EECS 20

Lecture 5 (January 26, 2001)

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Quiz

1. $\forall$ set x, $x \subseteq P(x)$

   false

2. $\exists$ function f, $\{ x \in \text{domain}(f) \mid x = f(x) \}$

   not well-formed

3. $\forall n \in \text{Nats}$, $n = 2 \Rightarrow (n, n+1) \in \{1, 2, 3\}^2$

   true

4. $\exists f \in [\text{Nats} \to \text{Nats}]$, $f(x) = x^2$

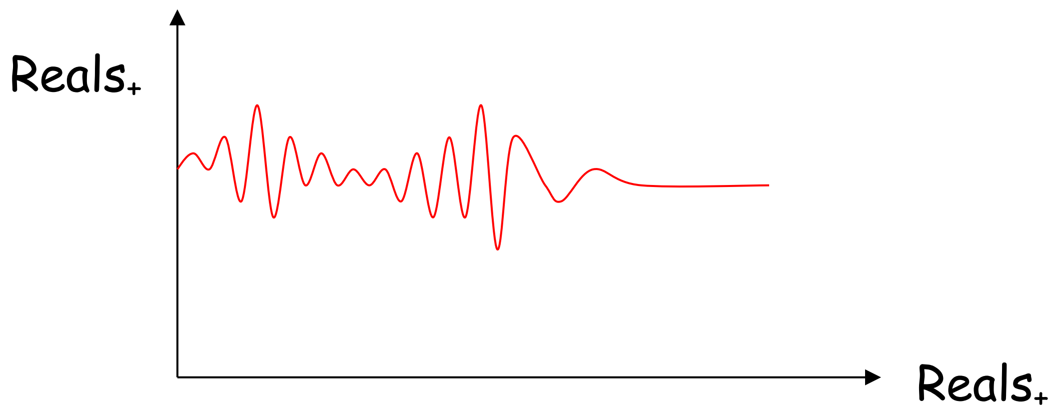
   free x

1 Systems are functions

2 Signals are functions

Audio Signals

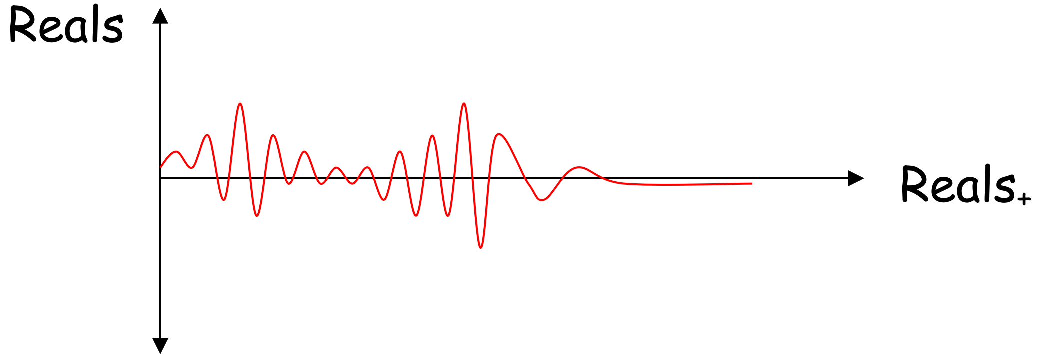
$$\text{sound} : \text{ContinuousTime} \to \text{AirPressure}$$



Let ContinuousTime = $\text{Reals}_+ = \{ x \in \text{Reals} \mid x \geq 0 \}$.

