Safety Control

EECS 20

Lecture 36 (April 23, 2001)

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Revised by Matteo Zavatteri and Tiziano Villa, Fall 2020

The Control Problem

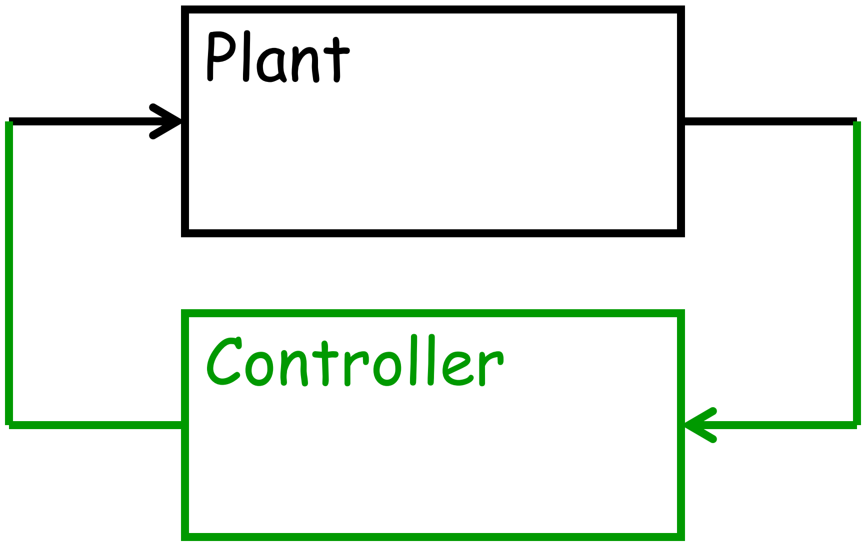
Given

1.

2. Objective

The Control Problem

Find



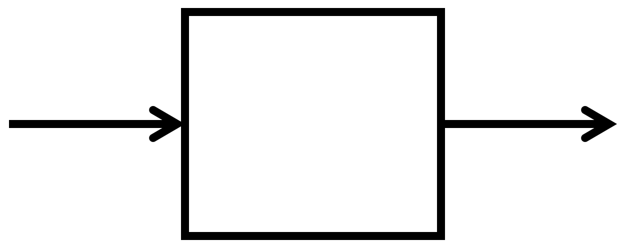
such that the composite ("closed-loop") system satisfies the Objective

Simple Control Problems

1. LTI Plant
2. Finite-State Plant

Even Simple Linear Systems are Not Finite-State

$x: Nats_0 \to Reals$

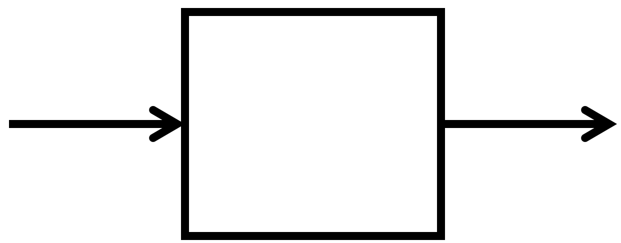


$y: Nats_0 \to Reals$

$$\forall\ z \in Nats_0,\ \ y(z) = \begin{cases} 0 & \text{if } z=0 \\ \frac{1}{2} \cdot (x(z-1) + x(z)) & \text{if } z>0 \end{cases}$$

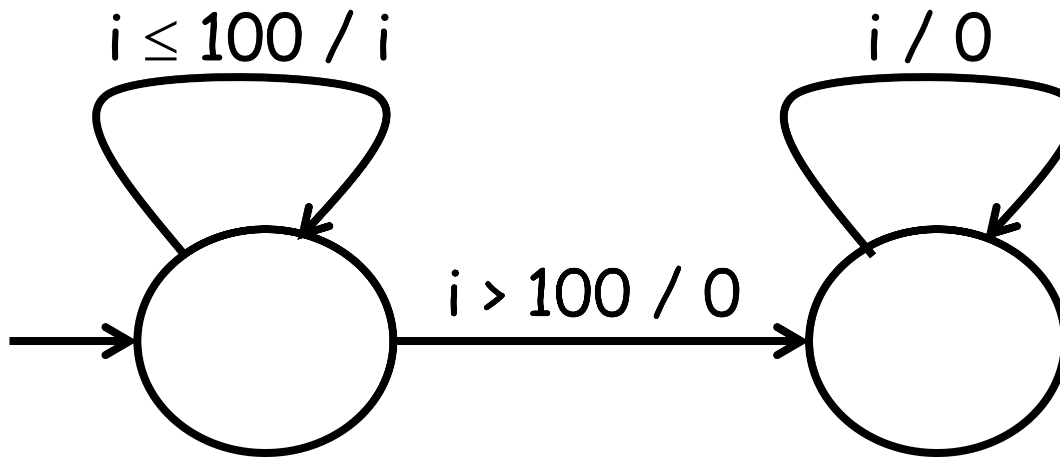
Even Simple Finite-State Systems are Not Linear

