Product Machines

EECS 20

Lecture 10 (February 7, 2001)

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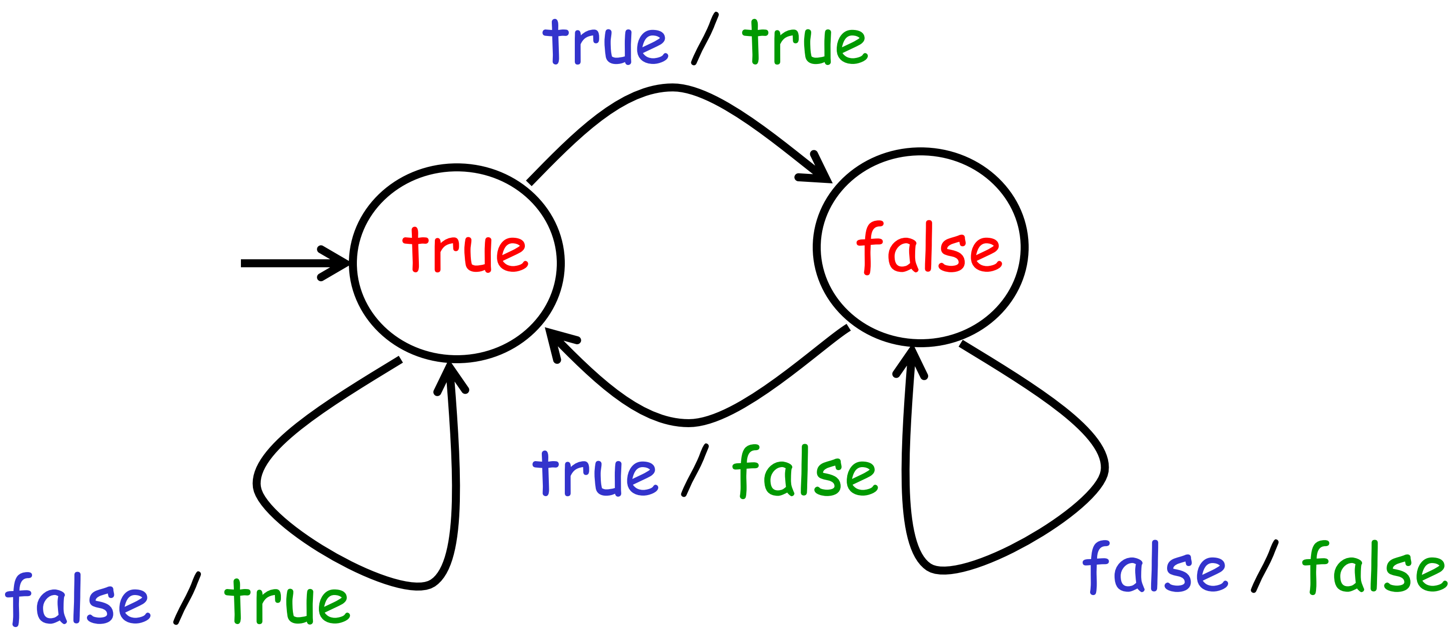
Revised by Matteo Zavatteri and Tiziano Villa, Fall 2020

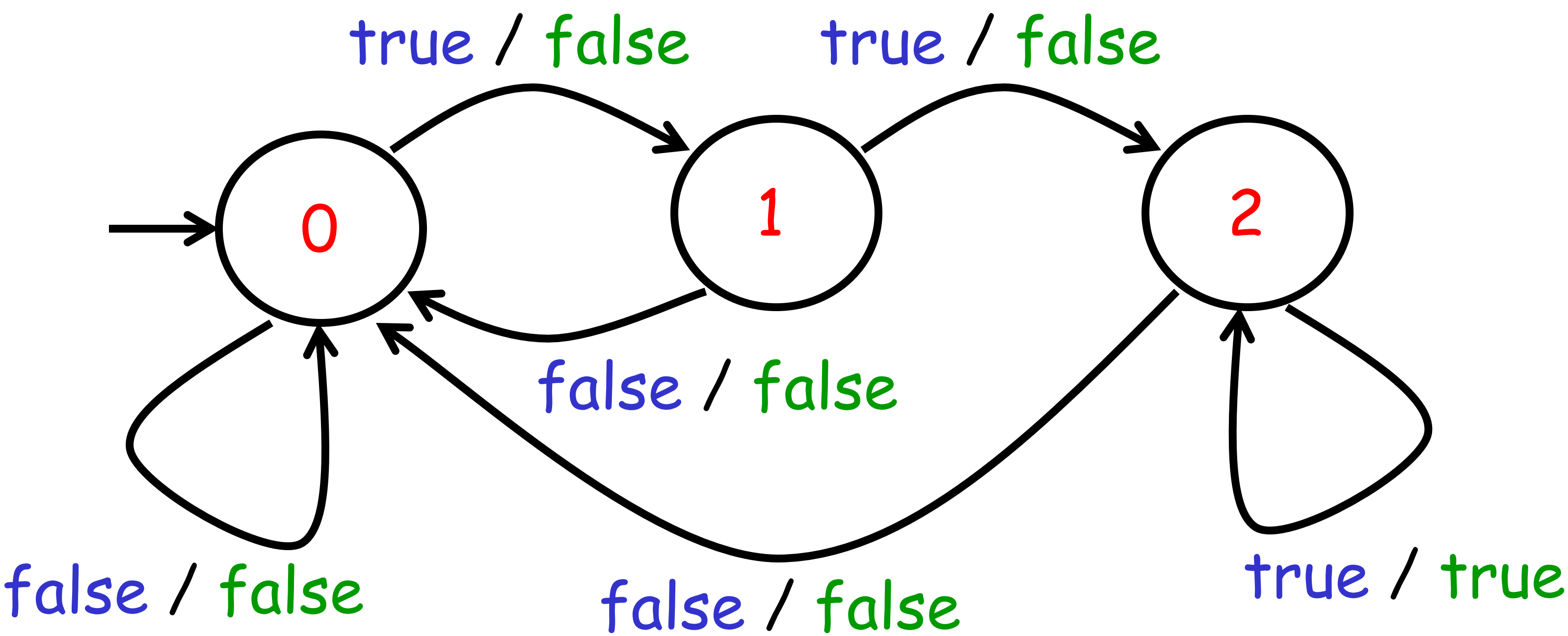
Composition of State Machines

Transition Diagram of the Parity System



States = Bools
Inputs = Bools
Outputs = Bools

Transition Diagram of the LastThree System



States = { 0, 1, 2 }
Inputs = Bools
Outputs = Bools

The Parity System :

$$\text{States}[\text{Parity}] = \{\text{true}, \text{false}\}$$

$$\text{initialState}[\text{Parity}] = \text{true}$$

$$\text{nextState}[\text{Parity}](q,x) = (q \neq x)$$

$$\text{output}[\text{Parity}](q,x) = q$$

The LastThree System :

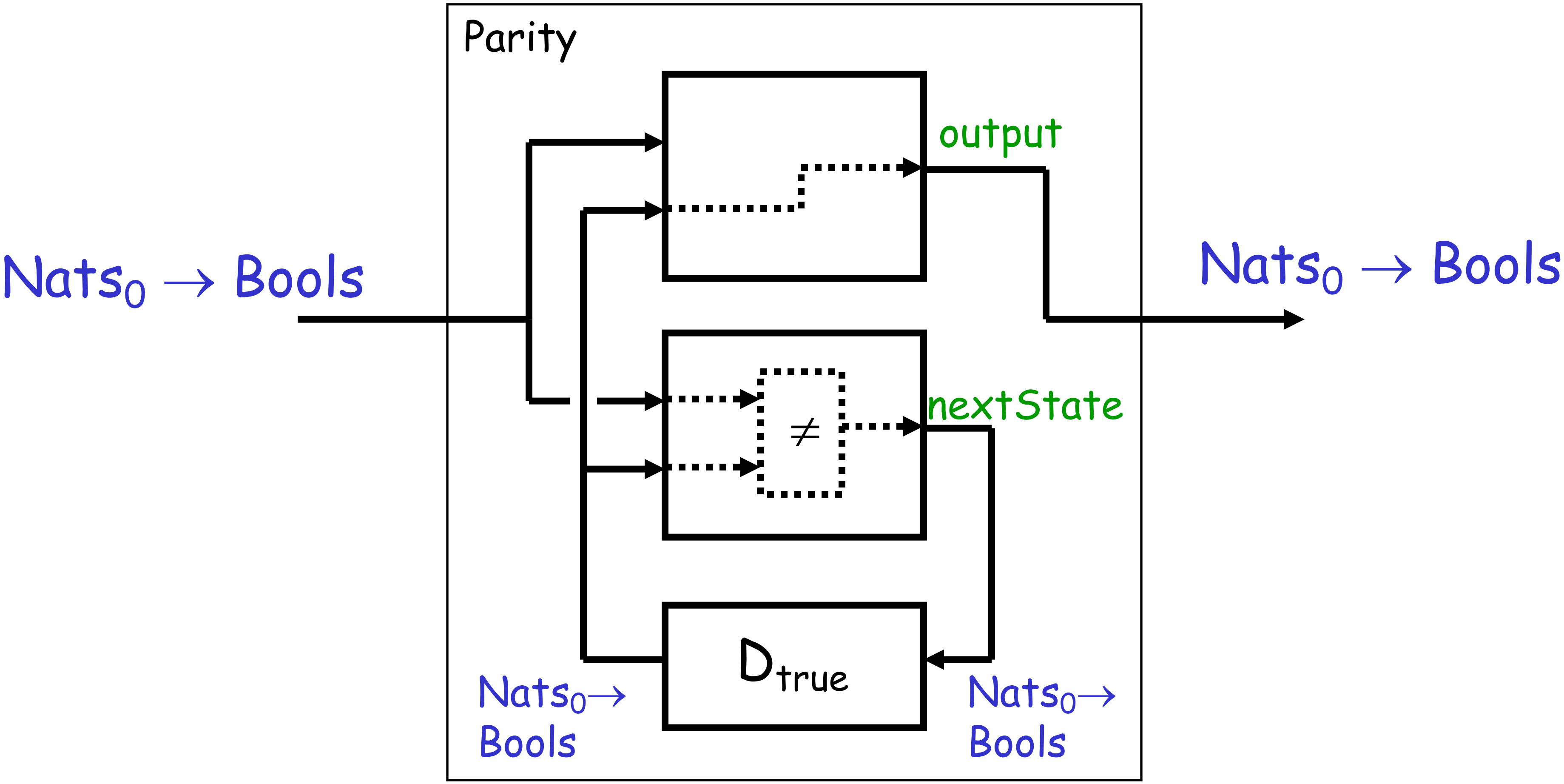
$$\text{States}[\text{LastThree}] = \{0, 1, 2\}$$

$$\text{initialState}[\text{LastThree}] = 0$$

$$\text{nextState}[\text{LastThree}](q,x) = \begin{cases} 0 & \text{if } \neg x \\ \min(q+1, 2) & \text{if } x \end{cases}$$

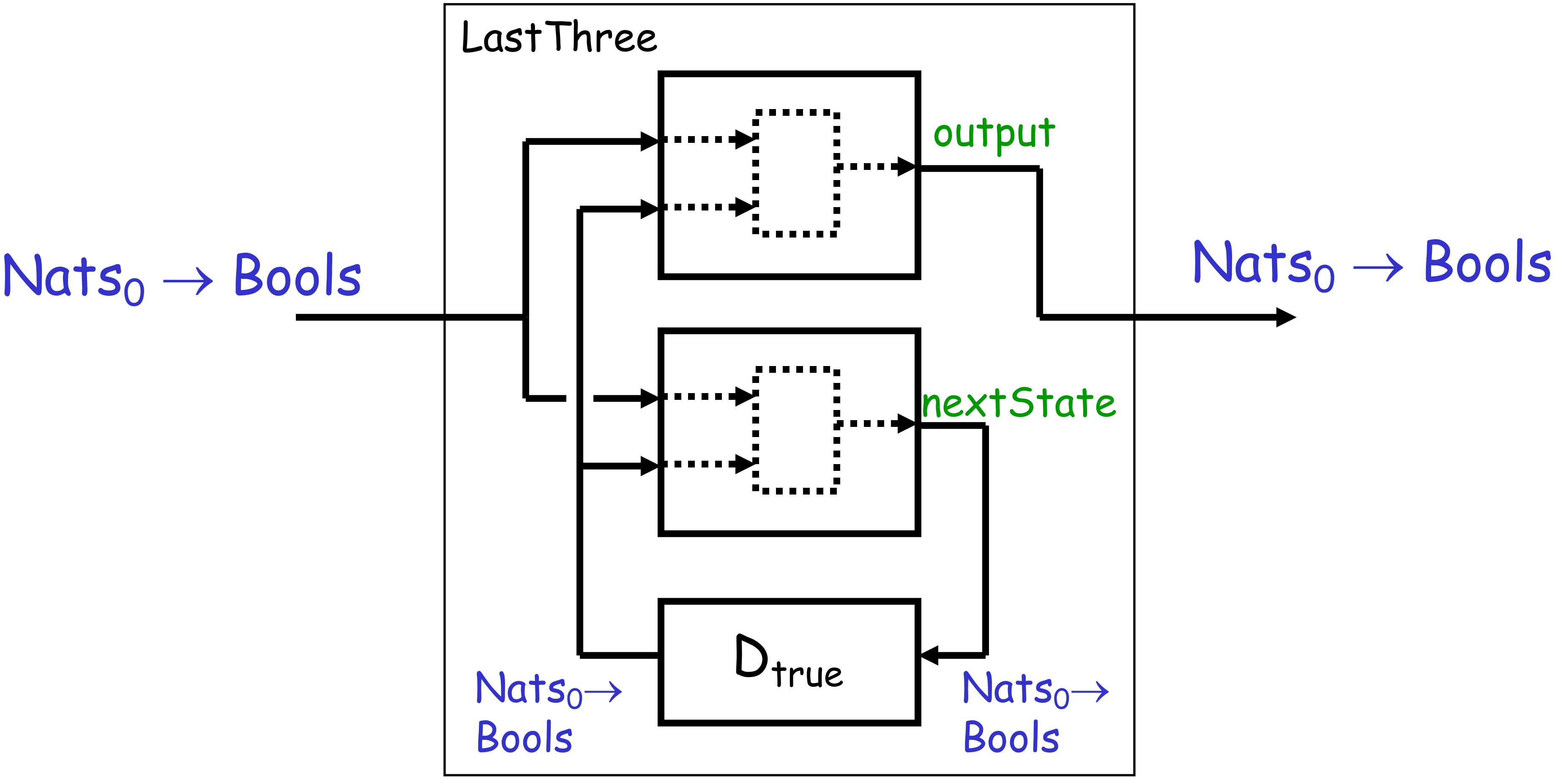
$$\text{output}[\text{LastThree}](q,x) = ((q = 2) \wedge x)$$

Block Diagram of Parity System

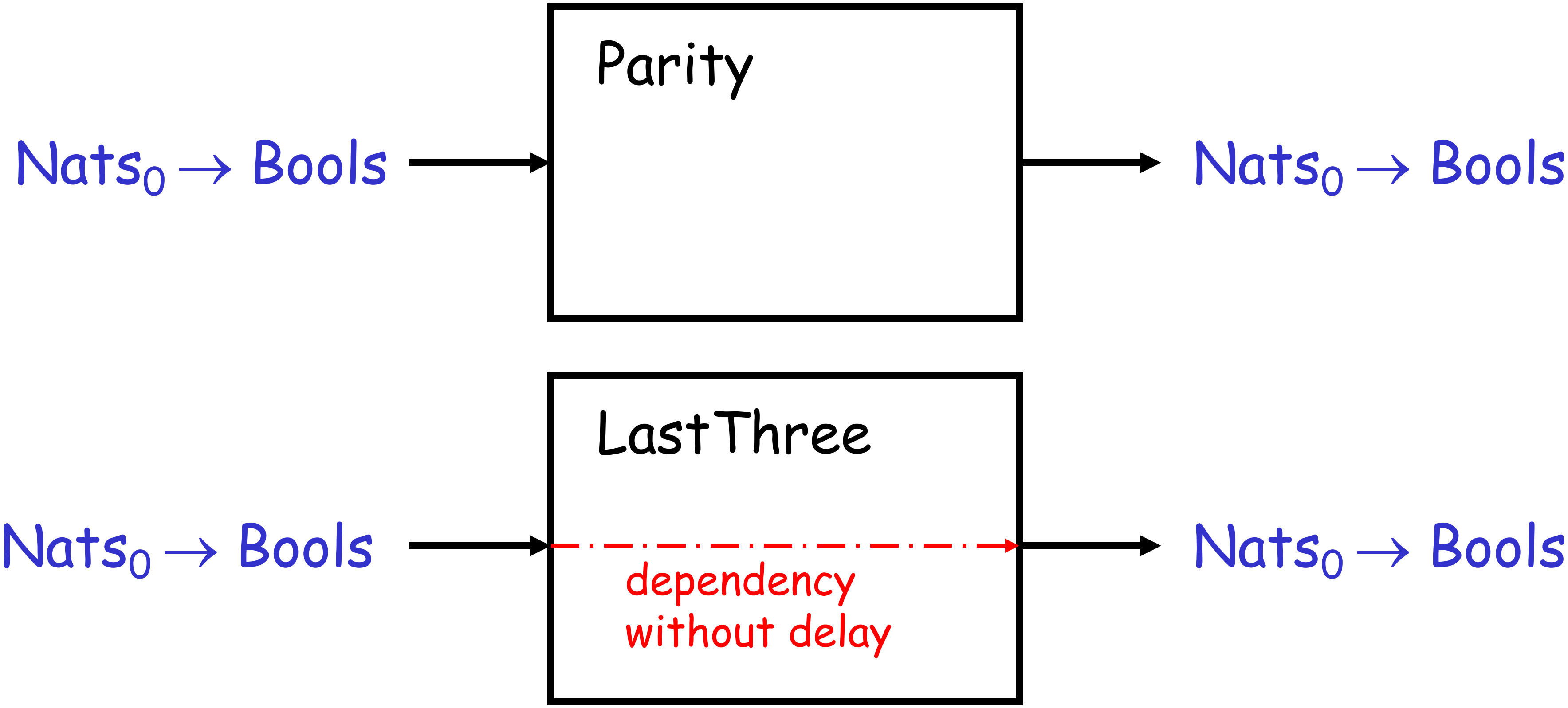


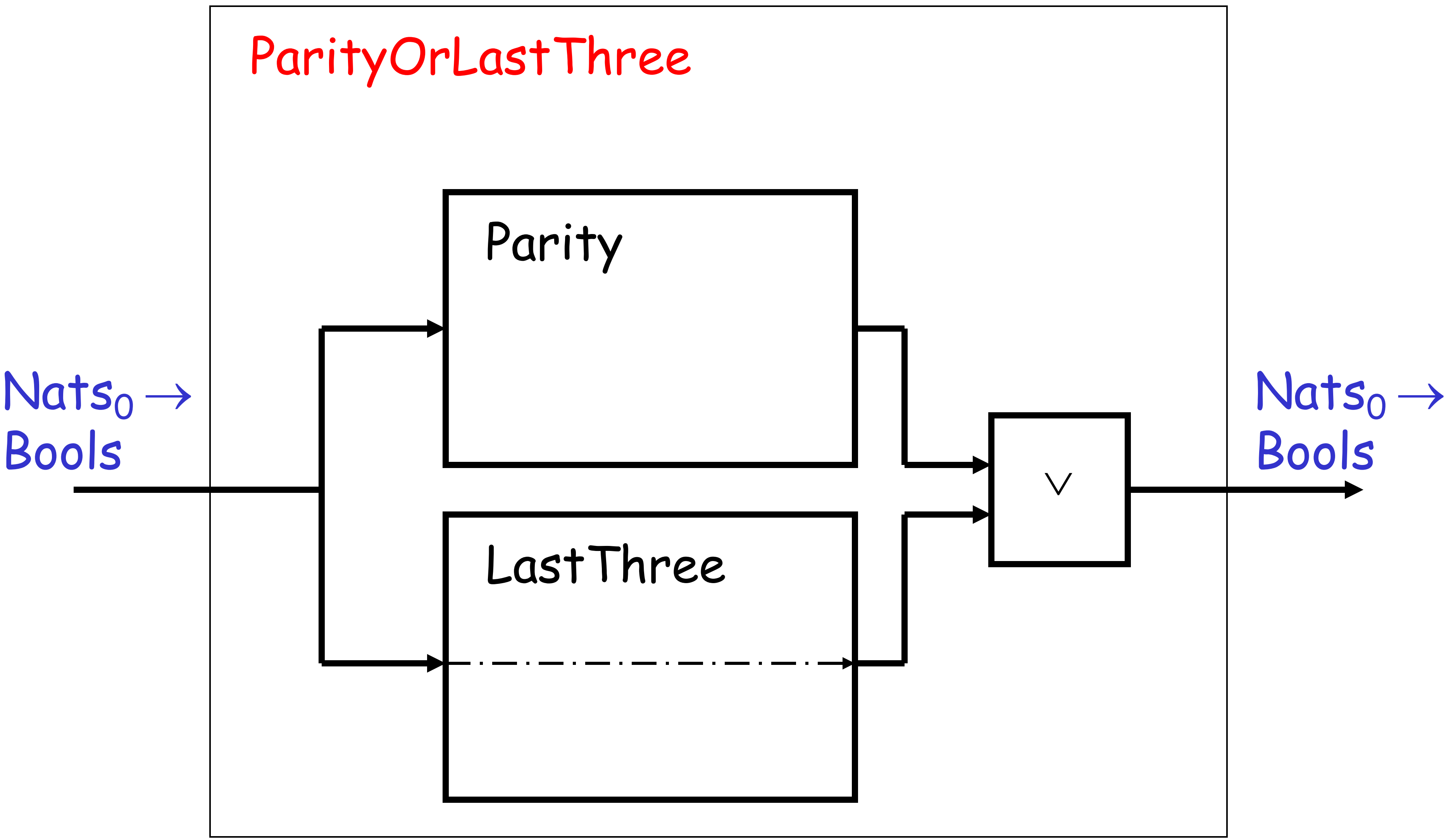
No path without delay from input to output.