Let AirPressure = Reals_+ .

$$\text{normalizedSound} : \text{ContinuousTime} \to \text{NormalizedPressure}$$

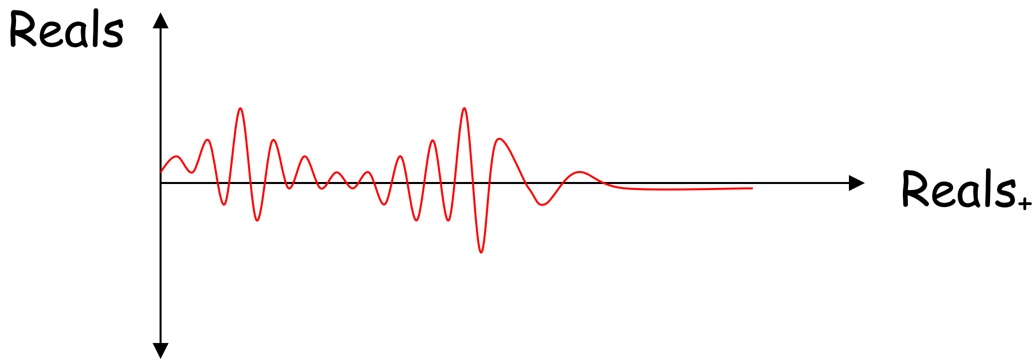


Let NormalizedPressure = Reals .

normalizedSound : ContinuousTime $\to$ NormalizedPressure

such that $\forall$ x $\in$ ContinuousTime,

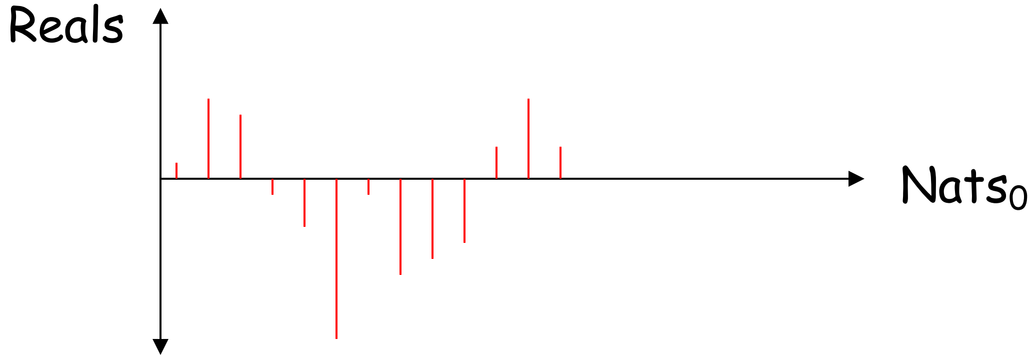
normalizedSound (x) = sound (x) - ambientAirPressure .



Let NormalizedPressure = Reals .

$$\text{sampledSound} : \text{DiscreteTime} \to \text{NormalizedPressure}$$

samplingPeriod (sec) = 1 / samplingFrequency (Hz)

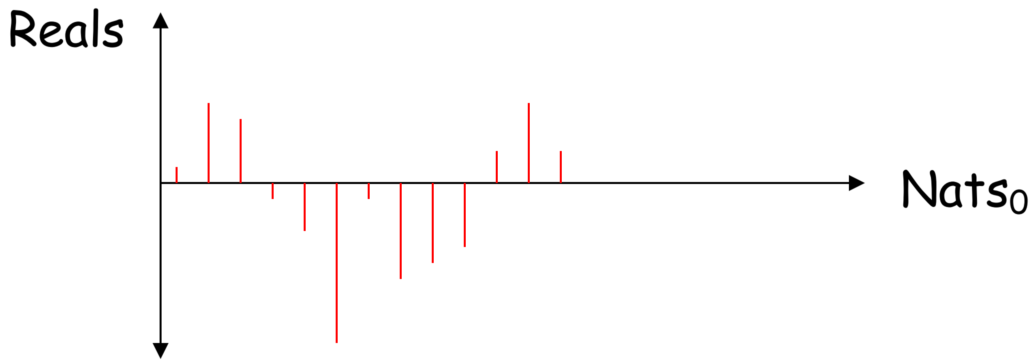


Let DiscreteTime = $Nats_0$.