$x: \text{Nats}_0 \to \text{Reals}$



$y: \text{Nats}_0 \to \text{Reals}$

$$\forall\, z \in \text{Nats}_0,\ \ y(z) = \begin{cases} x(z) & \text{if } \forall\, z' \le z,\ \ x(z') \le 100 \\ 0 & \text{if } \exists\, z' \le z,\ \ x(z') > 100 \end{cases}$$



("i" stands for any input value)

Simplest Finite-State Control Objective:

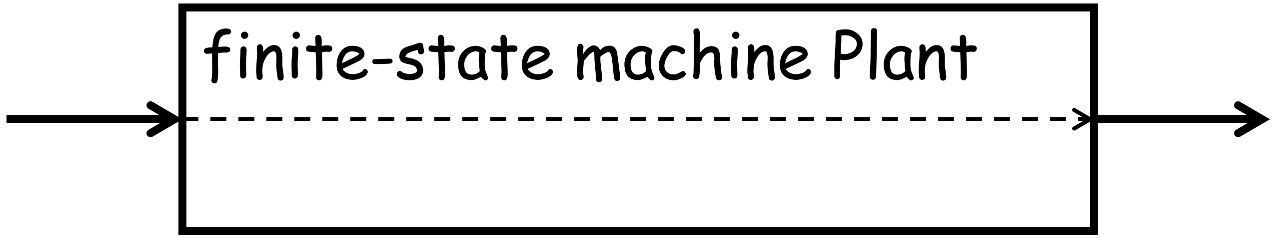
SAFETY

stay out of a set of undesirable plant states
(the “error” states)

The Finite-State Safety Control Problem

Given

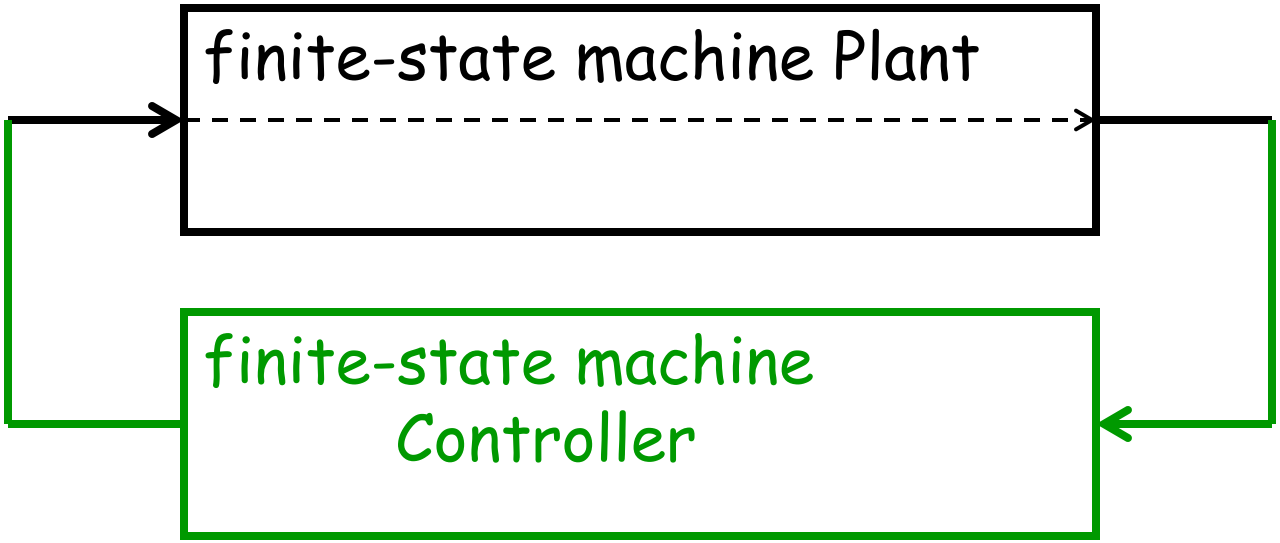
1.