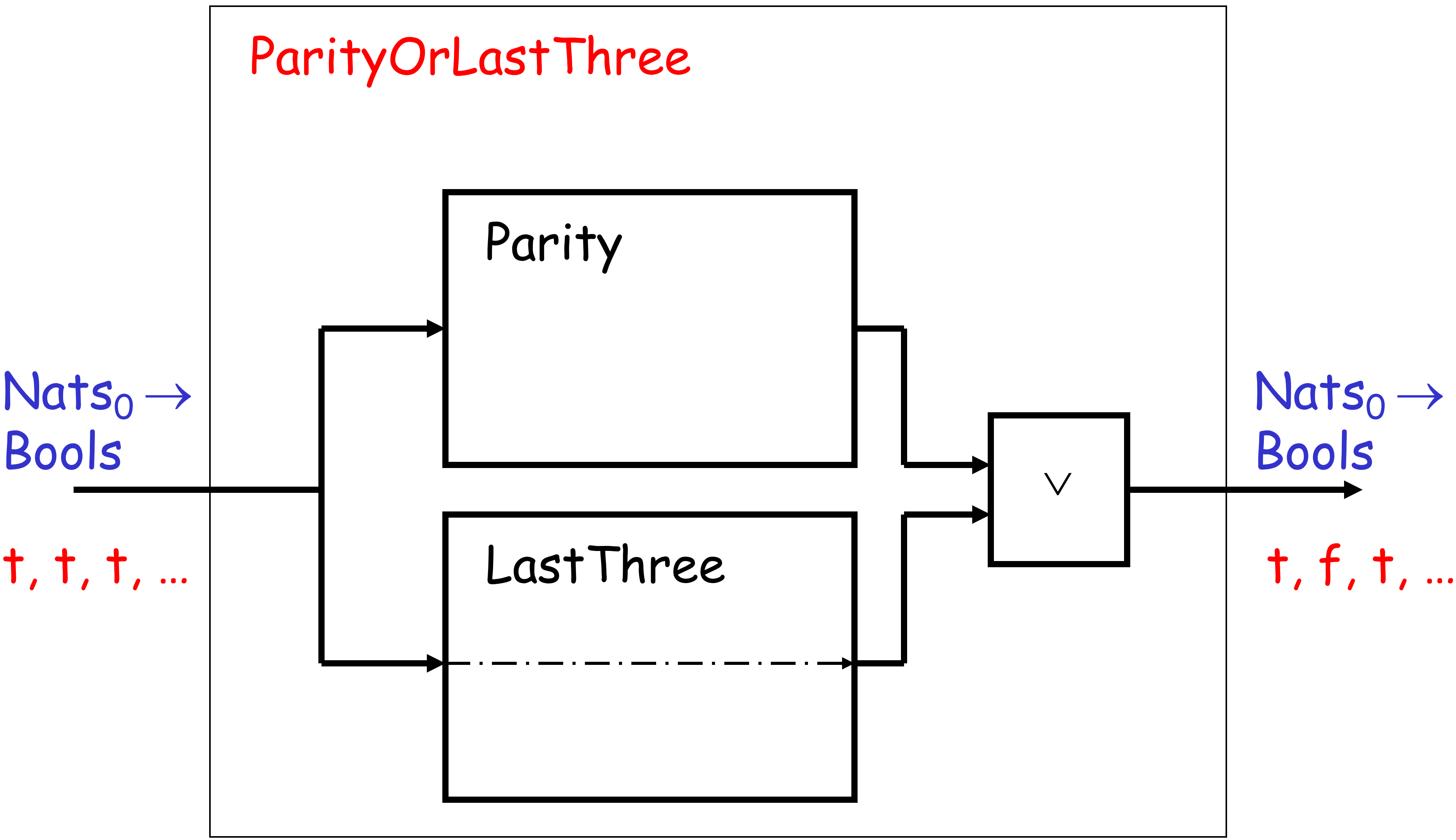
Block Diagram of LastThree System



Path without delay from input to output !


Parity
$Nats_0 \to Bools$
$Nats_0 \to Bools$
LastThree
$Nats_0 \to Bools$
$Nats_0 \to Bools$
dependency
without delay


ParityOrLastThree
Parity
LastThree
∨
$Nats_0 \to Bools$
$Nats_0 \to Bools$


ParityOrLastThree
Parity
LastThree
∨
Nats0 → Bools
t, t, t, ...
Nats0 → Bools
t, f, t, ...

The ParityOrLastThree System

Inputs [ParityOrLastThree] = Bools

Outputs [ParityOrLastThree] = Bools

States [ParityOrLastThree]

$= \text{States [Parity]} \times \text{States [LastThree]}$

$= \{ \text{true}, \text{false} \} \times \{ 0, 1, 2 \}$

initialState [ParityOrLastThree]

= (initialState [Parity], initialState [LastThree])

= (true, 0)

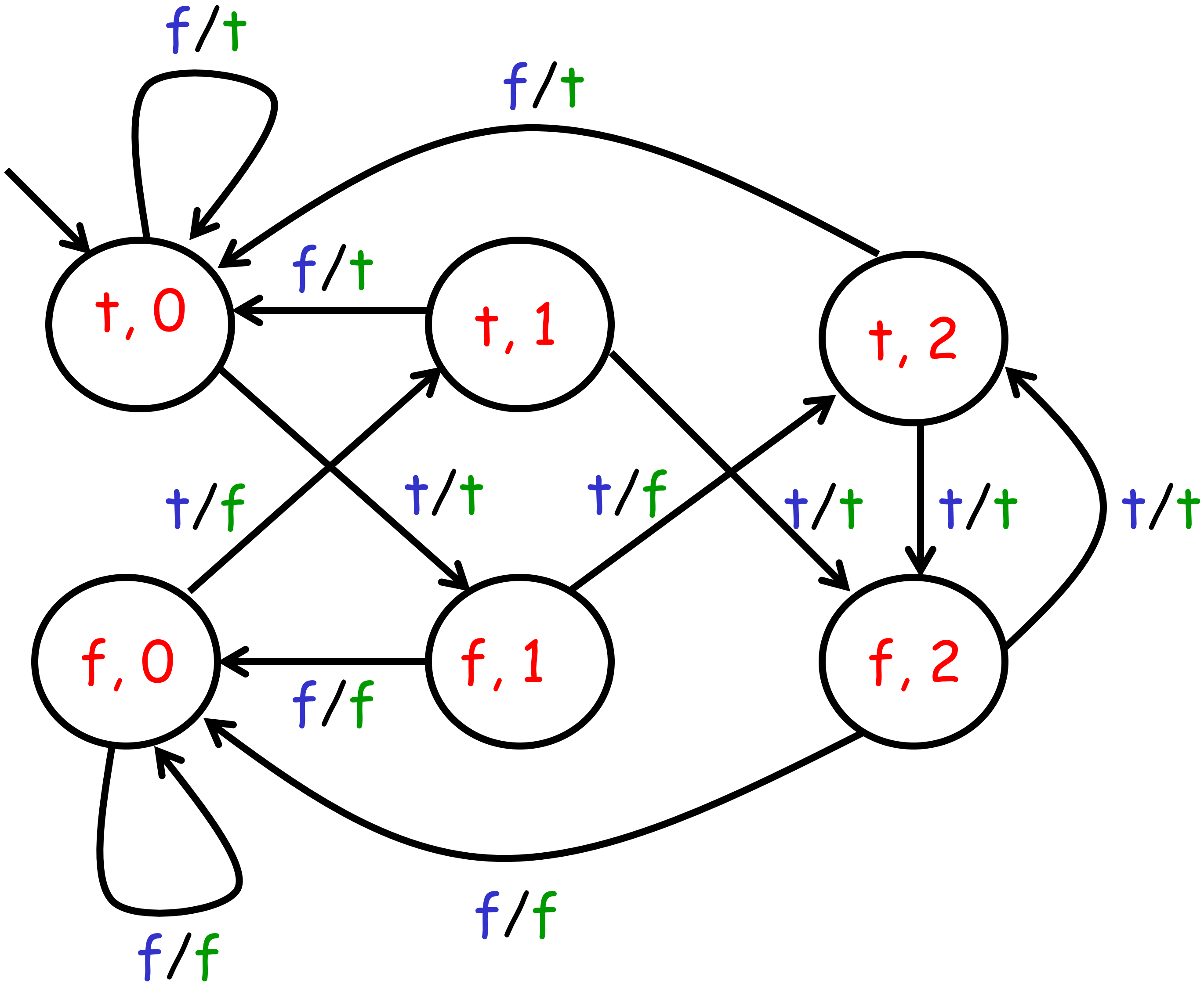
The ParityOrLastThree System, continued

nextState [ParityOrLastThree] ((q1, q2) , x)

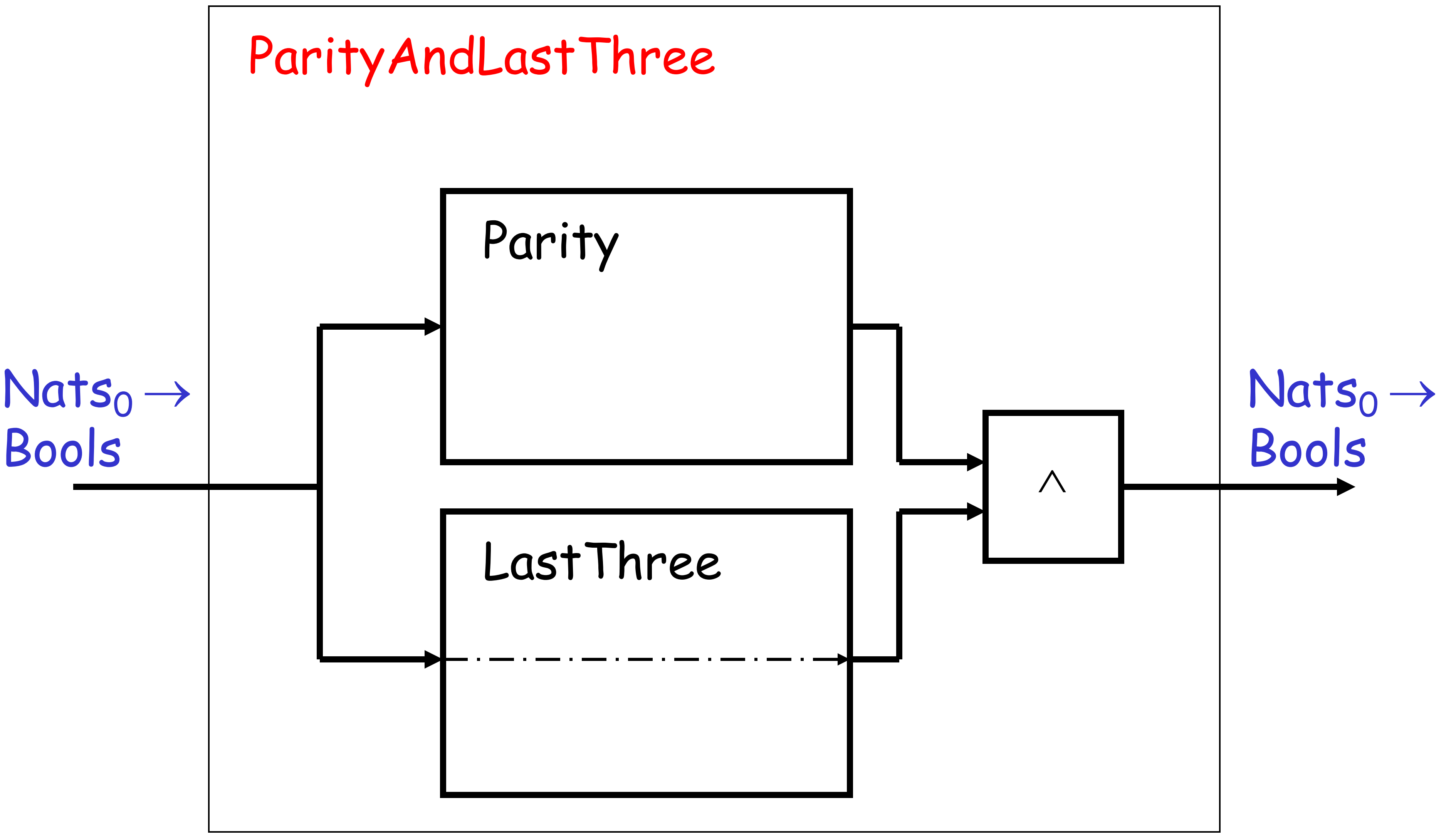
= (nextState [Parity] (q1, x) , nextState [LastThree] (q2, x))

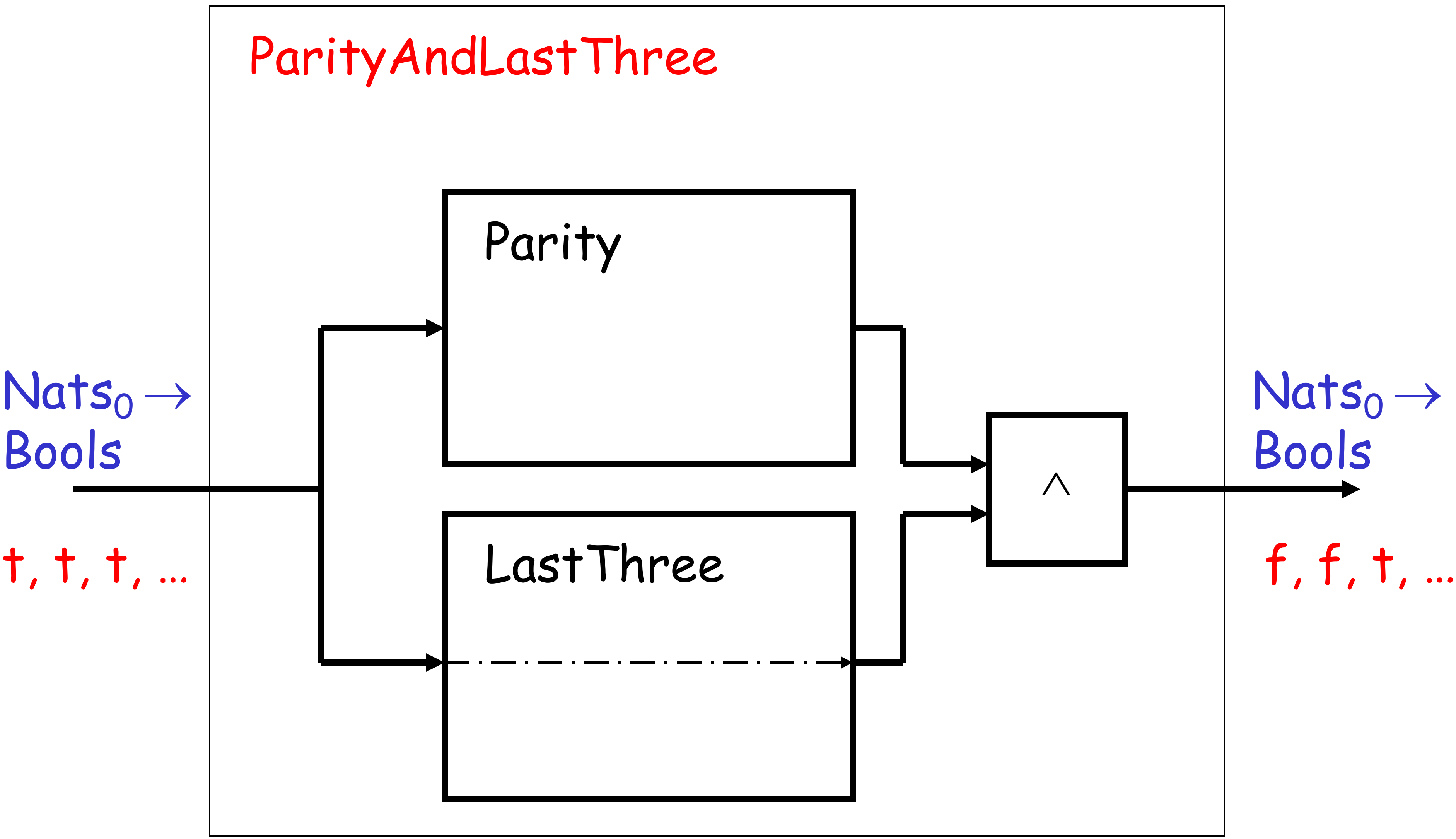
output [ParityOrLastThree] ((q1, q2) , x)

= output [Parity] (q1, x) $\vee$ output [LastThree] (q2, x)



t, 0
t, 1
t, 2
f, 0
f, 1
f, 2
f/t
f/t
f/t
t/f
t/t
t/f
t/t
t/t
t/t
f/f
f/f
f/f


ParityAndLastThree
Parity
LastThree
∧
Nats$_0$ →
Bools
Nats$_0$ →
Bools


ParityAndLastThree
Parity
LastThree
∧
$Nats_0 \to$ Bools
t, t, t, ...
$Nats_0 \to$ Bools
f, f, t, ...