$$\text{sampledSound} : \text{DiscreteTime} \to \text{NormalizedPressure}$$

such that $\forall\, x \in \text{DiscreteTime}$,

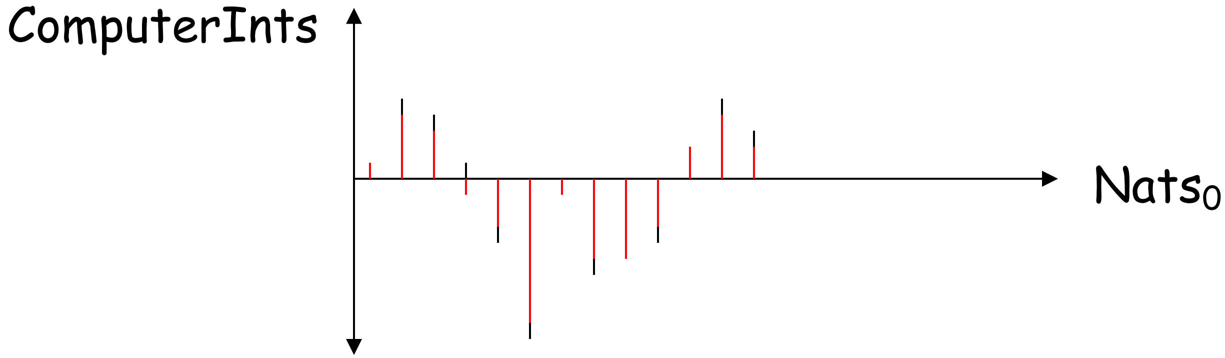
$$\text{sampledSound}(x) = \text{normalizedSound}(\text{samplingPeriod} \cdot x).$$



Let $\text{DiscreteTime} = \text{Nats}_0$.

$$\text{quantizedSound} : \text{DiscreteTime} \to \text{ComputerInts}$$

$$\text{maxint} = 2^{\text{wordsize} - 1}$$

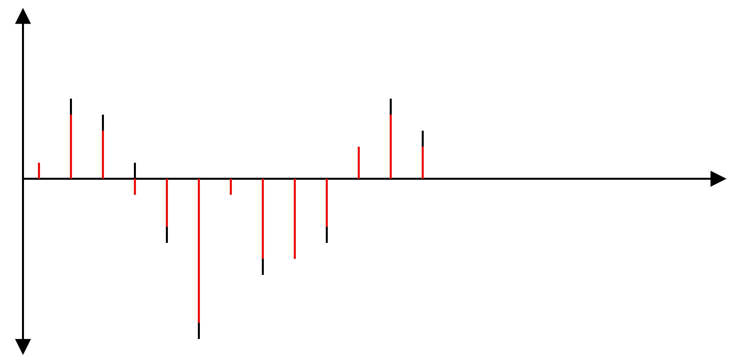


Let $\text{ComputerInts} = \{ x \in \text{Ints} \mid -\text{maxint} \leq x \leq \text{maxint} \}$.

$$\text{quantizedSound} : \text{DiscreteTime} \to \text{ComputerInts}$$

such that $\forall\, x \in \text{DiscreteTime}$,

$$\text{quantizedSound}(x) = \text{trunc}(\lfloor \text{sampledSound}(x) \rfloor, \text{maxint}).$$



$$\text{Let ComputerInts} = \{ x \in \text{Ints} \mid -\text{maxint} \le x \le \text{maxint} \}.$$

$\lfloor\ \rfloor$: Reals $\to$ Ints

such that $\forall$ x $\in$ Reals, $\lfloor x \rfloor = \max \{ y \in \text{Ints} \mid y \leq x \}$.

trunc : Ints $\times$ ComputerInts $\to$ ComputerInts

such that $\forall$ x $\in$ Ints, $\forall$ y $\in$ ComputerInts,

$$\text{trunc}(x,y) = \begin{cases} x & \text{if } -y \leq x \leq y \\ y & \text{if } x > y \\ -y & \text{if } x < -y\,. \end{cases}$$

$$\forall\, x \subseteq \text{Reals},\quad \forall\, y \in \text{Reals},$$

$$\text{let}\quad \max x = y \iff y \in x \;\wedge\; (\forall\, z \in x,\; z \le y)\,.$$