2. set Error of states of Plant

The Finite-State Safety Control Problem

Find



such that the composite system never enters a state in Error

Step 1:

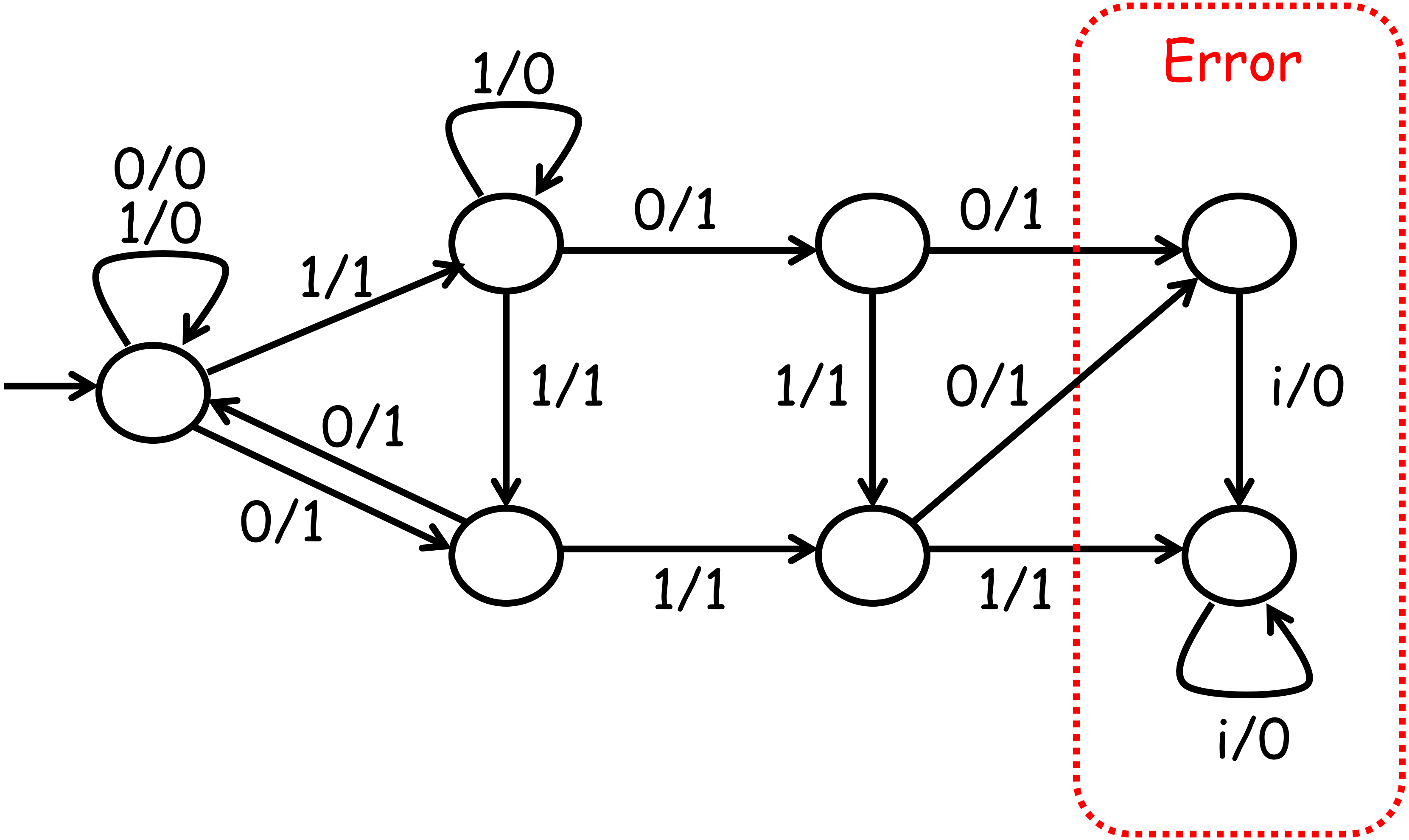
Compute the "uncontrollable" states of Plant