The ParityAndLastThree System

Inputs [ParityAndLastThree] = Bools

Outputs [ParityAndLastThree] = Bools

States [ParityAndLastThree]

= States [Parity] $\times$ States [LastThree]

= { true, false } $\times$ { 0, 1, 2 }

initialState [ParityAndLastThree]

= (initialState [Parity], initialState [LastThree])

= (true, 0)

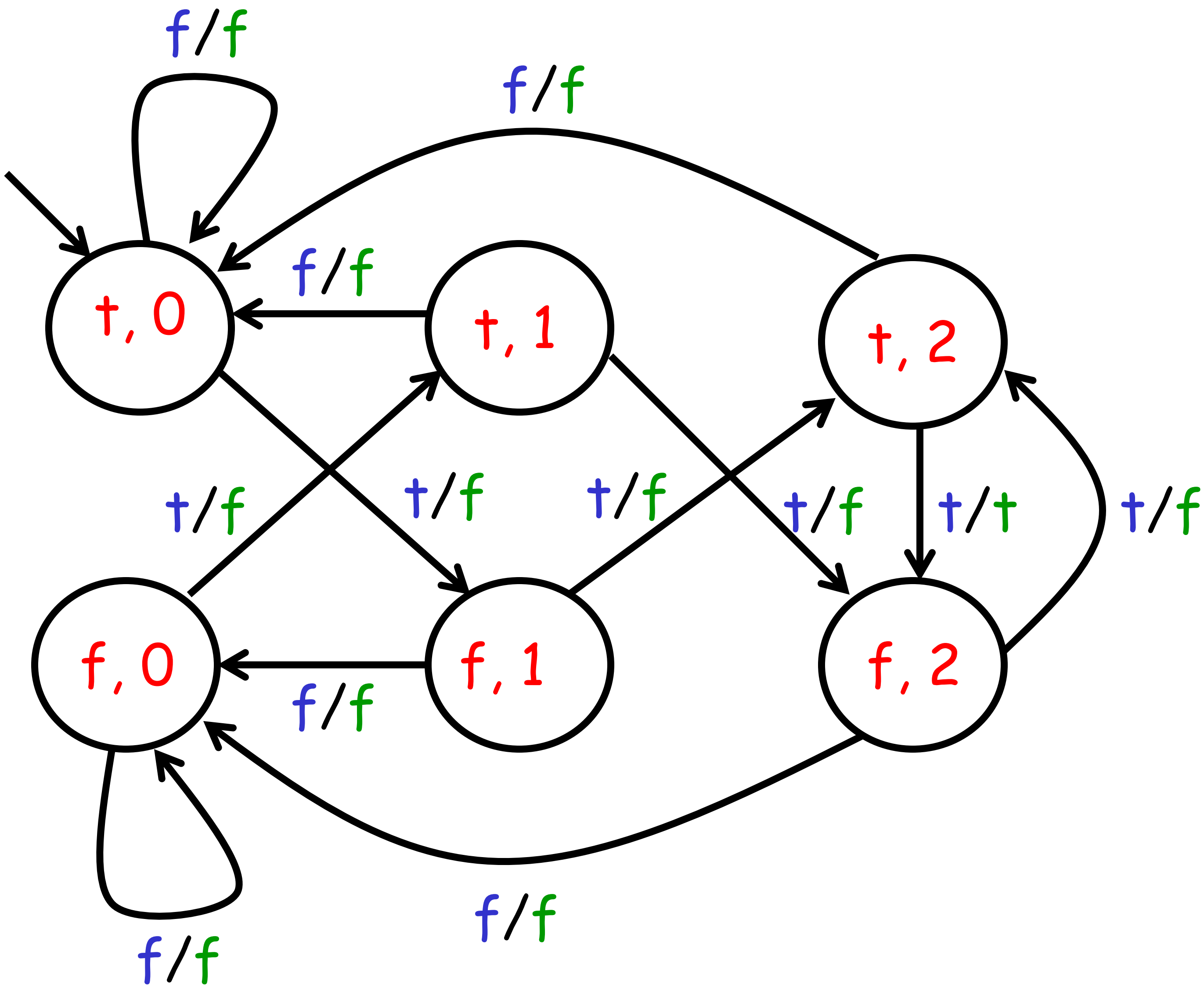
The ParityAndLastThree System, continued

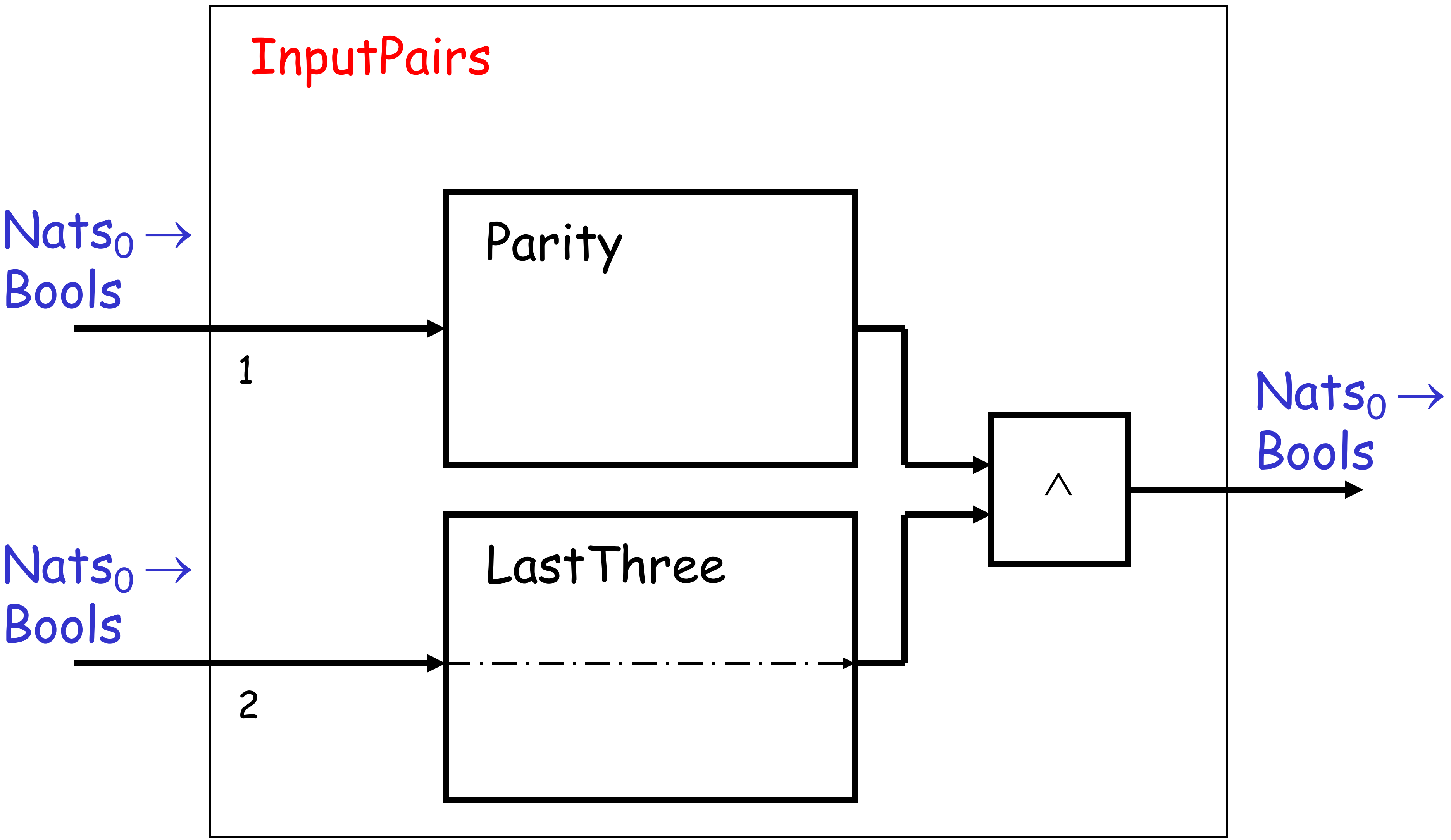
nextState [ParityAndLastThree] ((q1, q2) , x)

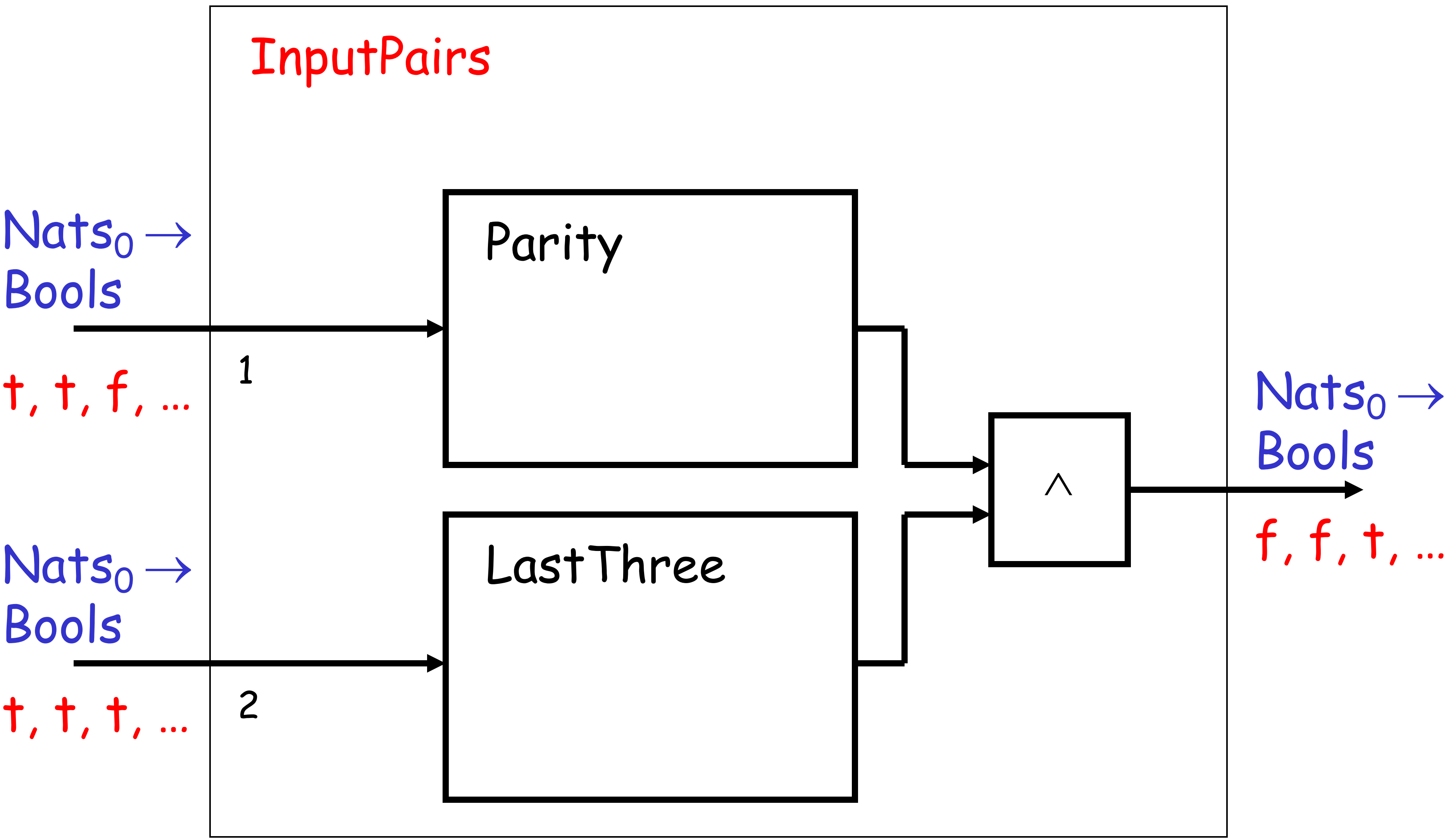
= (nextState [Parity] (q1, x) , nextState [LastThree] (q2, x))

output [ParityAndLastThree] ((q1, q2) , x)

= output [Parity] (q1, x) $\wedge$ output [LastThree] (q2, x)


f/f
f/f
f/f
t, 0
t, 1
t, 2
t/f
t/f
t/f
t/f
t/t
t/f
f, 0
f, 1
f, 2
f/f
f/f
f/f


InputPairs
Nats$_0$ → Bools
Parity
1
Nats$_0$ → Bools
LastThree
2
∧
Nats$_0$ → Bools


InputPairs
Nats$_0$ → Bools
t, t, f, ...
1
Parity
Nats$_0$ → Bools
t, t, t, ...
2
LastThree
∧
Nats$_0$ → Bools
f, f, t, ...

The InputPairs System

Inputs [InputPairs] = Bools × Bools

Outputs [InputPairs] = Bools

States [InputPairs]

= States [Parity] × States [LastThree]

= { true, false } × { 0, 1, 2 }

initialState [InputPairs]

= (initialState [Parity], initialState [LastThree])

= (true, 0)

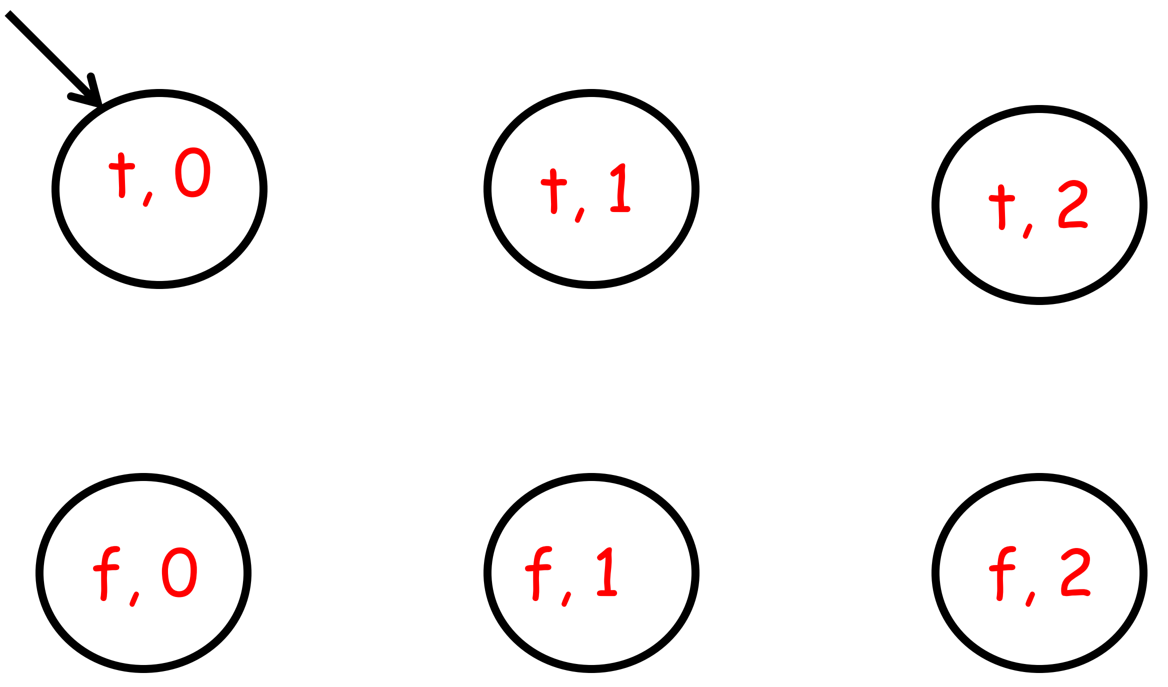
The InputPairs System, continued

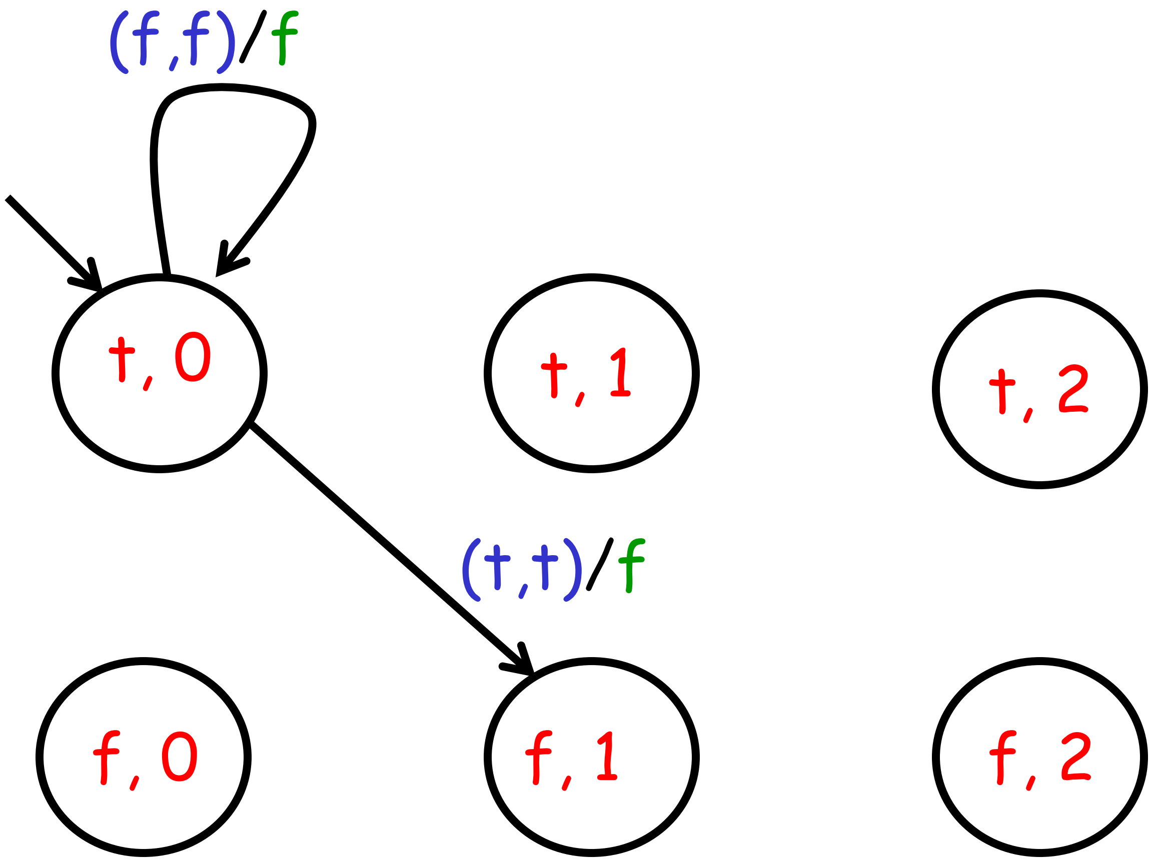
$$\text{nextState}\,[\,\text{InputPairs}\,]\,((q1, q2), (x1, x2))$$

$$= (\text{nextState}\,[\,\text{Parity}\,]\,(q1, x1), \text{nextState}\,[\,\text{LastThree}\,]\,(q2, x2))$$

