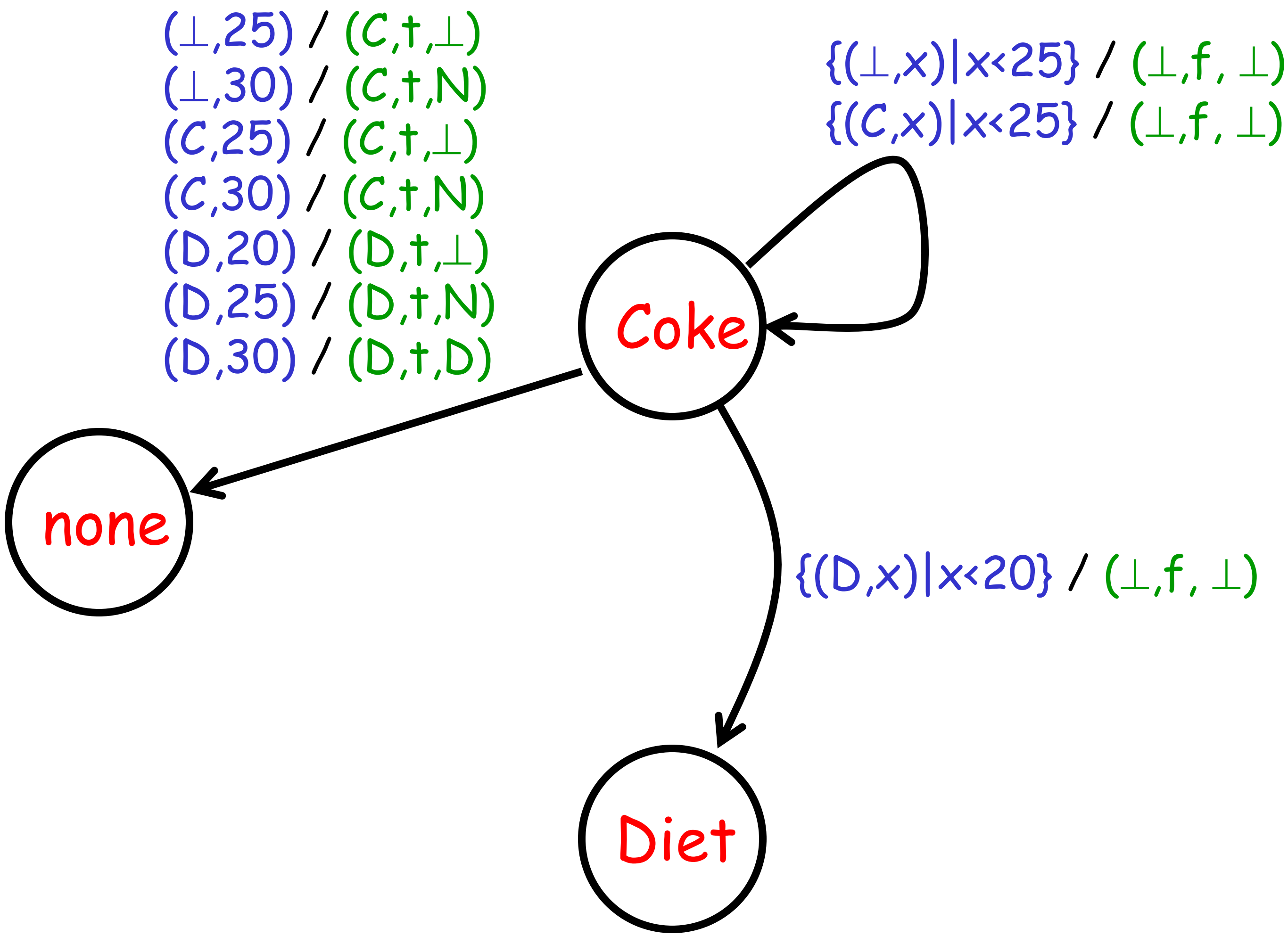
Let $Select = \{ selectCoke, selectDiet \}$.

Let $Dispense = \{ dispenseCoke, dispenseDiet \}$.