1. Every state in Error is uncontrollable.
2. For all states s,

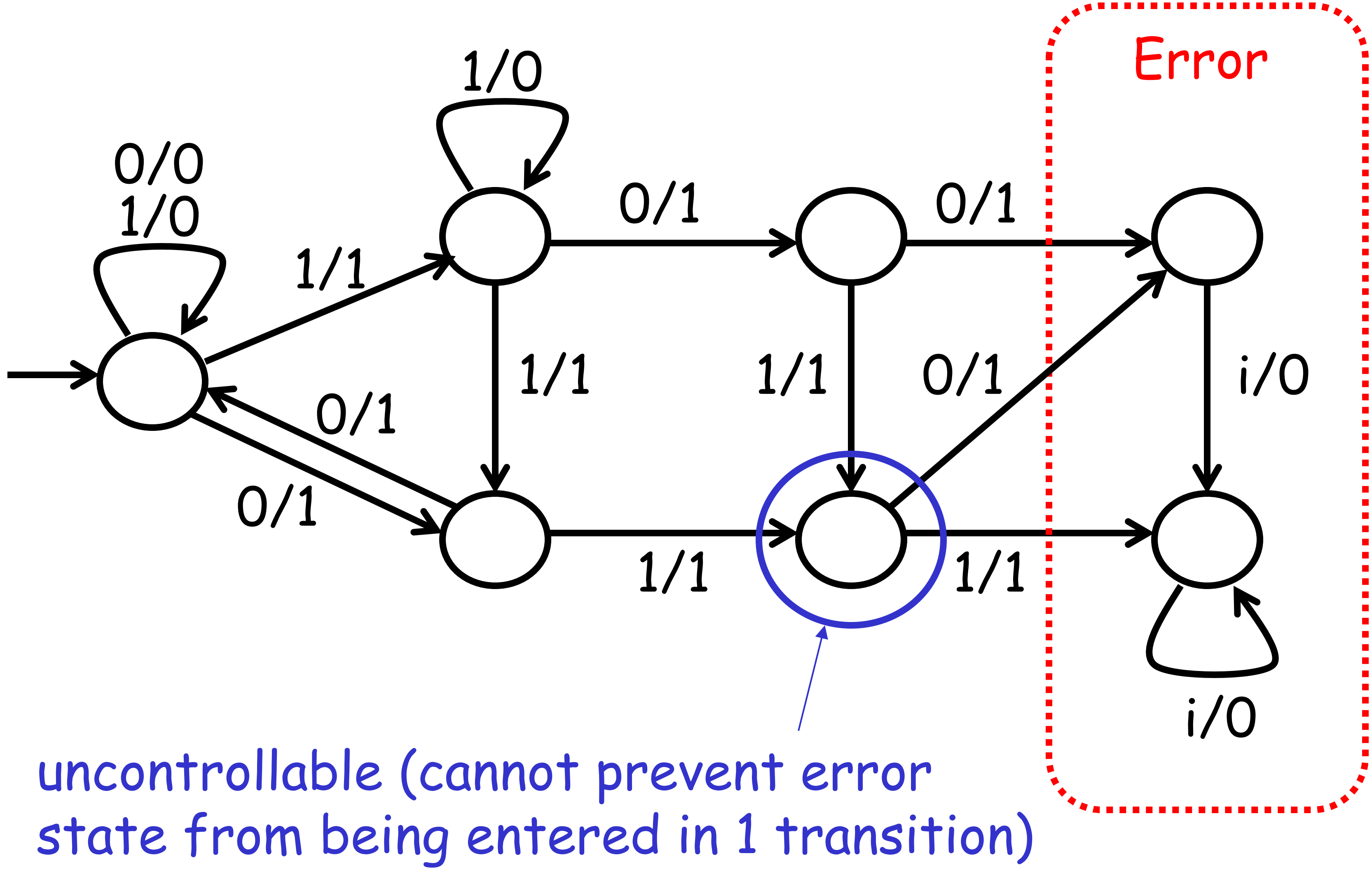
   if for all inputs i
   there exist an uncontrollable state s'
   and an output o
   such that (s',o) ∈ possibleUpdates (s,i)

   then s is uncontrollable.