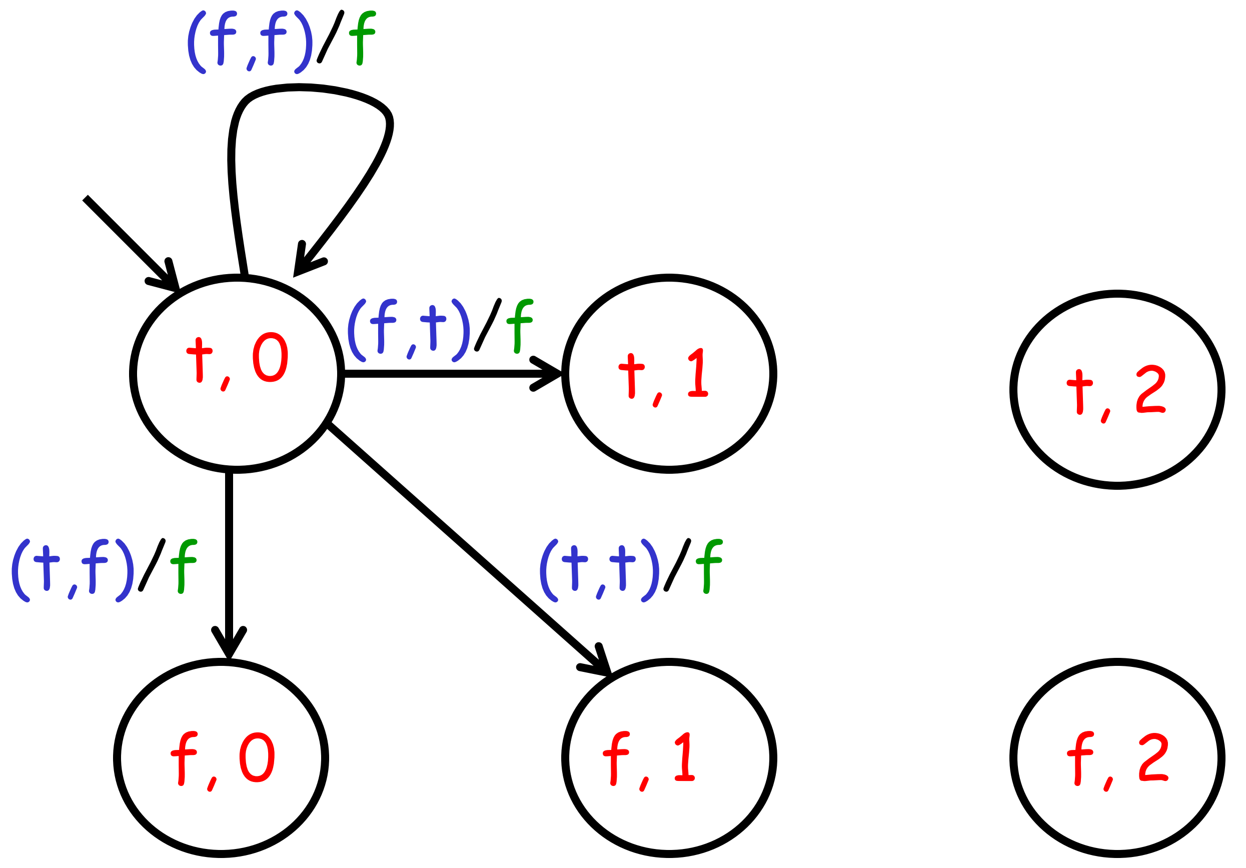
$$\text{output}\,[\,\text{InputPairs}\,]\,((q1, q2), (x1, x2))$$

$$= \text{output}\,[\,\text{Parity}\,]\,(q1, x1) \wedge \text{output}\,[\,\text{LastThree}\,]\,(q2, x2)$$

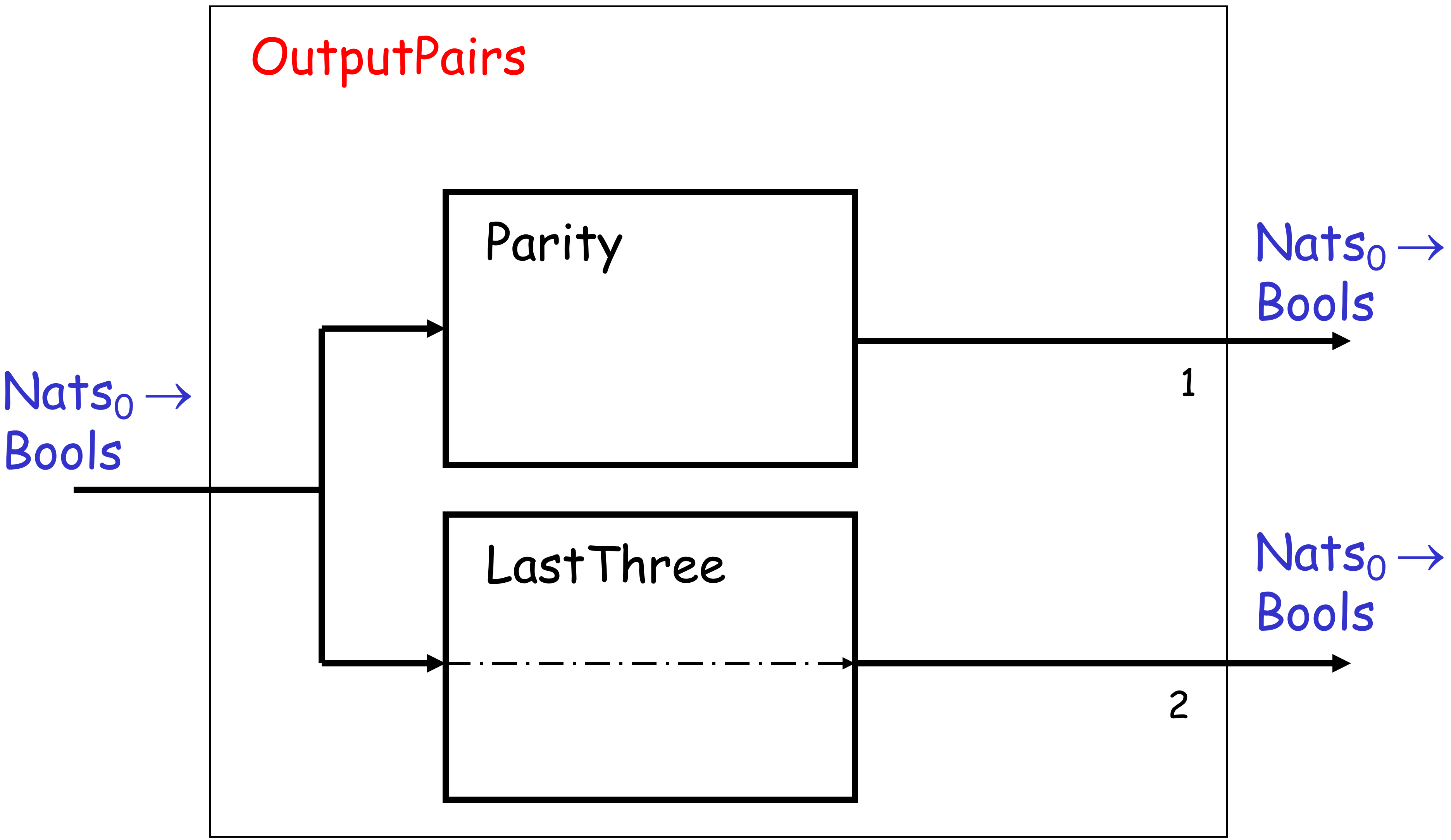

t, 0
t, 1
t, 2
f, 0
f, 1
f, 2

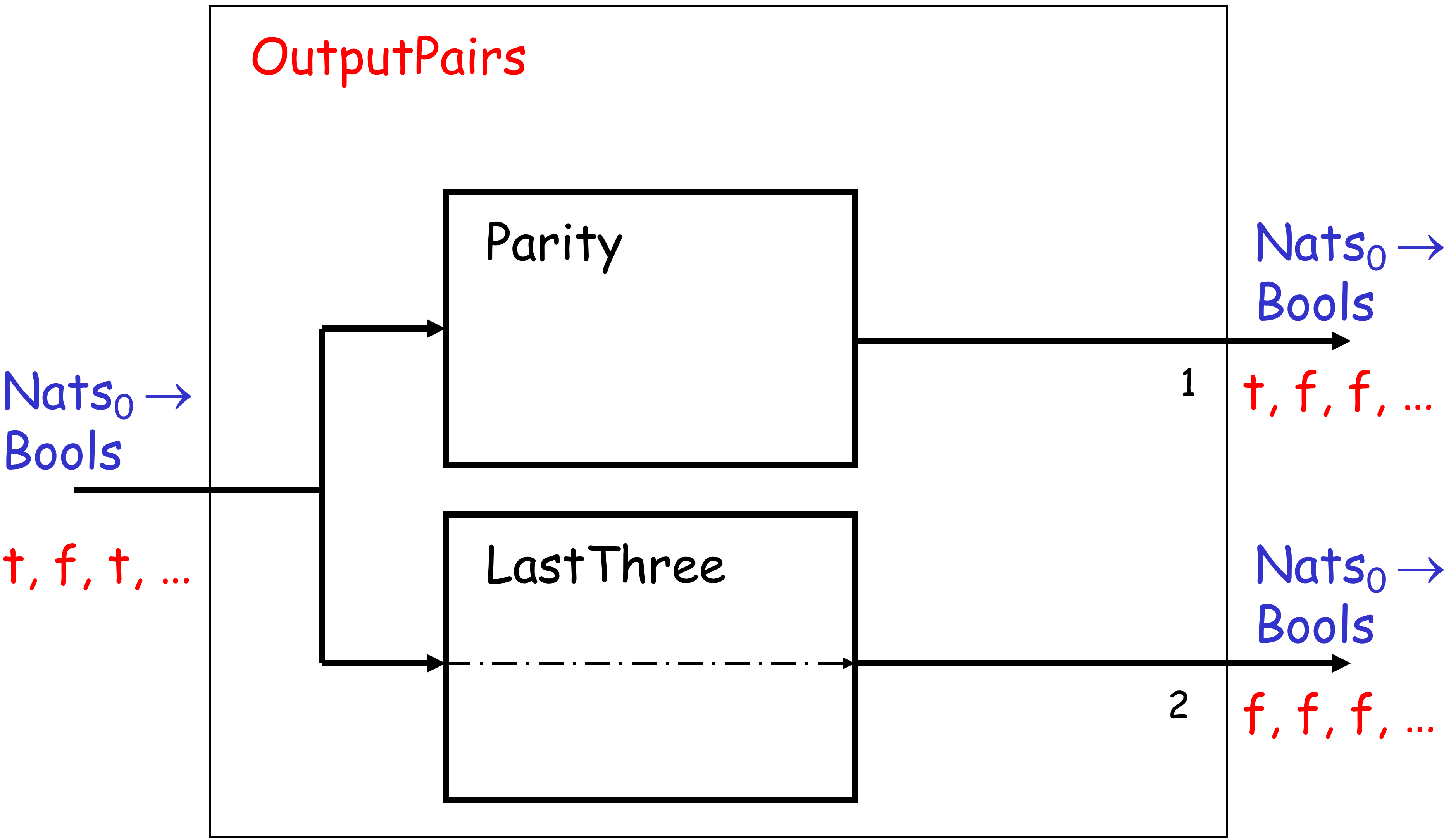


(f,f)/f
t, 0
t, 1
t, 2
(t,t)/f
f, 0
f, 1
f, 2



(f,f)/f
(f,t)/f
(t,f)/f
(t,t)/f
t, 0
t, 1
t, 2
f, 0
f, 1
f, 2


OutputPairs
Parity
LastThree
Nats0 → Bools
Nats0 → Bools
Nats0 → Bools
1
2



OutputPairs
Parity
LastThree
Nats$_0$ → Bools
t, f, t, …
Nats$_0$ → Bools
1
t, f, f, …
Nats$_0$ → Bools
2
f, f, f, …

The OutputPairs System

$$\text{Inputs}[\text{OutputPairs}] = \text{Bools}$$

$$\text{Outputs}[\text{OutputPairs}] = \text{Bools} \times \text{Bools}$$

$$\begin{aligned}\text{States}[\text{OutputPairs}] &\\ &= \text{States}[\text{Parity}] \times \text{States}[\text{LastThree}] \\ &= \{\text{true}, \text{false}\} \times \{0, 1, 2\}\end{aligned}$$

$$\begin{aligned}\text{initialState}[\text{OutputPairs}] &\\ &= (\text{initialState}[\text{Parity}], \text{initialState}[\text{LastThree}]) \\ &= (\text{true}, 0)\end{aligned}$$

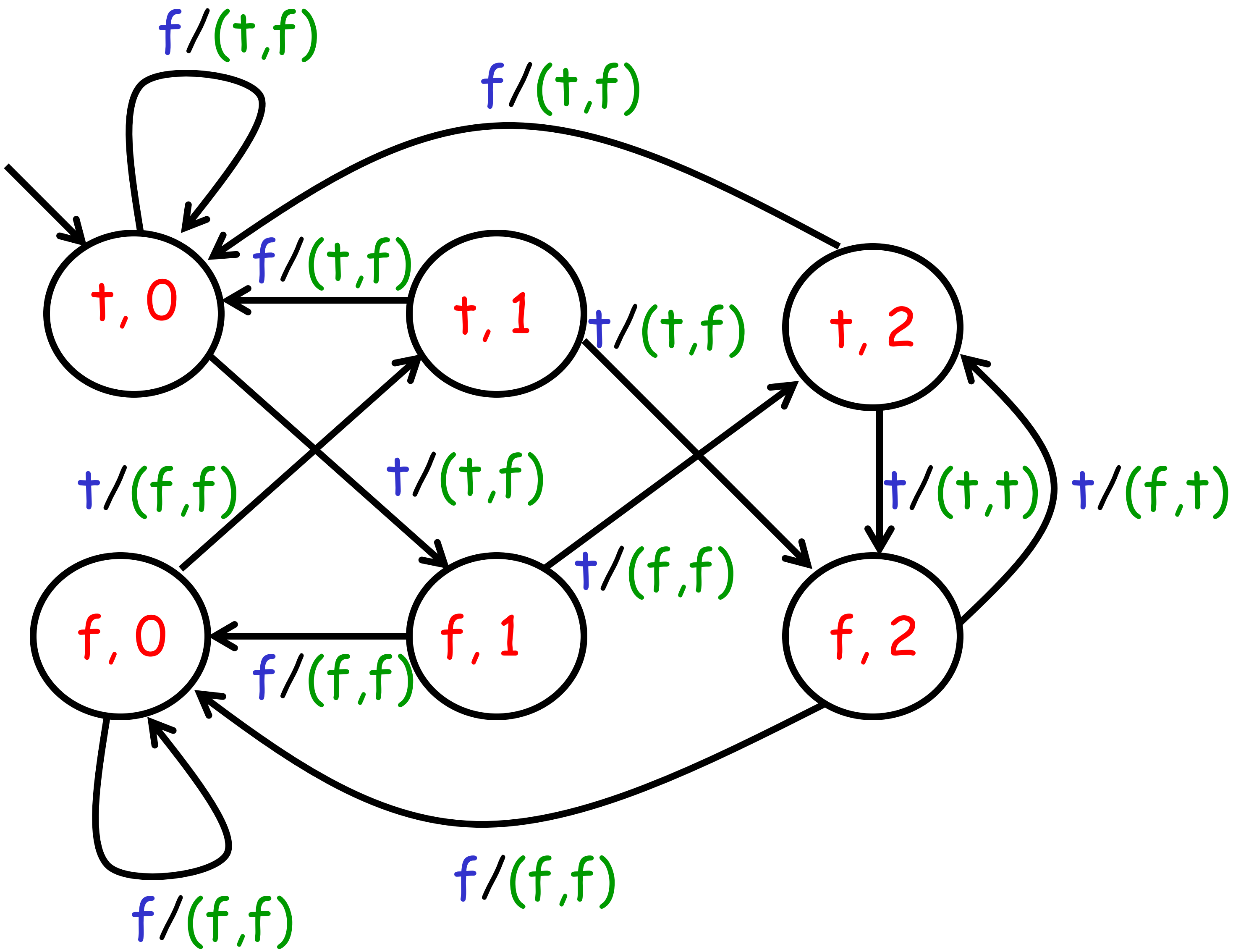
The OutputPairs System, continued

nextState [OutputPairs] ((q1, q2) , x)

= (nextState [Parity] (q1, x) , nextState [LastThree] (q2, x))

output [OutputPairs] ((q1, q2) , x)

= (output [Parity] (q1, x) , output [LastThree] (q2, x))



t, 0
t, 1
t, 2
f, 0
f, 1
f, 2
f/(t,f)
f/(t,f)
f/(t,f)
t/(t,f)
t/(f,f)
t/(t,f)
t/(f,f)
t/(t,t)
t/(f,t)
f/(f,f)
f/(f,f)
f/(f,f)

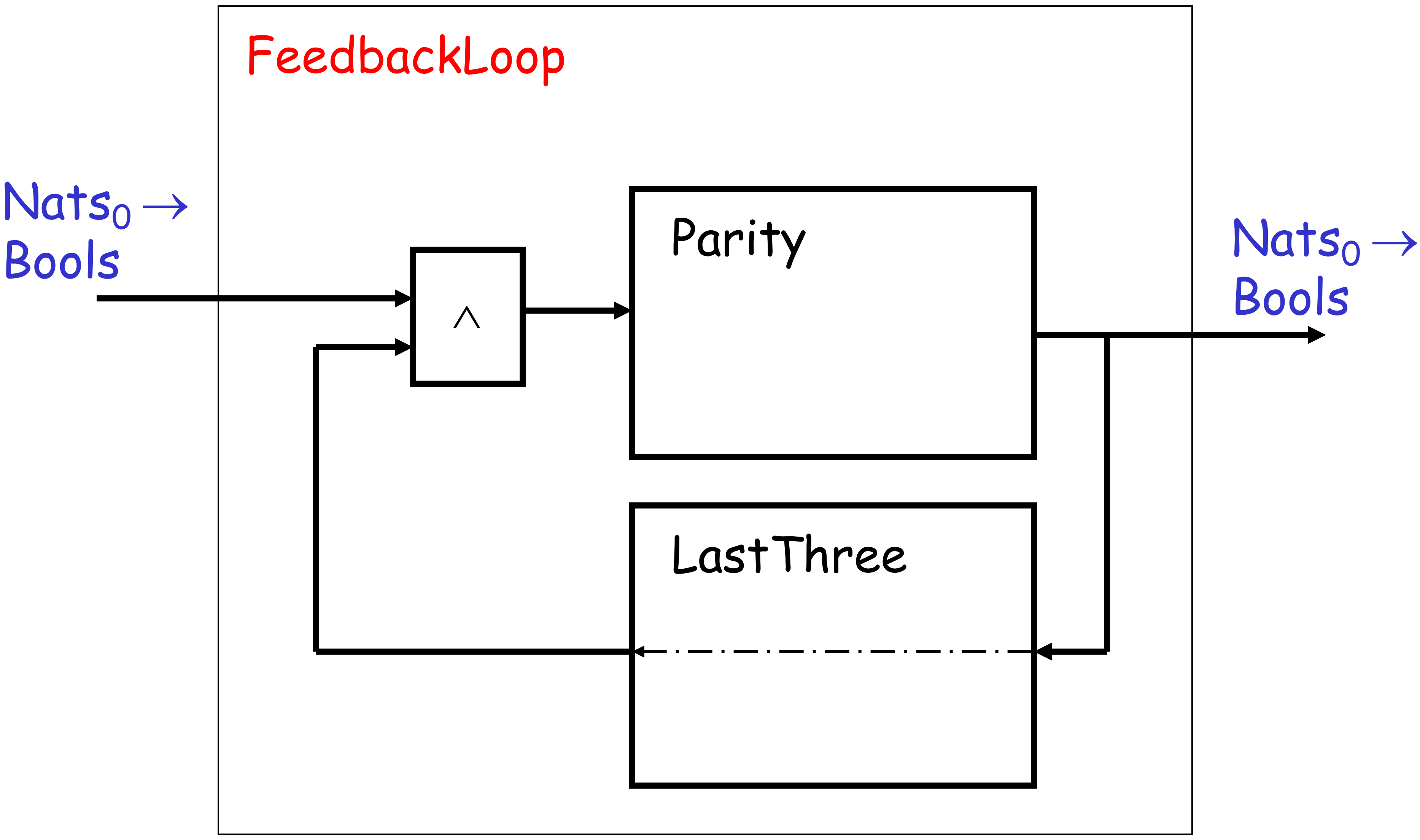
Any block diagram of N state machines with the state spaces