$$\forall x \in \text{Ints},\; \forall\, y \in \text{ComputerInts},$$

$$(\,-y \le x \le y \;\Rightarrow\; \text{trunc}\,(x,y) = x\,) \;\wedge$$

$$(\,x > y \qquad\quad \Rightarrow\; \text{trunc}\,(x,y) = y\,) \;\wedge$$

$$(\,x < -y \qquad \Rightarrow\; \text{trunc}\,(x,y) = -y\,)\,.$$

$sound: Reals_+ \to Reals$ analog signal

$quantizedSound: Nats_0 \to Ints$ digital signal

Video Signals

$$\text{movie}: \ \text{DiscreteTime} \to \text{Frames}$$

(typical frequency = 30 Hz)

$$\begin{aligned}\text{AnalogFrames} = [\ &\text{DiscreteVerticalSpace} \times \\ &\text{HorizontalSpace} \to \\ &\text{Intensity}\ ]\end{aligned}$$

$$\begin{aligned}\text{DigitalFrames} = [\ &\text{DiscreteVerticalSpace} \times \\ &\text{DiscreteHorizontalSpace} \to \\ &\text{DiscreteIntensity}\ ]\end{aligned}$$

Sheet of paper : VerticalSpace = [0, 11]

HorizontalSpace = [0, 8.5]

TV : DiscreteVerticalSpace = { 1, 2, … , 525 }

LCD : DiscreteVerticalSpace = { 1, 2, … , 1024 }

DiscreteHorizontalSpace = { 1, 2, … , 1280 }

DiscreteIntensity = ComputerInts

ColorIntensity = Intensity^3

Currying

For all sets A, B, C,

$$[\, A \times B \to C \,] = [\, A \to [\, B \to C \,]\,].$$

Frames =

$$[\, \mathrm{VerticalSpace} \times \mathrm{HorizontalSpace} \to \mathrm{Intensity} \,] =$$

$$[\, \mathrm{VerticalSpace} \to [\, \mathrm{HorizontalSpace} \to \mathrm{Intensity} \,]\,]$$

Currying

For all sets A, B, C,

$$[\, A \times B \to C \,] = [\, A \to [\, B \to C \,]\,].$$

movie $\in$

$$[\, DiscreteTime \to [\, VSpace \to [\, HSpace \to Intensity \,]]] =$$

$$[\, DiscreteTime \times VSpace \times HSpace \to Intensity \,]$$

More Signals

$\text{position} : \text{Time} \to \text{Space}$

$$
\begin{aligned}
\text{ContinuousTime} &= \text{Reals}_+ \, . \\
\text{DiscreteTime} &= \text{Nats}_0 \, . \\
\text{DiscTwoSpace} &= \text{Ints}^2 \, . \\
\text{ContThreeSpace} &= \text{Reals}^3 \, .
\end{aligned}
$$

More Signals

$$\text{position} : \text{Time} \to \text{Space}$$

$$\begin{aligned}
\text{ContinuousTime} &= \text{Reals}_{+} . \\
\text{DiscreteTime} &= \text{Nats}_{0} . \\
\text{DiscTwoSpace} &= \text{Ints}^{2} . \\
\text{ContThreeSpace} &= \text{Reals}^{3} .
\end{aligned}$$

$$\text{velocity} : \text{Time} \to \text{DerivativeSpace}$$

$$\text{DerivativeSpace} = \text{Space} .$$

More Signals

$\text{position} : \text{Time} \to \text{Space}$

$$\begin{aligned}
\text{ContinuousTime} &= \text{Reals}_+ \,. \\
\text{DiscreteTime} &= \text{Nats}_0 \,. \\
\text{DiscTwoSpace} &= \text{Ints}^2 \,. \\
\text{ContThreeSpace} &= \text{Reals}^3 \,.
\end{aligned}$$

$\text{velocity} : \text{Time} \to \text{DerivativeSpace}$

$$\text{DerivativeSpace} = \text{Space} \,.$$

$\text{positionVelocity}: \text{Time} \to \text{Space} \times \text{DerivativeSpace}$

such that $\forall\, x \in \text{Time}$,
$\text{positionVelocity}(x) = (\text{position}(x), \text{velocity}(x))$.