Plant

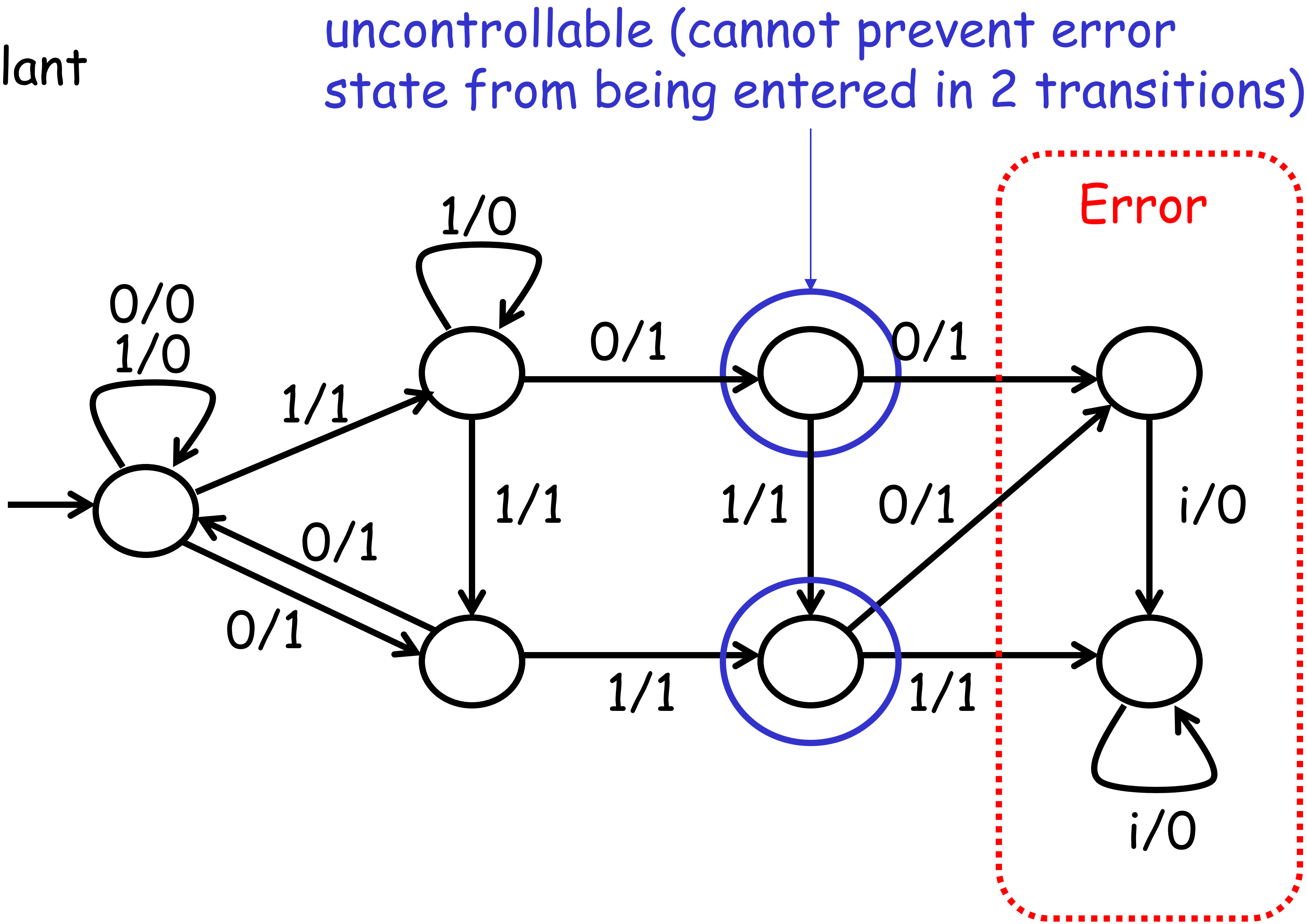


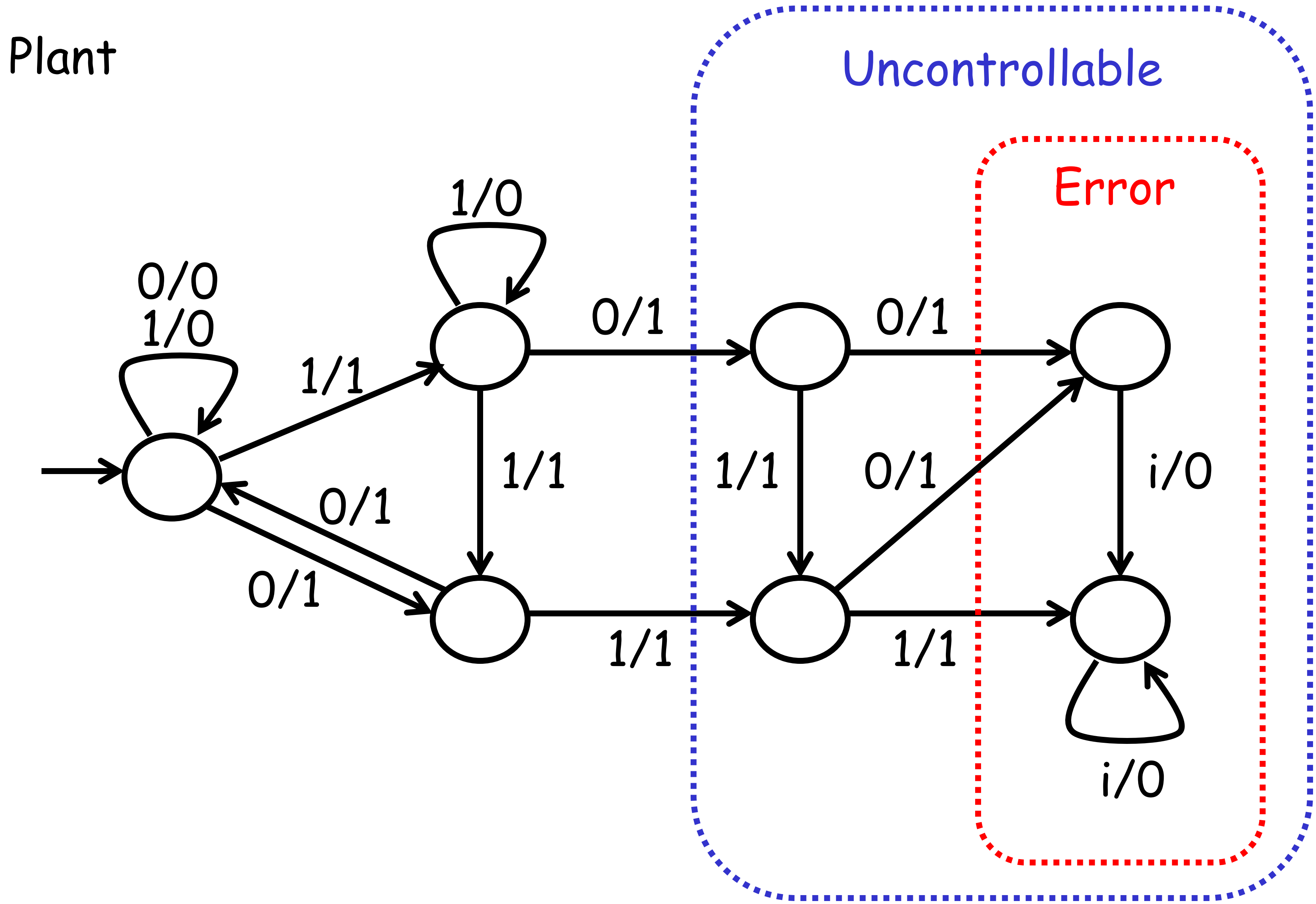
Plant

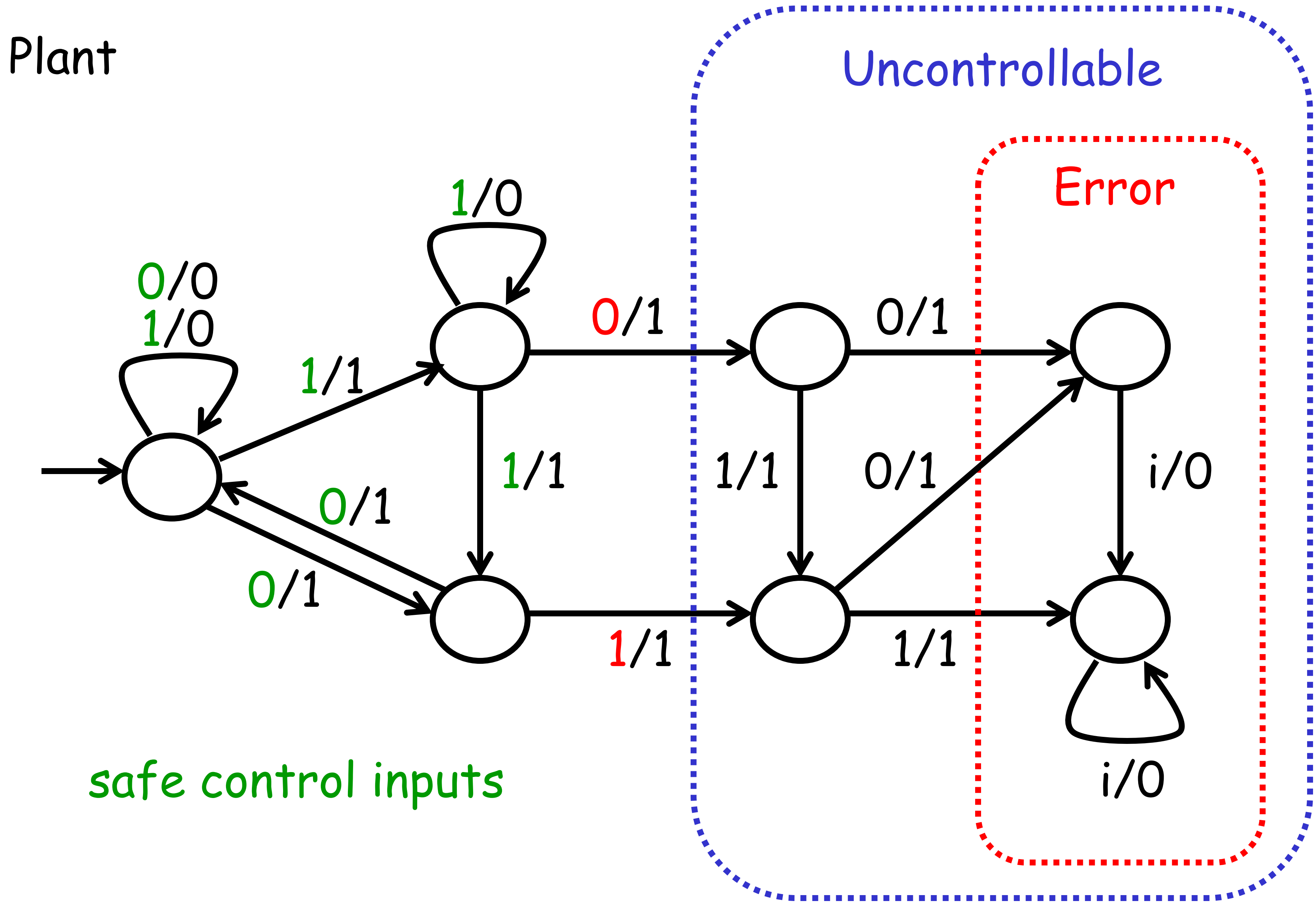


Error
0/0
1/0
1/0
1/1
0/1
0/1
0/1
0/1
1/1
1/1
0/1
i/0
1/1
1/1
i/0
uncontrollable (cannot prevent error state from being entered in 1 transition)

Plant


uncontrollable (cannot prevent error state from being entered in 2 transitions)
Error
0/0
1/0
1/0
1/1
0/1
0/1
0/1
1/1
1/1
1/1
0/1
0/1
1/1
i/0
i/0


Plant
Uncontrollable
Error
1/0
0/0
1/0
1/1
0/1
0/1
1/1
0/1
0/1
1/1
0/1
i/0
1/1
1/1
i/0