States1, States2, … StatesN

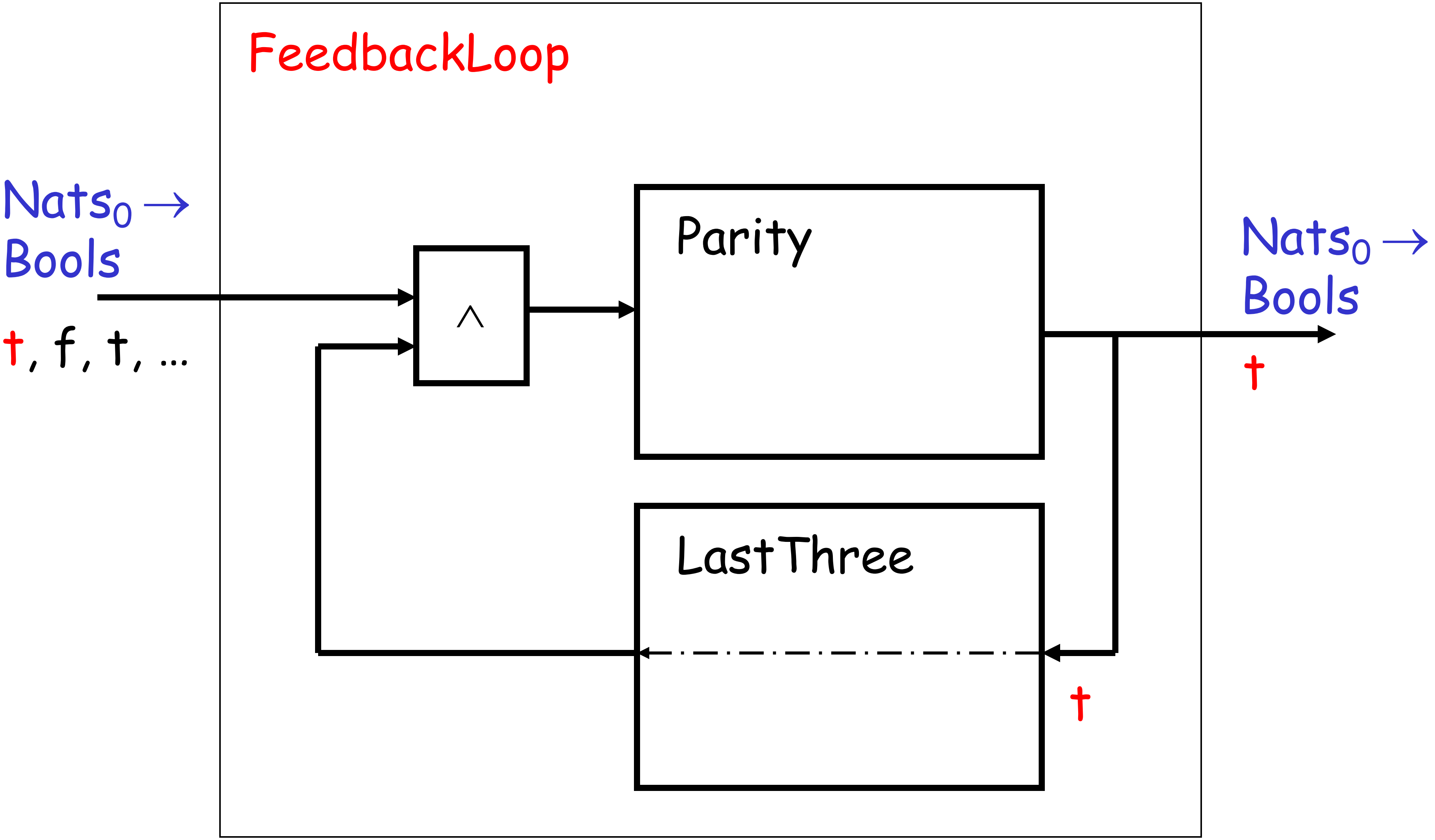
can be implemented by a single state machine with the state space

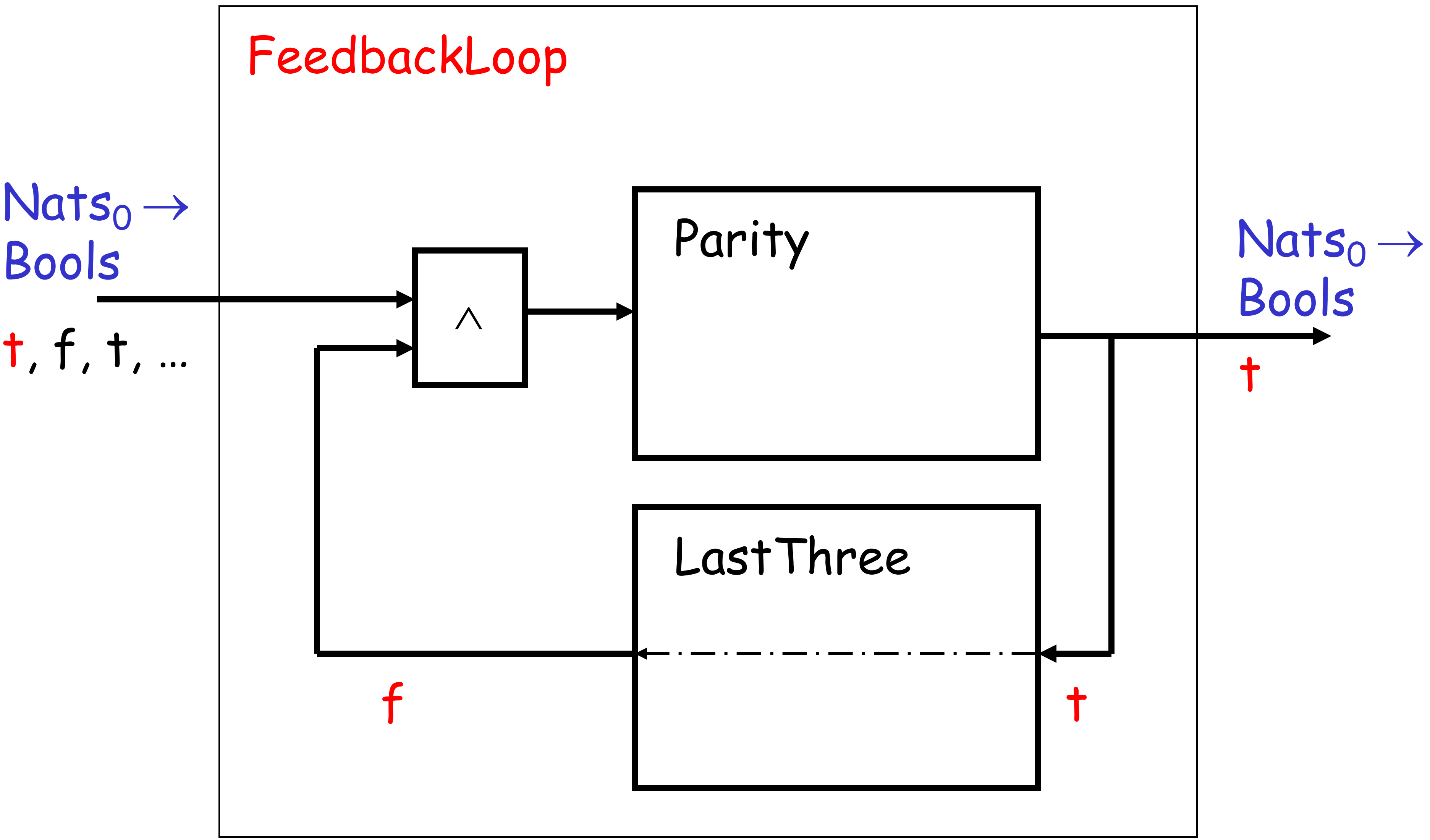
States1 $\times$ States2 $\times$ … $\times$ StatesN .

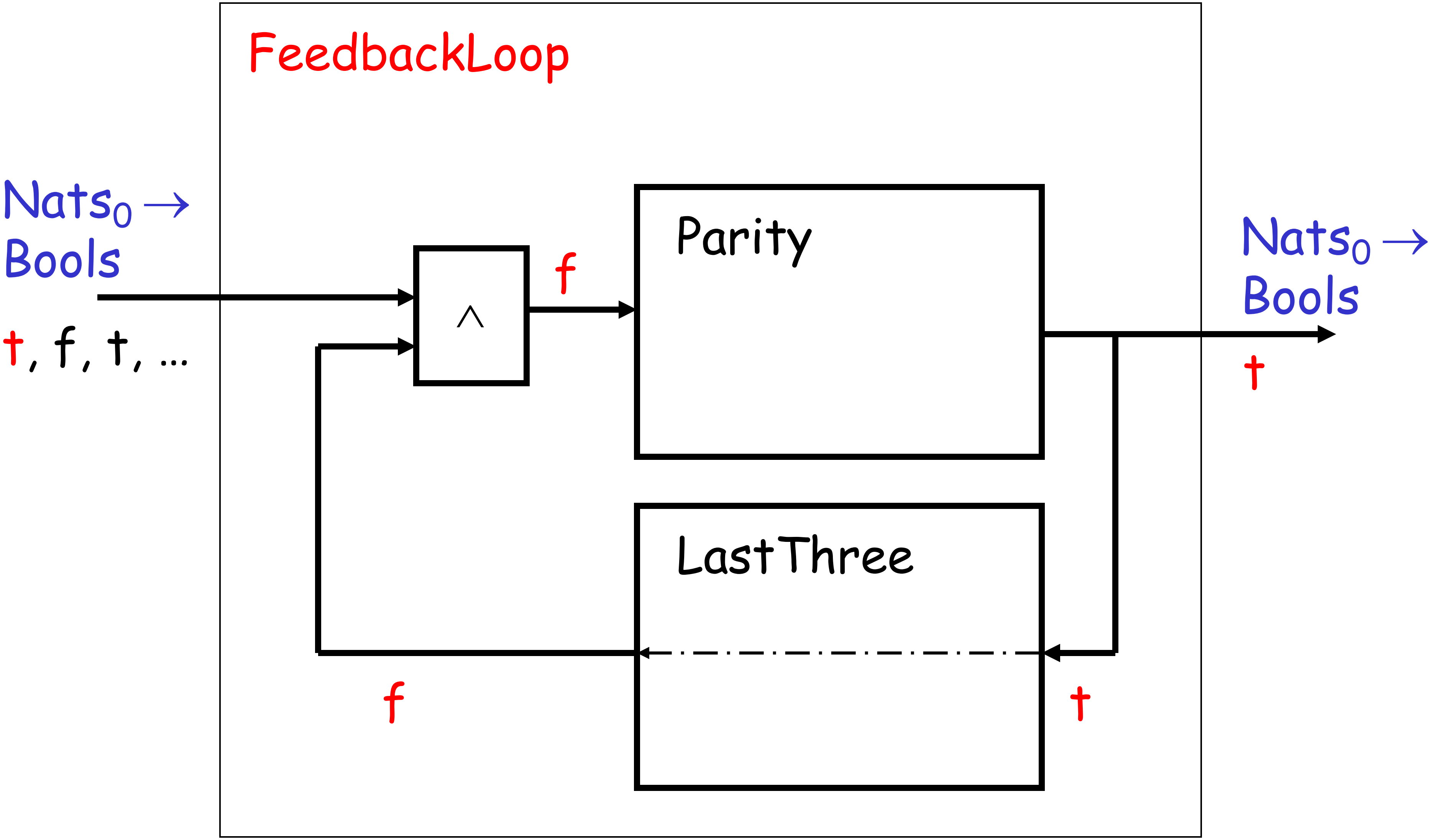
This is called a "product machine".



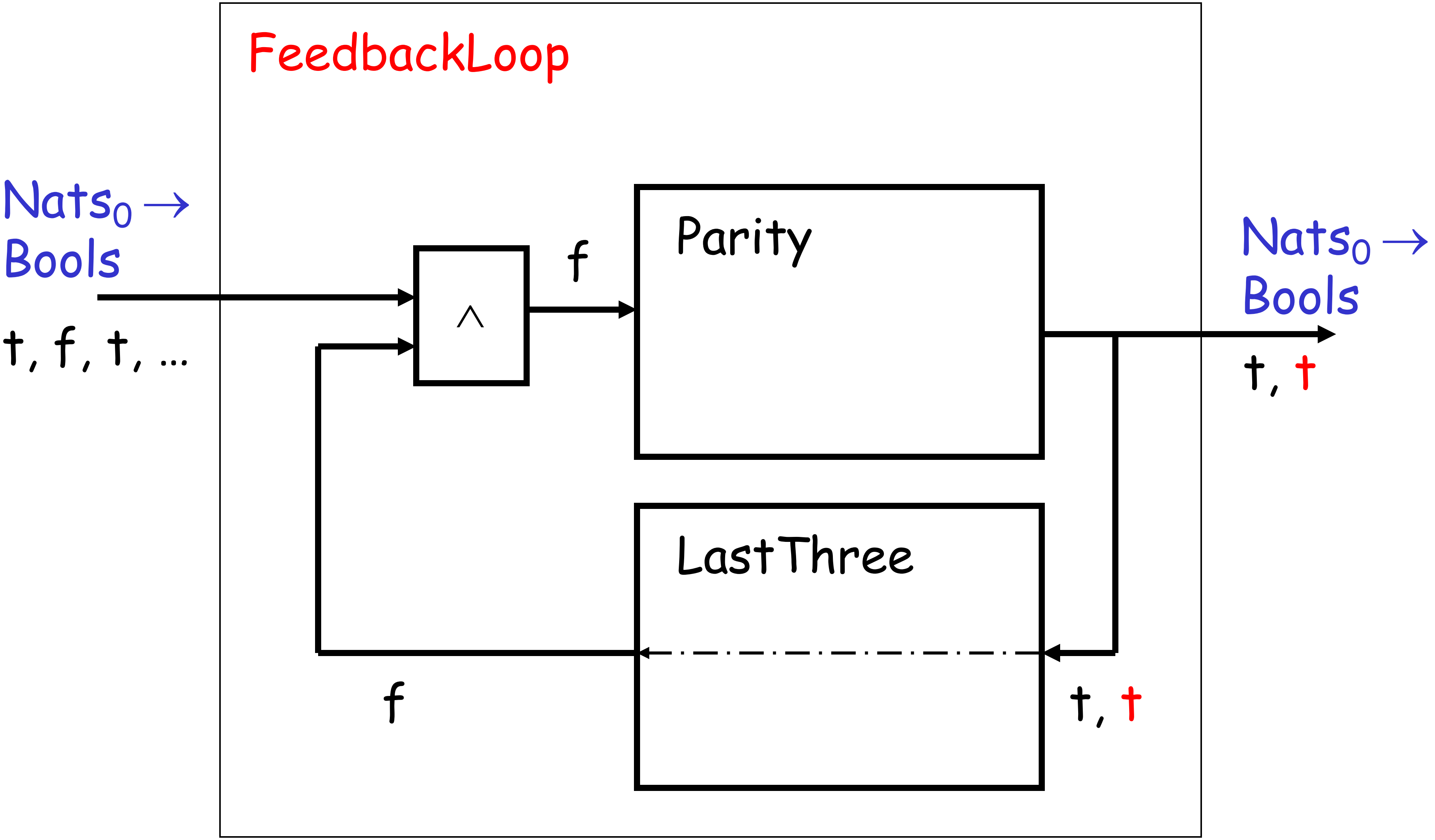
This block diagram is ok, because every cycle contains a delay.