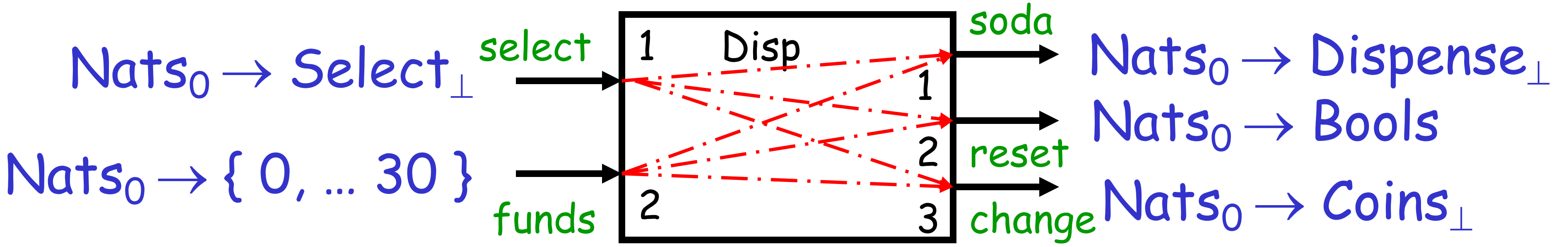
Soda Dispenser



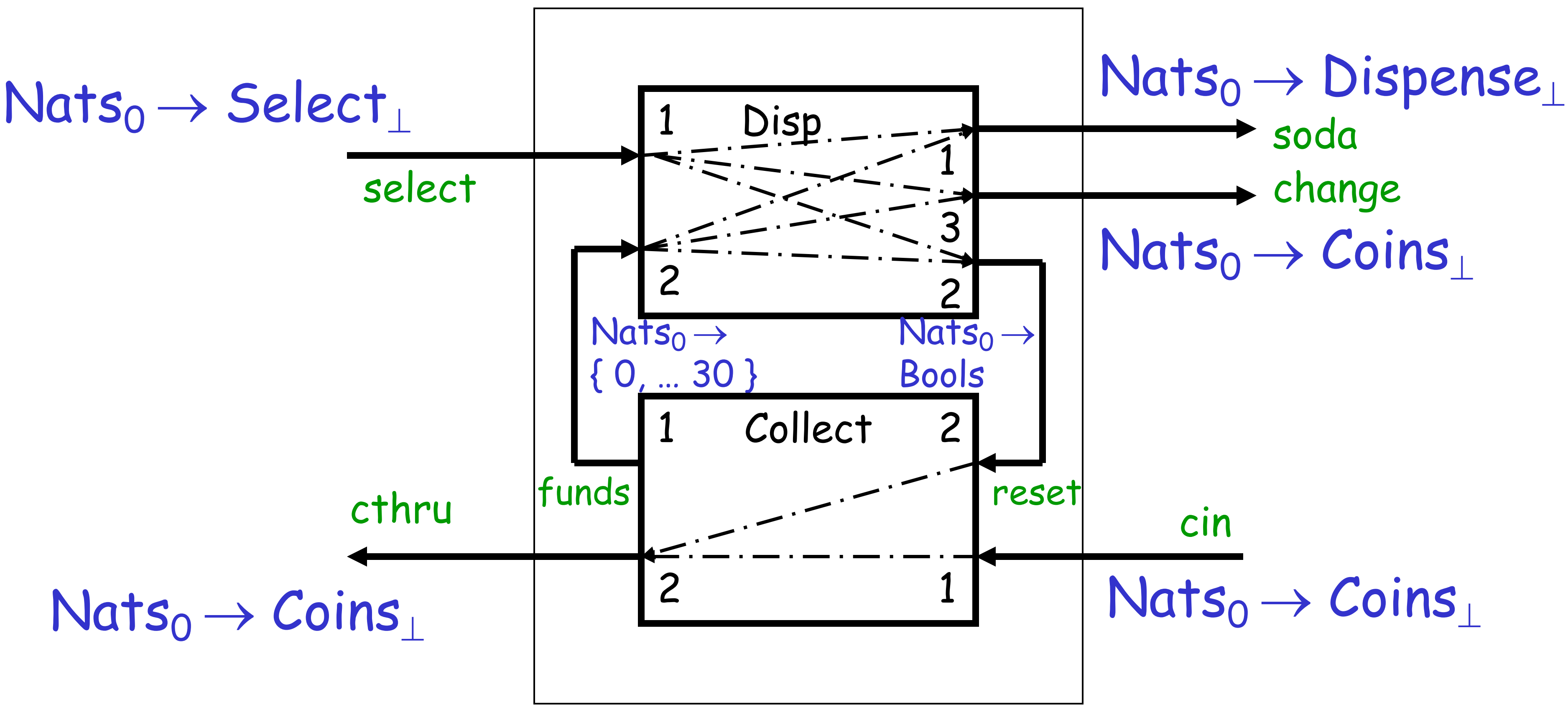
Soda Dispenser



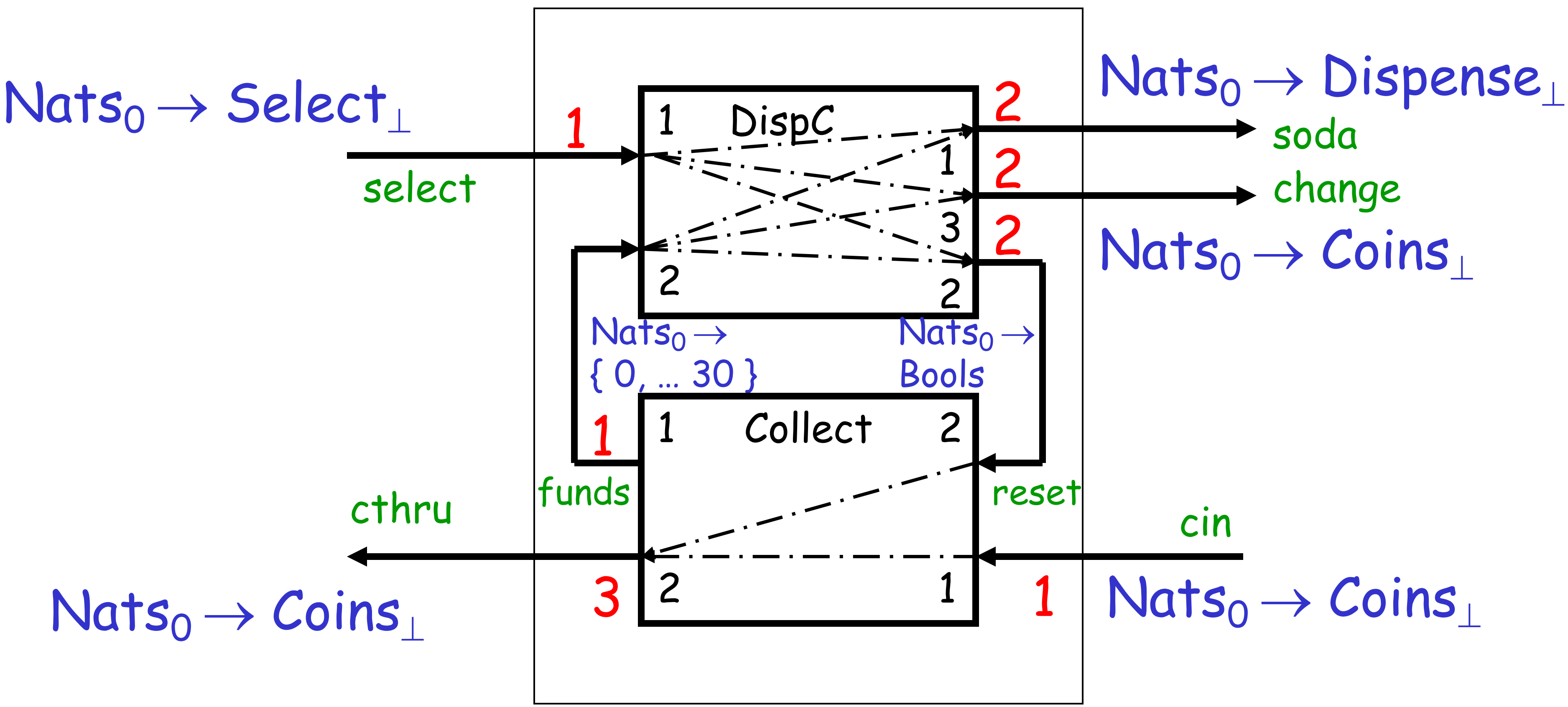
Soda Dispenser



Vending Machine



Vending Machine



	Time		0	1	2	3	4
	cin		N	$\perp$	Q	Q	D
	select		$\perp$	C	$\perp$	D	C
1	funds						
2	soda						
2	change						
2	reset						
3	cthru						
	State Coll	0					
	State Disp	n					

Time	0	1	2	3	4
cin	N	$\perp$	Q	Q	D
select	$\perp$	C	$\perp$	D	C
funds	0	5	5	0	0
soda	$\perp$	$\perp$	C	D	$\perp$
change	$\perp$	$\perp$	N	N	$\perp$
reset	f	f	t	t	f
cthru	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$

State Coll	0	5	5	0	0	10
State Disp	n	n	C	n	n	C

Quiz

1. Draw the transition diagram of the system

$$Delay_0 : [\, Nats_0 \to Bins \,] \to [\, Nats_0 \to Bins \,]$$

$$\forall\, x \in [\, Nats_0 \to Bins \,],\ \forall\, y \in Nats_0,$$

$$(\,Delay_0\,(x))\,(y) = \begin{cases} 0 & \text{if } y = 0 \\ x\,(y-1) & \text{if } y > 0 \end{cases}$$

2. Draw the transition diagram of the system