FeedbackLoop
Nats0 → Bools
t, f, t, ...
∧
Parity
LastThree
Nats0 → Bools
t
t

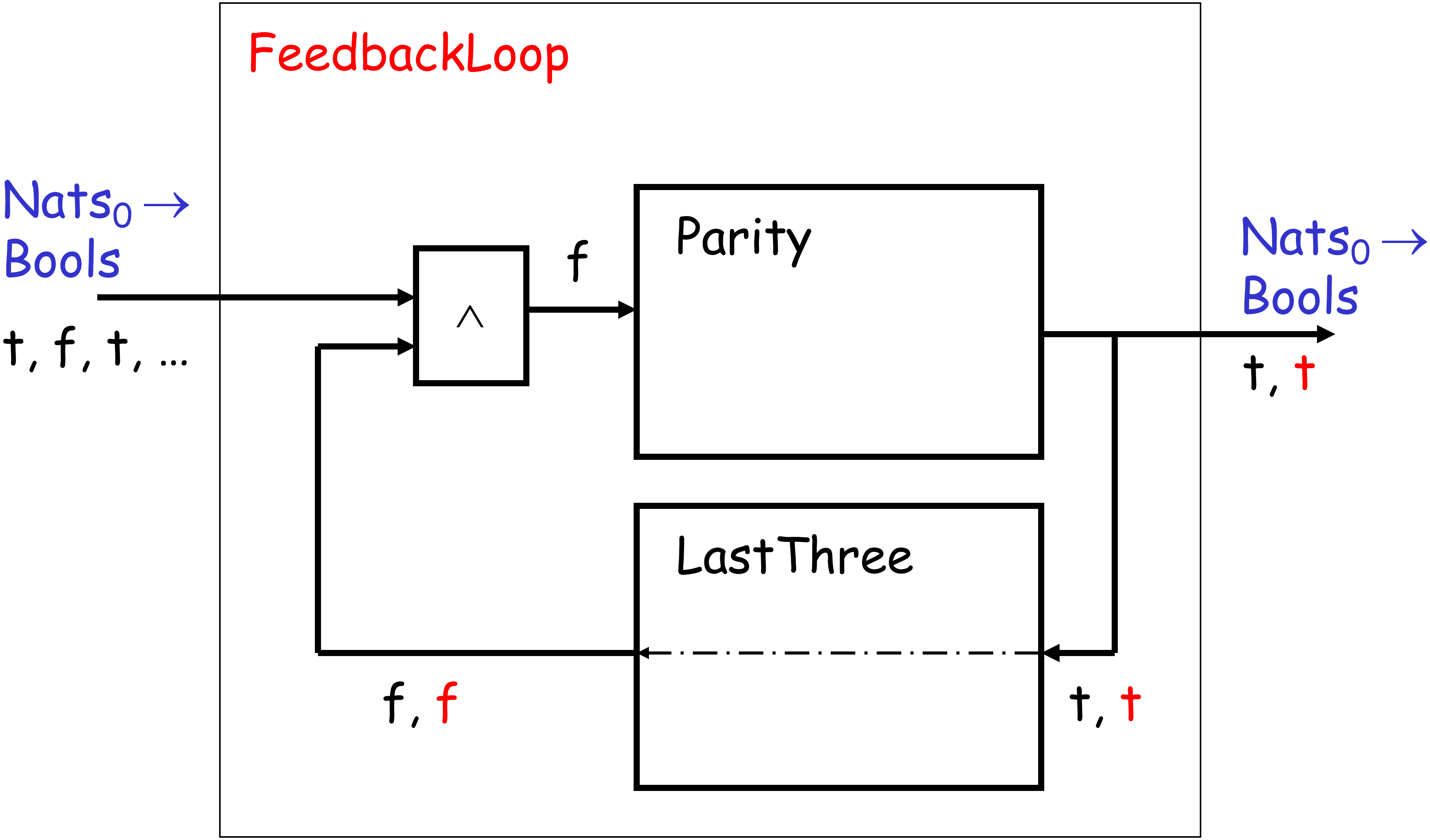

FeedbackLoop
Nats0 → Bools
t, f, t, ...
∧
Parity
Nats0 → Bools
t
LastThree
f
t

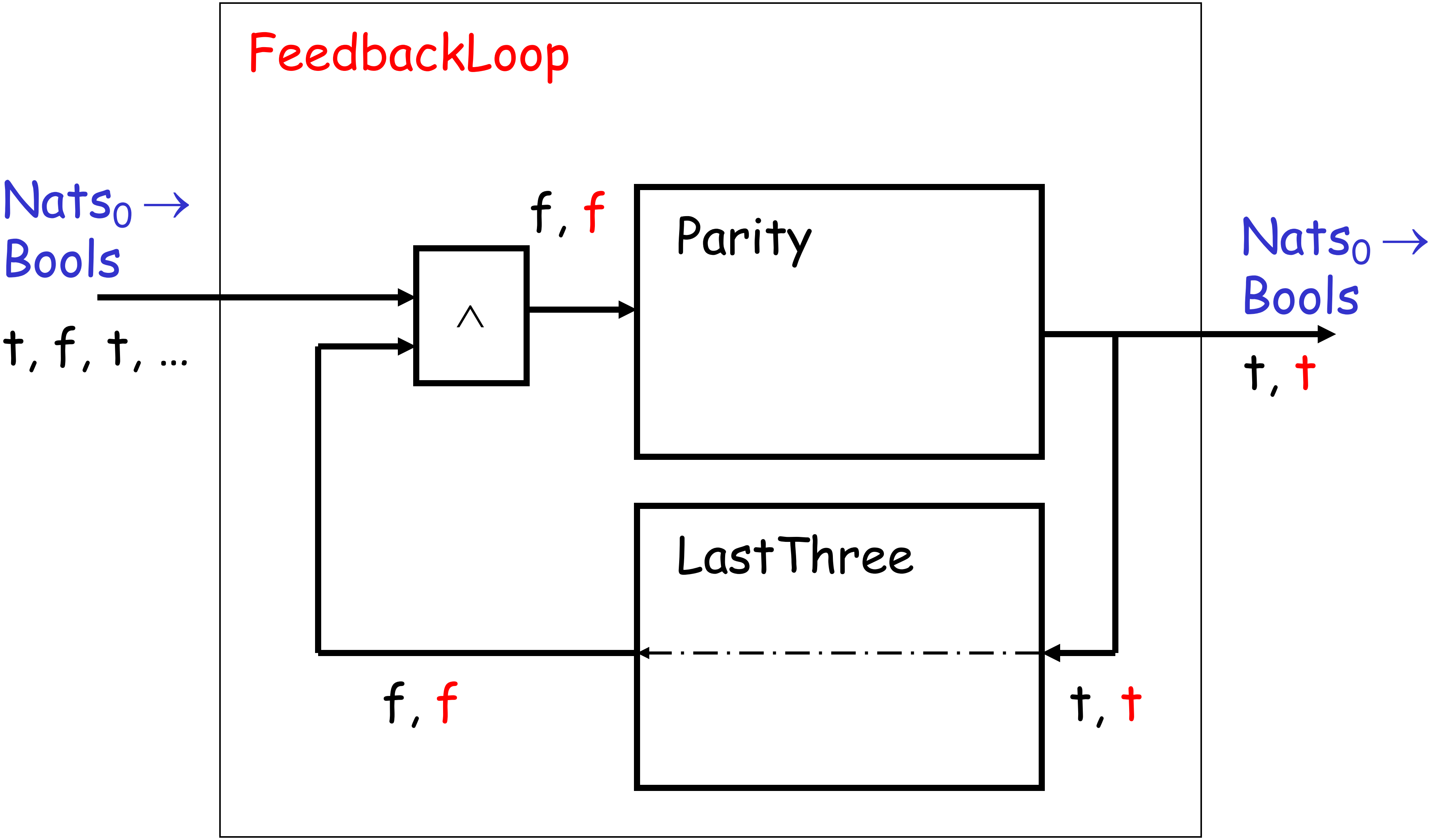


FeedbackLoop
Nats0 → Bools
t, f, t, ...
∧
f
Parity
Nats0 → Bools
t
LastThree
f
t

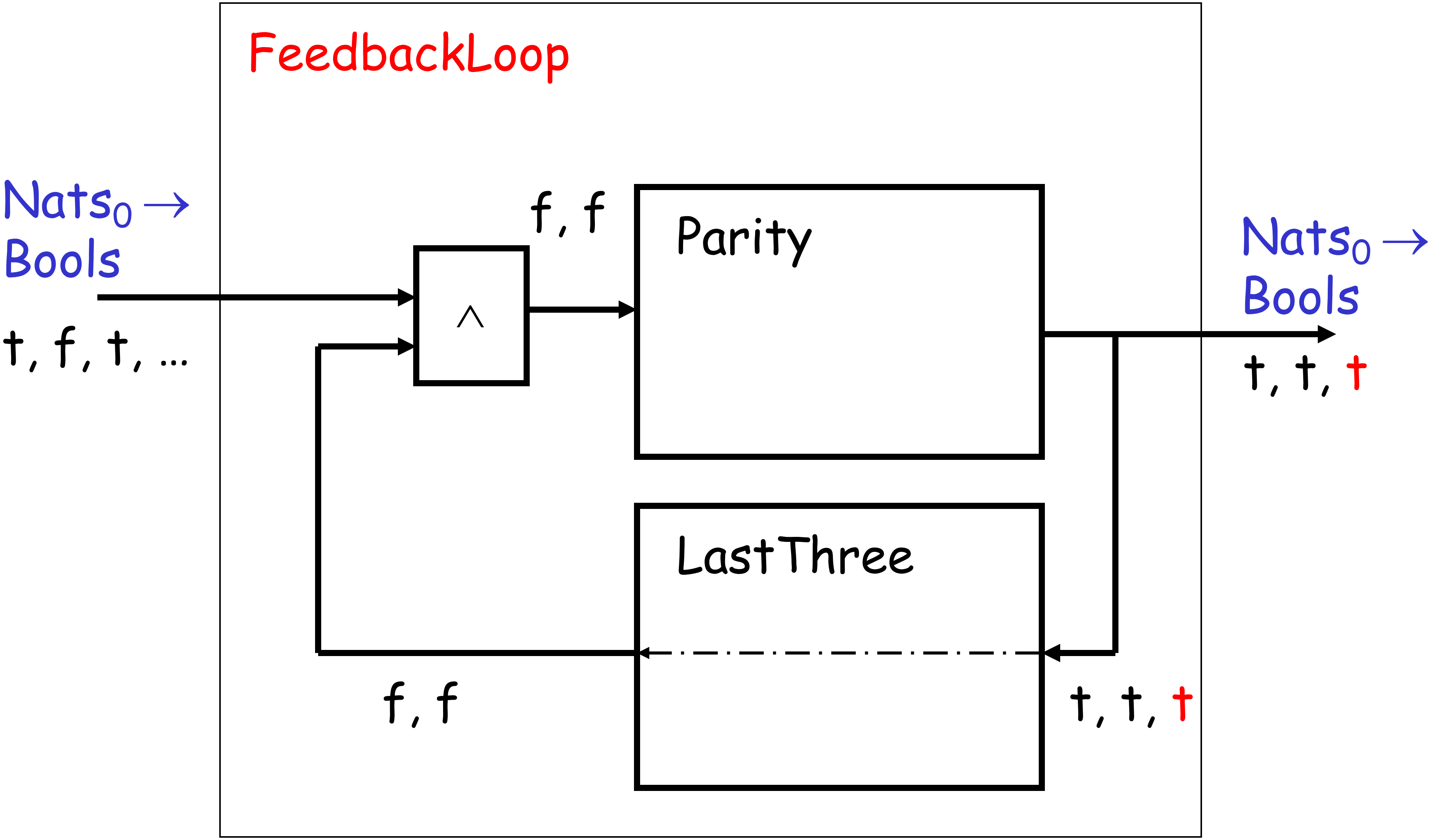


FeedbackLoop
$Nats_0 \rightarrow Bools$
t, f, t, ...
∧
f
Parity
$Nats_0 \rightarrow Bools$
t, t
LastThree
f
t, t

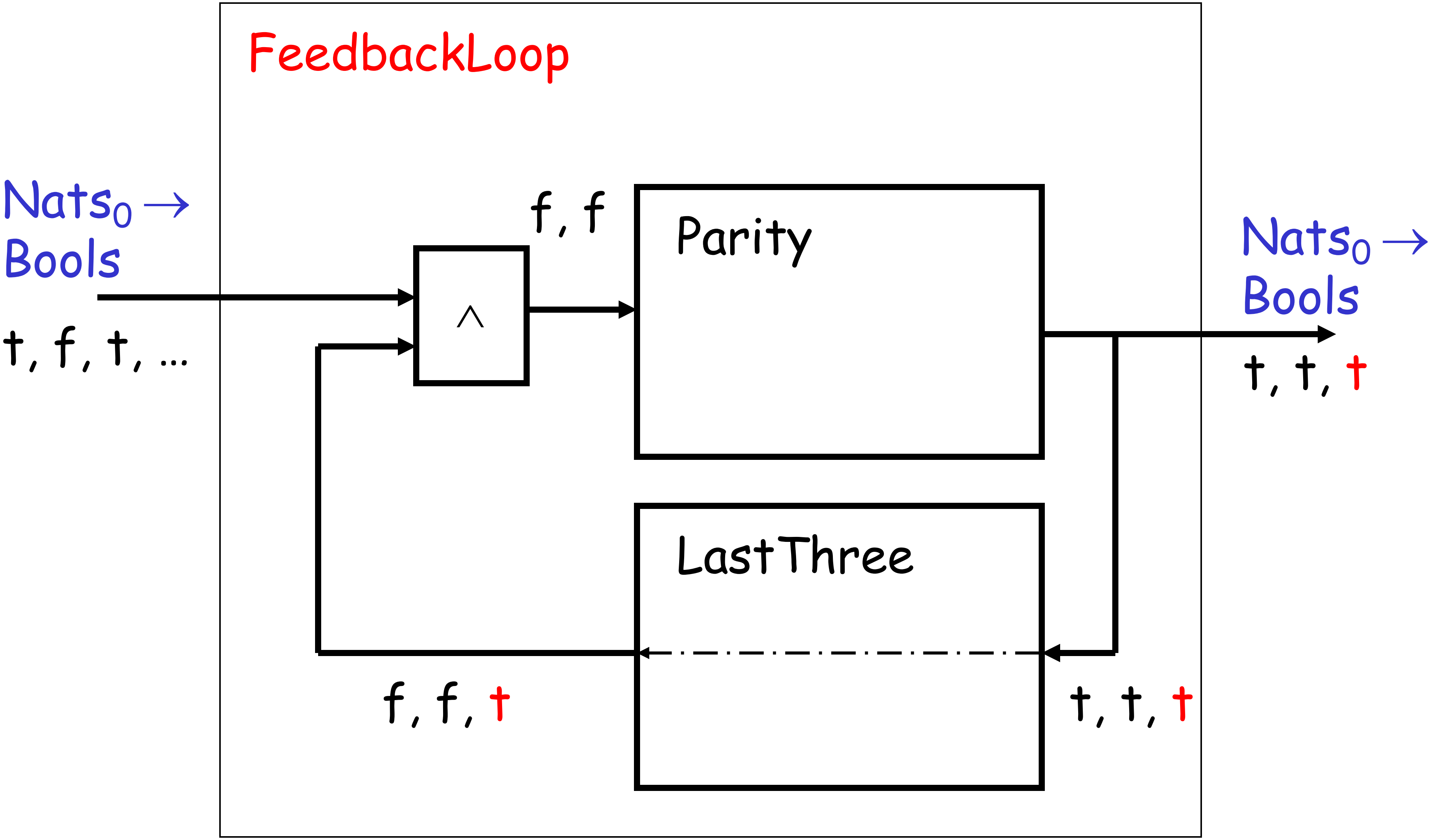

FeedbackLoop
Nats$_0$ → Bools
t, f, t, ...
∧
f
Parity
Nats$_0$ → Bools
t, t
LastThree
f, f
t, t

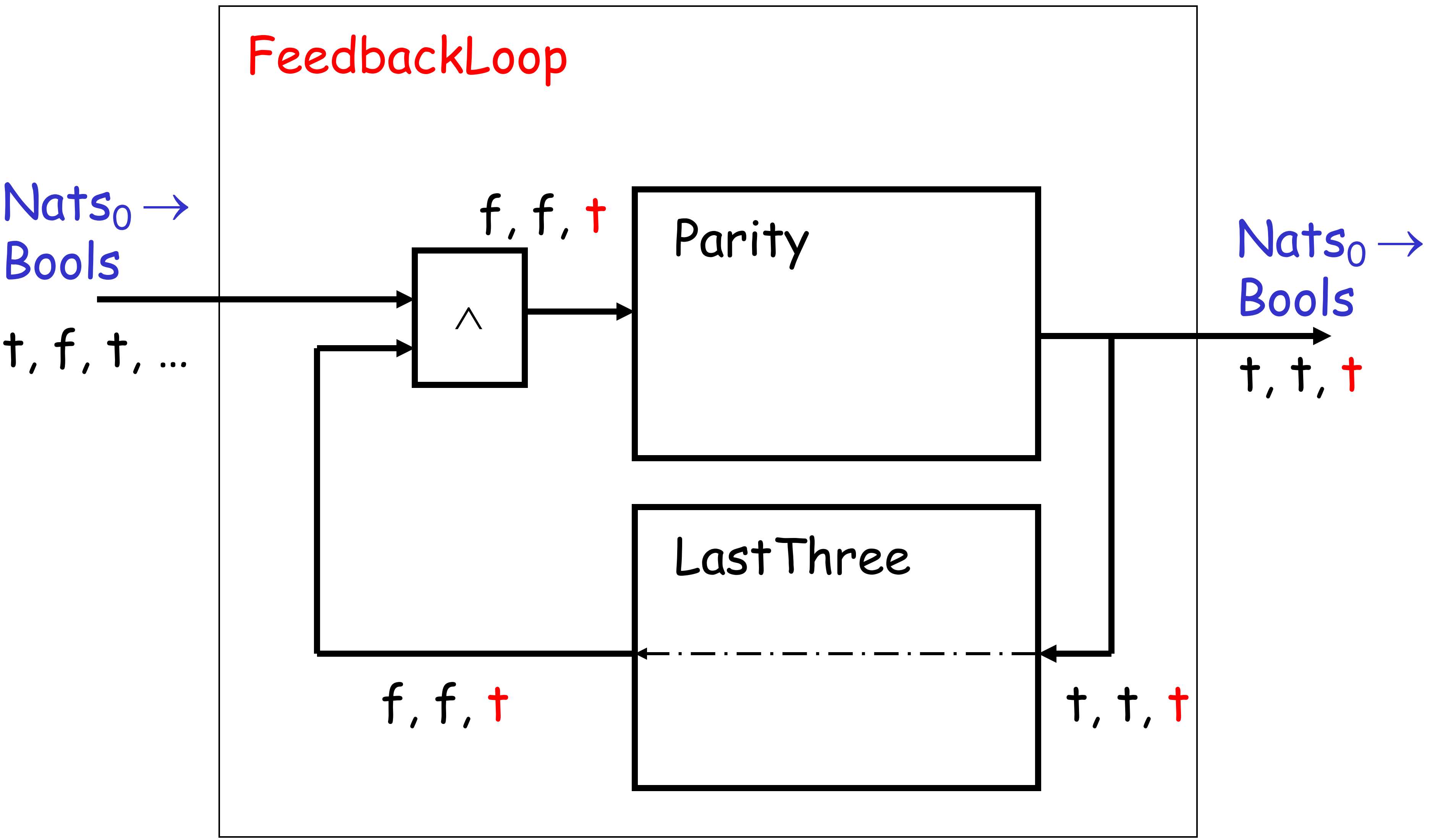


FeedbackLoop
$Nats_0 \rightarrow$ Bools
t, f, t, ...
f, f
$\wedge$
Parity
$Nats_0 \rightarrow$ Bools
t, t
LastThree
f, f
t, t

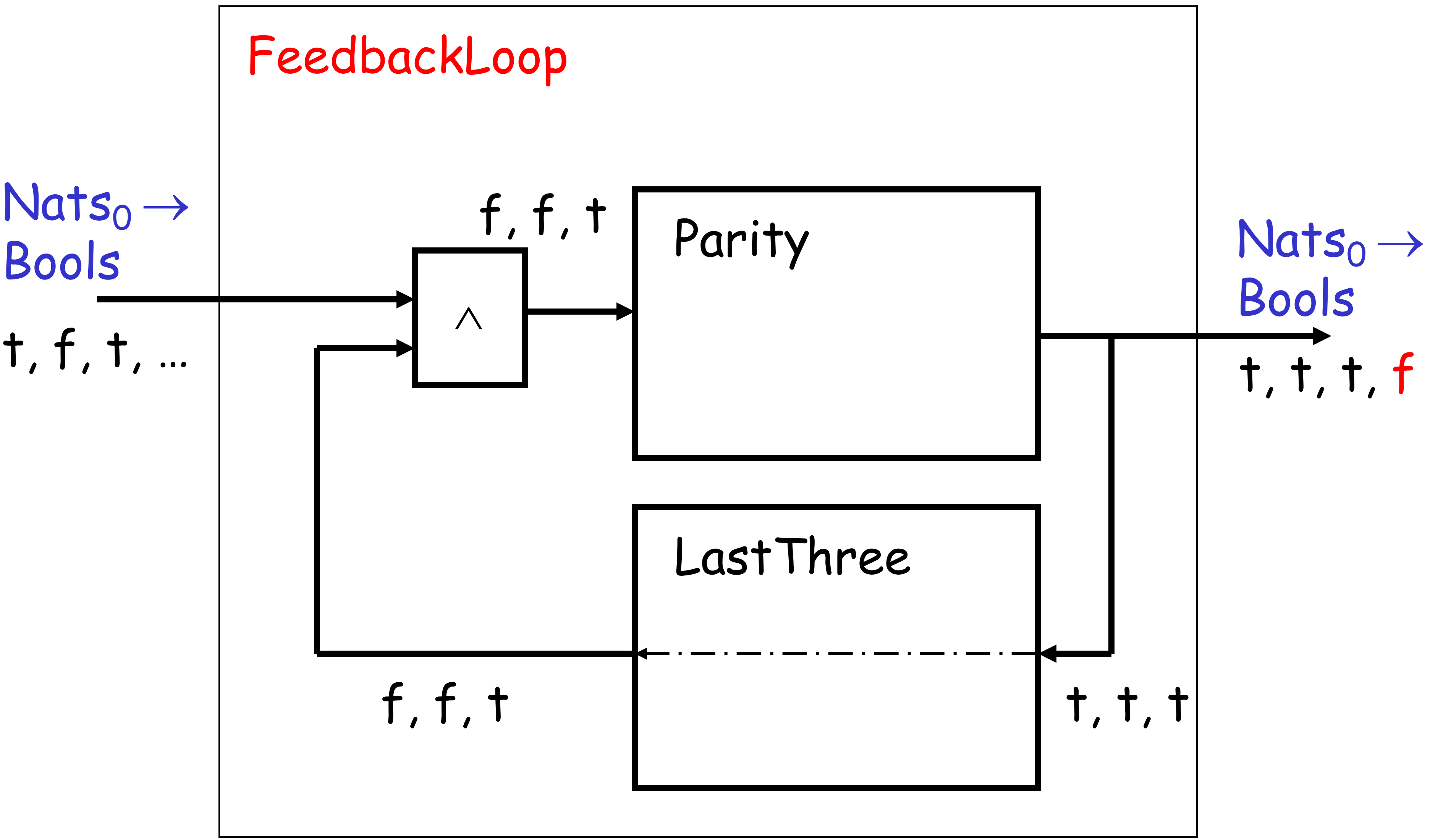


FeedbackLoop
$Nats_0 \to$ Bools
t, f, t, …
∧
f, f
Parity
$Nats_0 \to$ Bools
t, t, t
LastThree
f, f
t, t, t


FeedbackLoop
Nats0 → Bools
t, f, t, ...
f, f
∧
Parity
Nats0 → Bools
t, t, t
LastThree
f, f, t
t, t, t



FeedbackLoop
$Nats_0 \rightarrow Bools$
t, f, t, ...
∧
f, f, t
Parity
$Nats_0 \rightarrow Bools$
t, t, t
LastThree
f, f, t
t, t, t


FeedbackLoop
Nats0 → Bools
t, f, t, ...
f, f, t
∧
Parity
Nats0 → Bools
t, t, t, f
LastThree
f, f, t
t, t, t

Example: Vending Machine

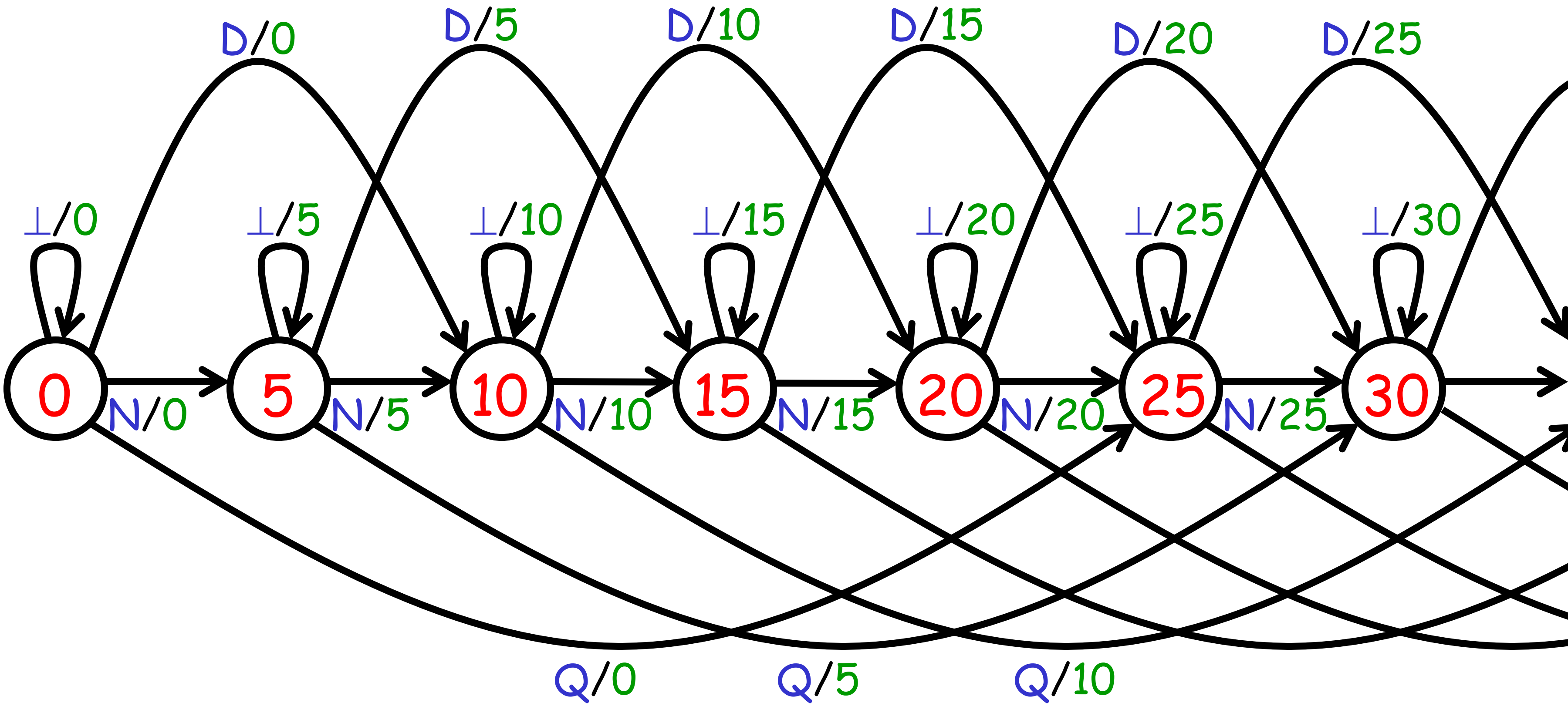
Coin Collector

$$(\mathrm{Nats}_0 \to \mathrm{Coins}_\perp) \longrightarrow \boxed{\text{Collect}} \longrightarrow (\mathrm{Nats}_0 \to \mathrm{Nats}_0)$$

Let $\mathrm{Coins} = \{ \mathrm{Nickel}, \mathrm{Dime}, \mathrm{Quarter} \}$.

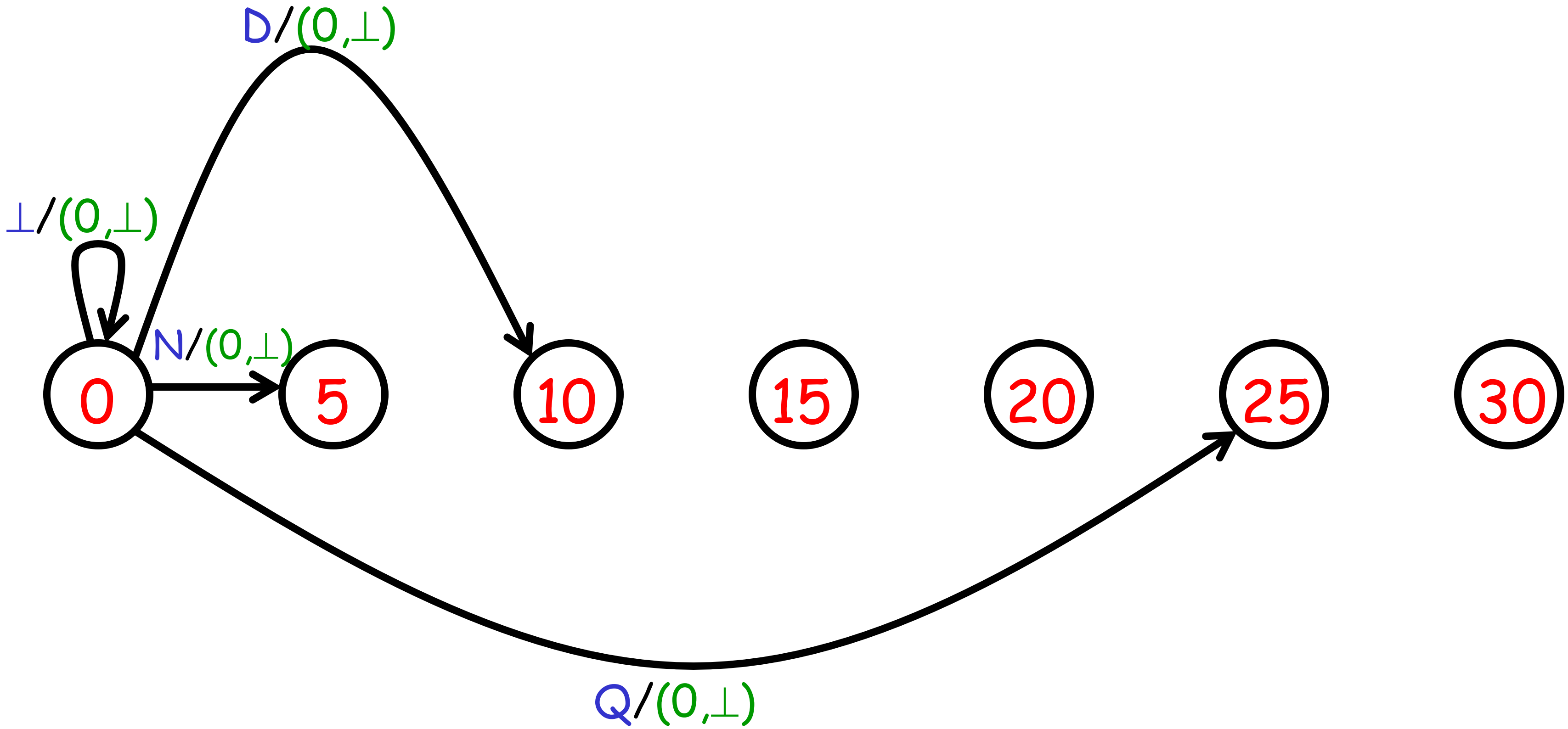
Let $\mathrm{Coins}_\perp = \mathrm{Coins} \cup \{ \perp \}$.   ($\perp$ stands for "no input.")

Coin Collector

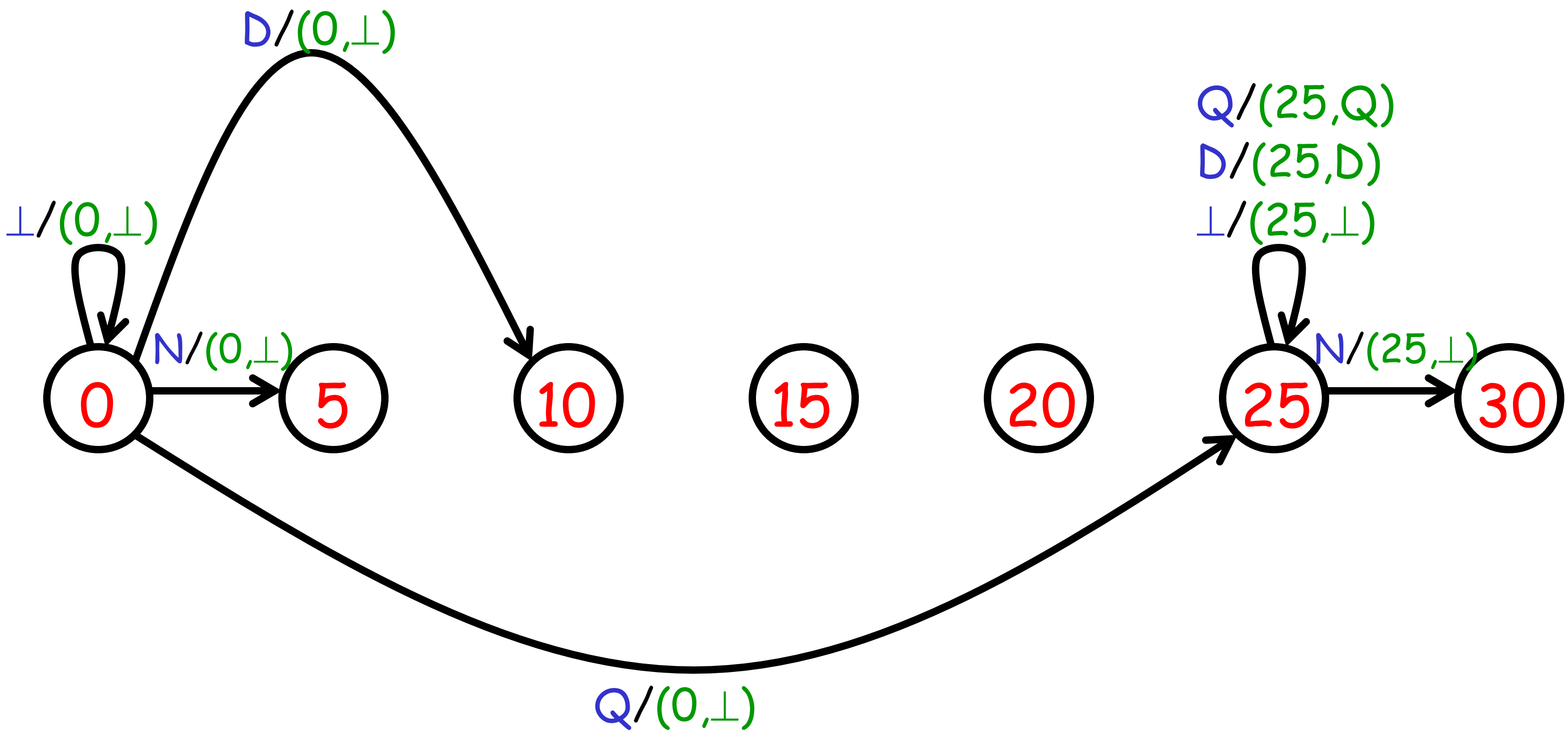


Finite-State Coin Collector

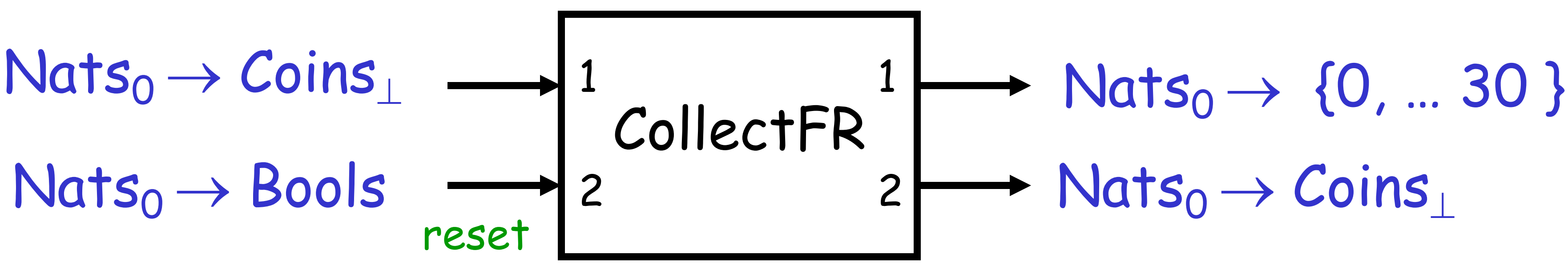
Finite-State Coin Collector



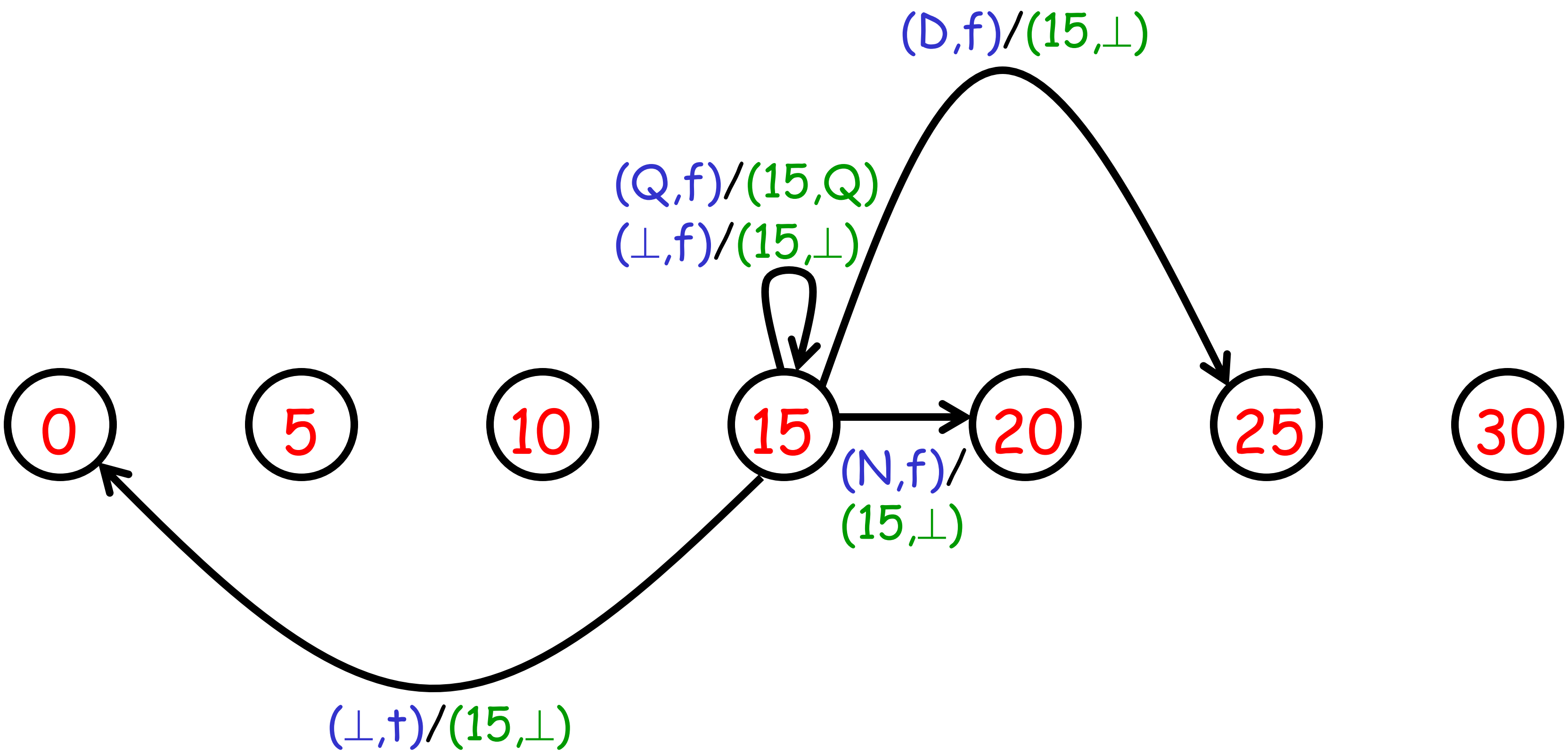
Finite-State Coin Collector



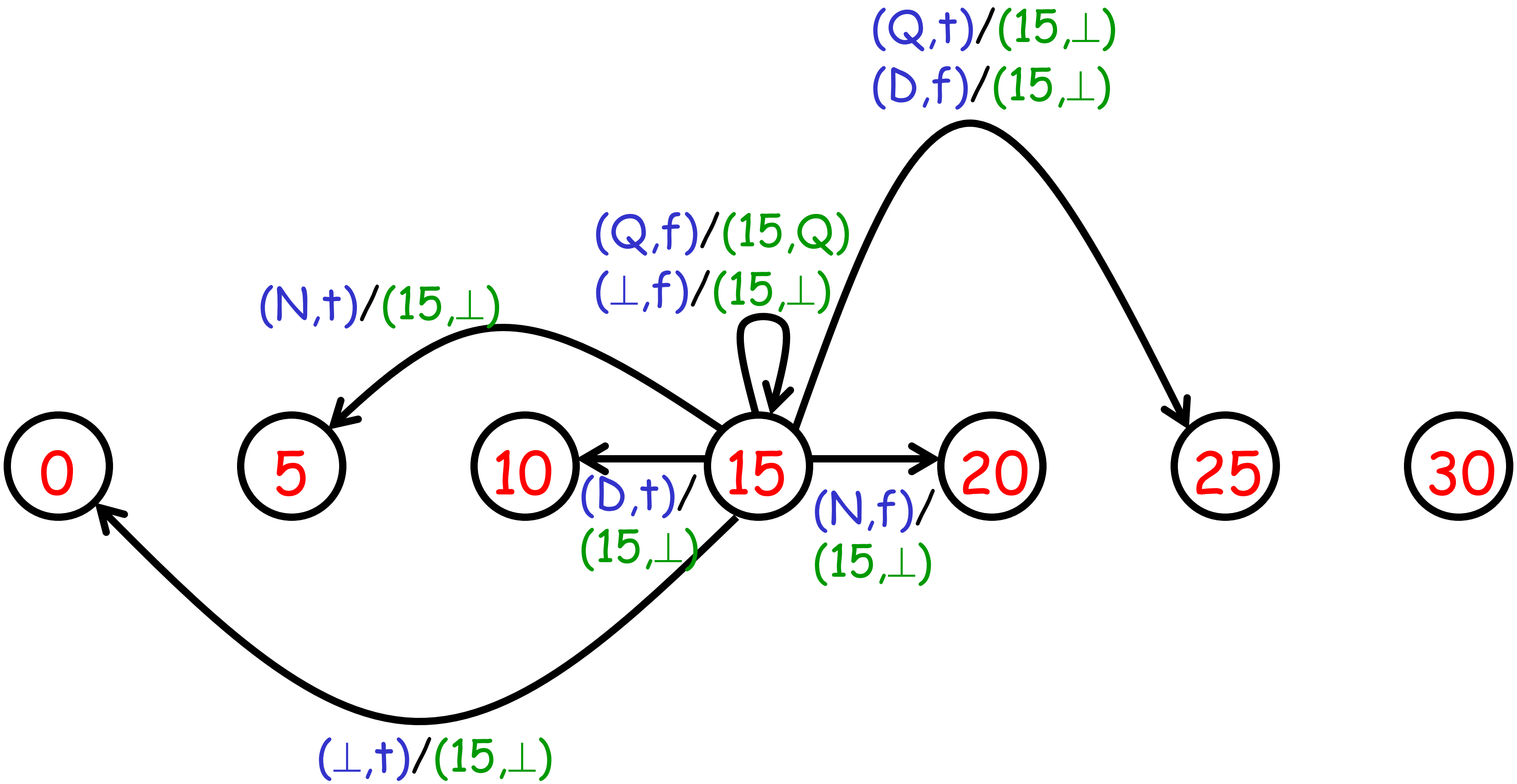
Coin Collector with Reset



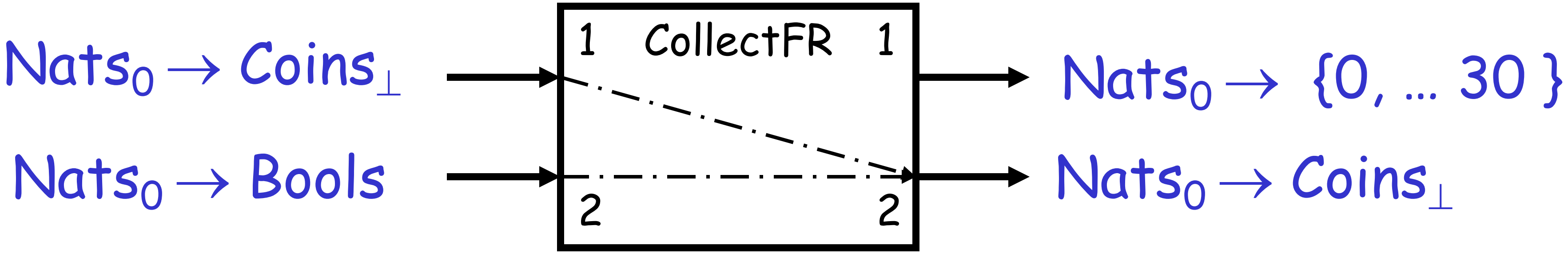
Coin Collector with Reset



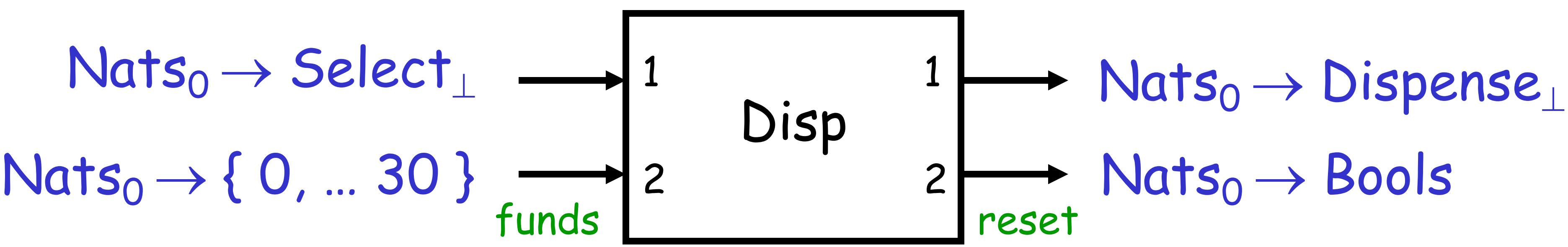
Coin Collector with Reset



Coin Collector with Reset



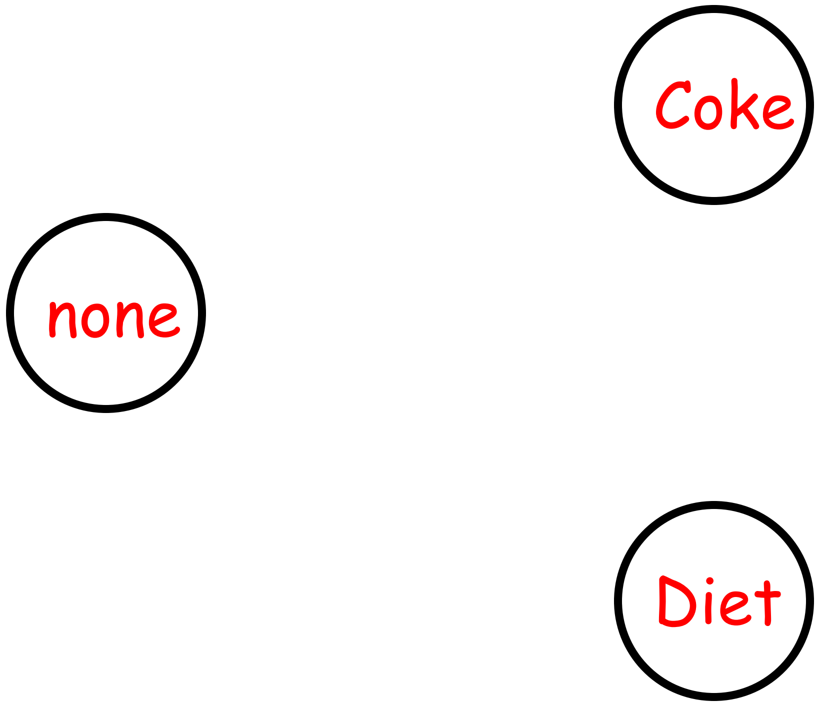
Soda Dispenser



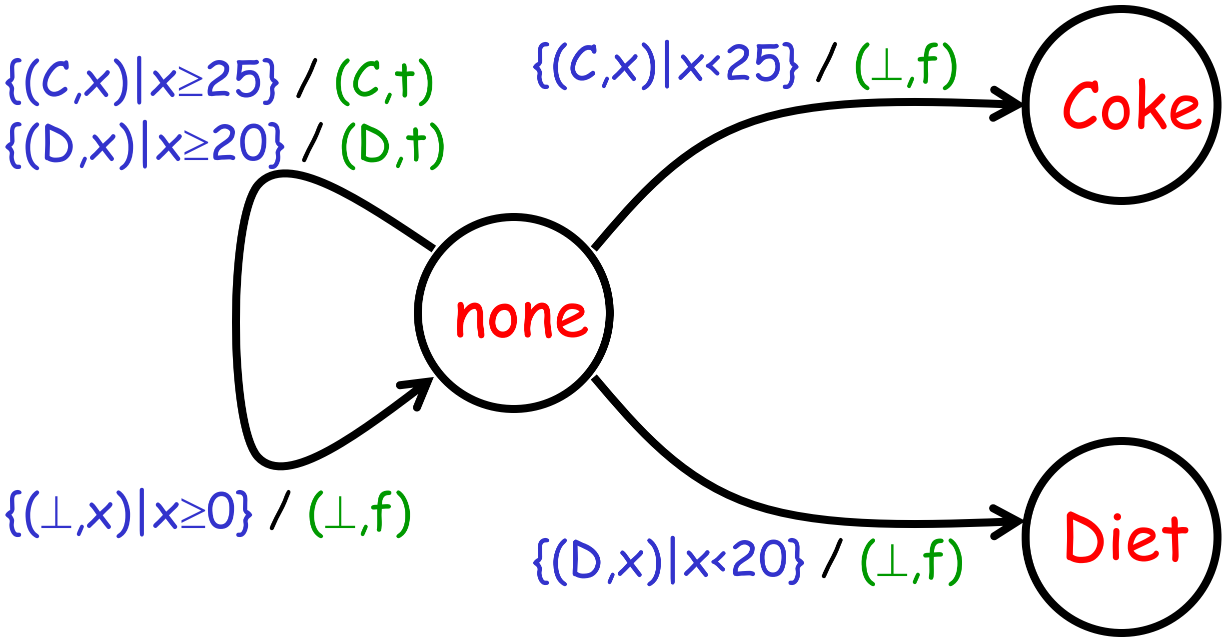
Let $Select = \{ selectCoke, selectDiet \}$.

Let $Dispense = \{ dispenseCoke, dispenseDiet \}$.

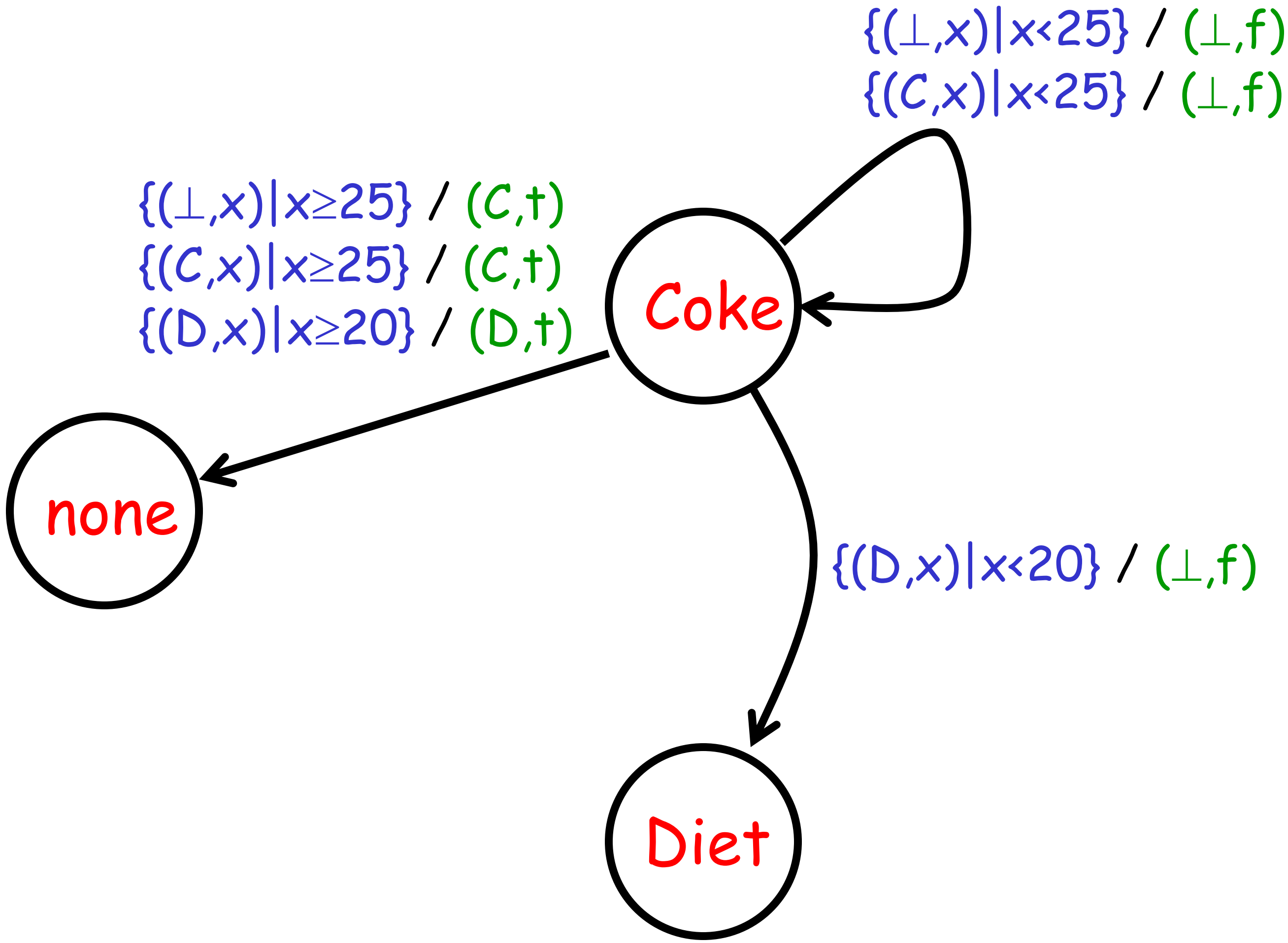
Soda Dispenser



Soda Dispenser

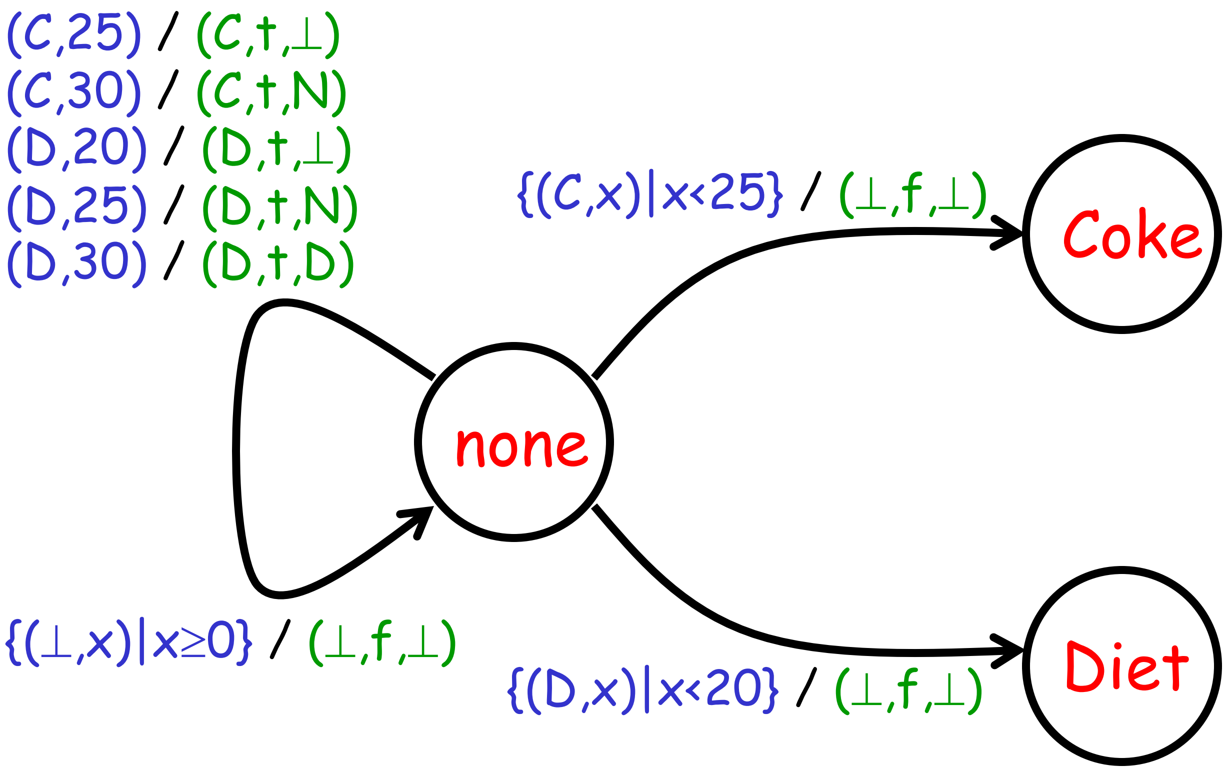


Soda Dispenser

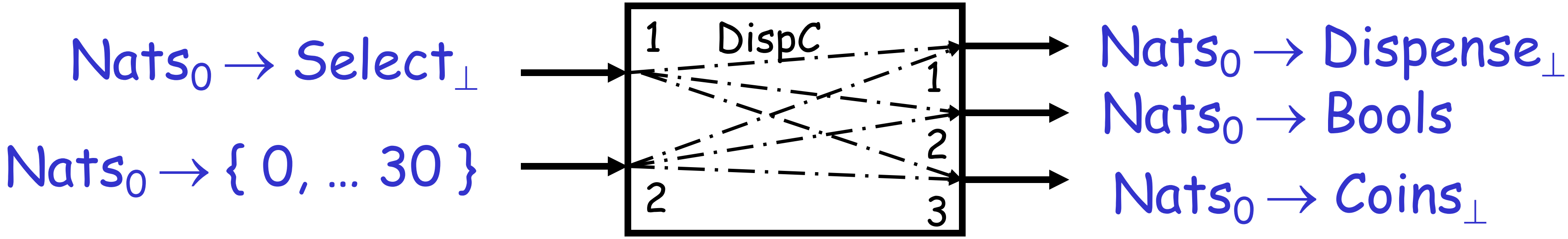


Soda Dispenser with Change

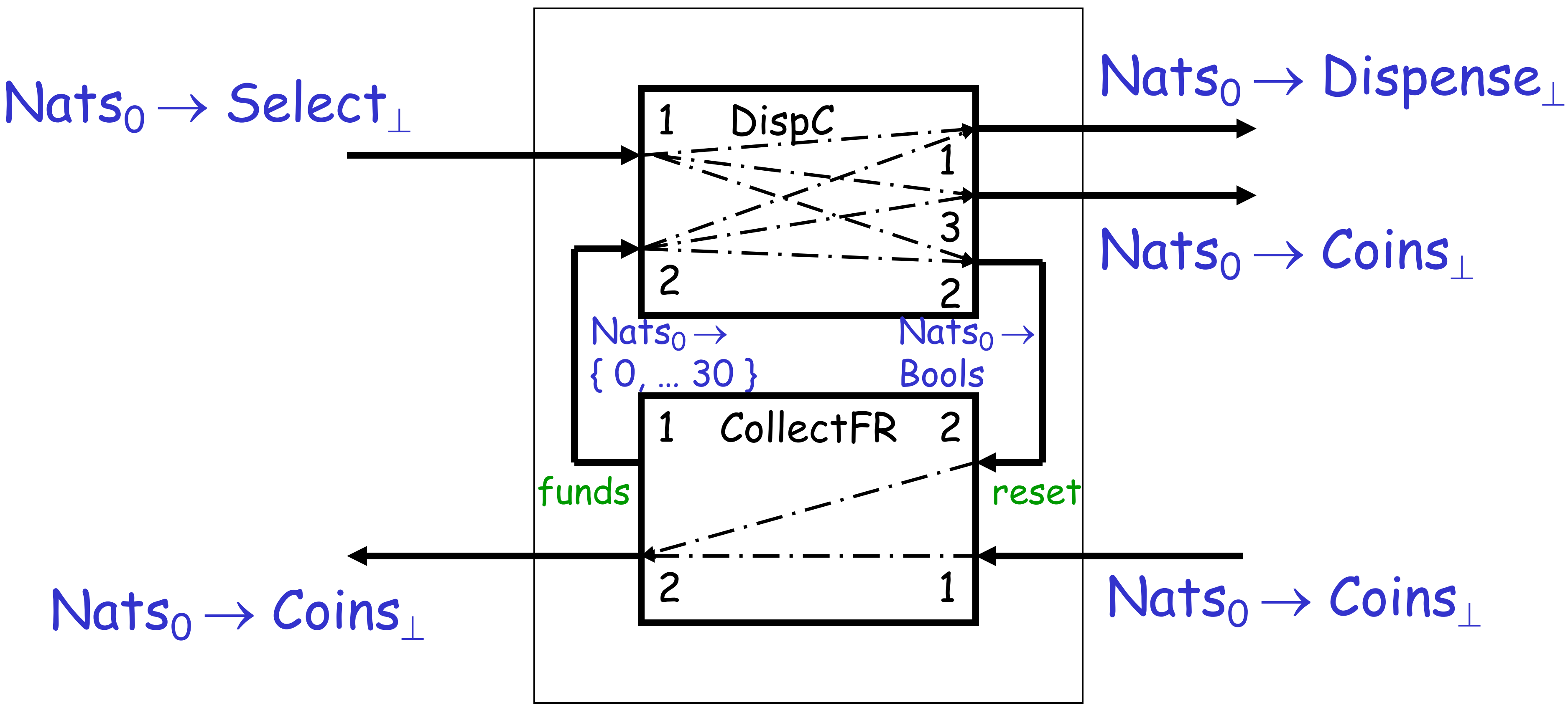
Soda Dispenser with Change



Soda Dispenser with Change



Vending Machine



State Space of Vending Machine

$$\{ 0, 5, 10, 15, 20, 25, 30 \} \times \{ \text{none}, \text{Coke}, \text{Diet} \}$$

21 states