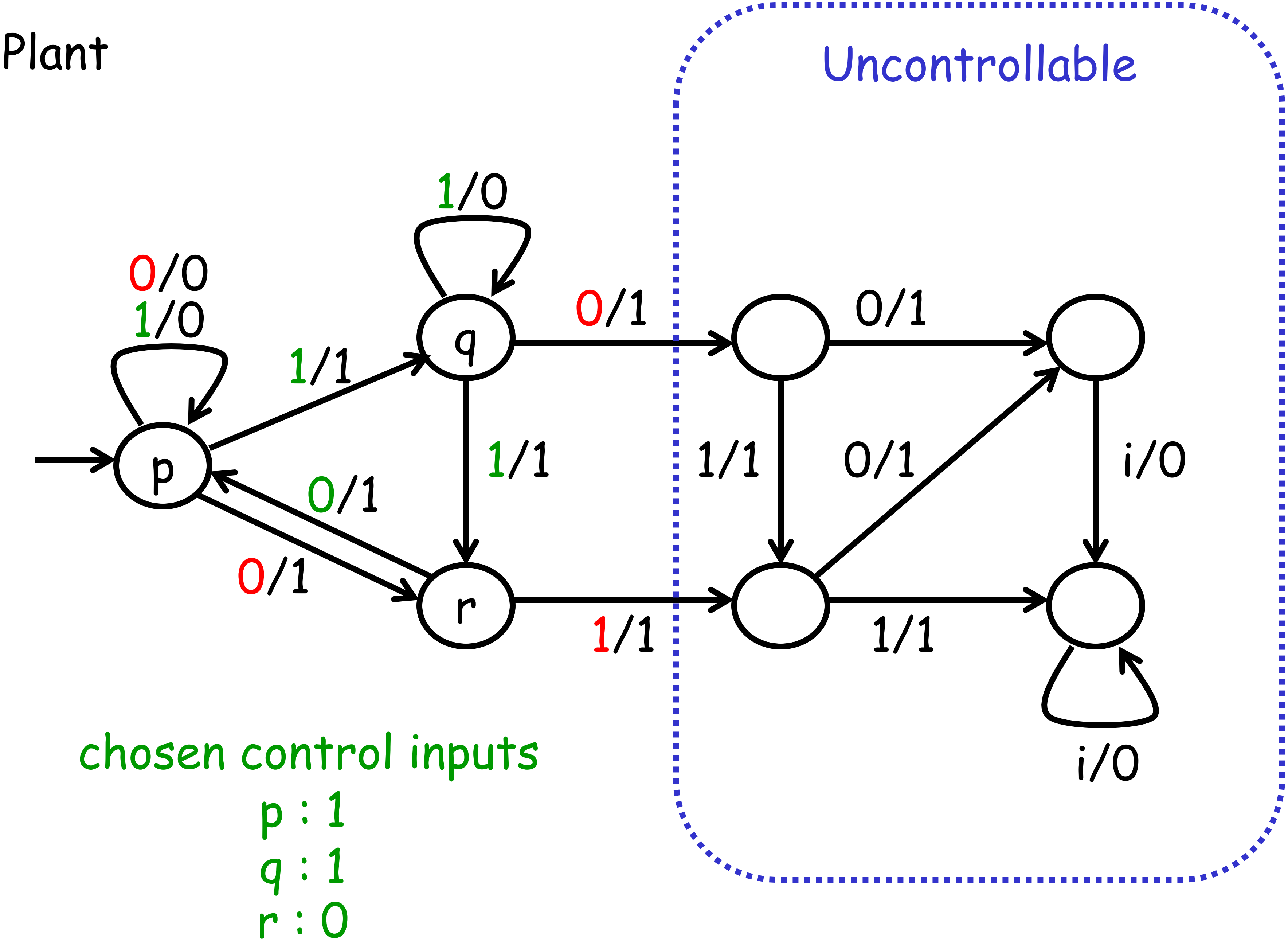


Plant
Uncontrollable
Error
0/0
1/0
1/0
1/1
0/1
0/1
0/1
1/1
1/1
0/1
1/1
0/1
1/1
i/0
i/0
safe control inputs

Step 2:

Design the Controller

1. For each controllable state s of the plant, choose one input i so that possibleUpdates (s,i) contains only controllable states.



Plant
Uncontrollable
0/0
1/0
1/0
1/1
0/1
0/1
1/1
0/1
1/1
0/1
1/1
0/1
i/0
1/1
i/0
p
q
r
chosen control inputs
p : 1
q : 1
r : 0

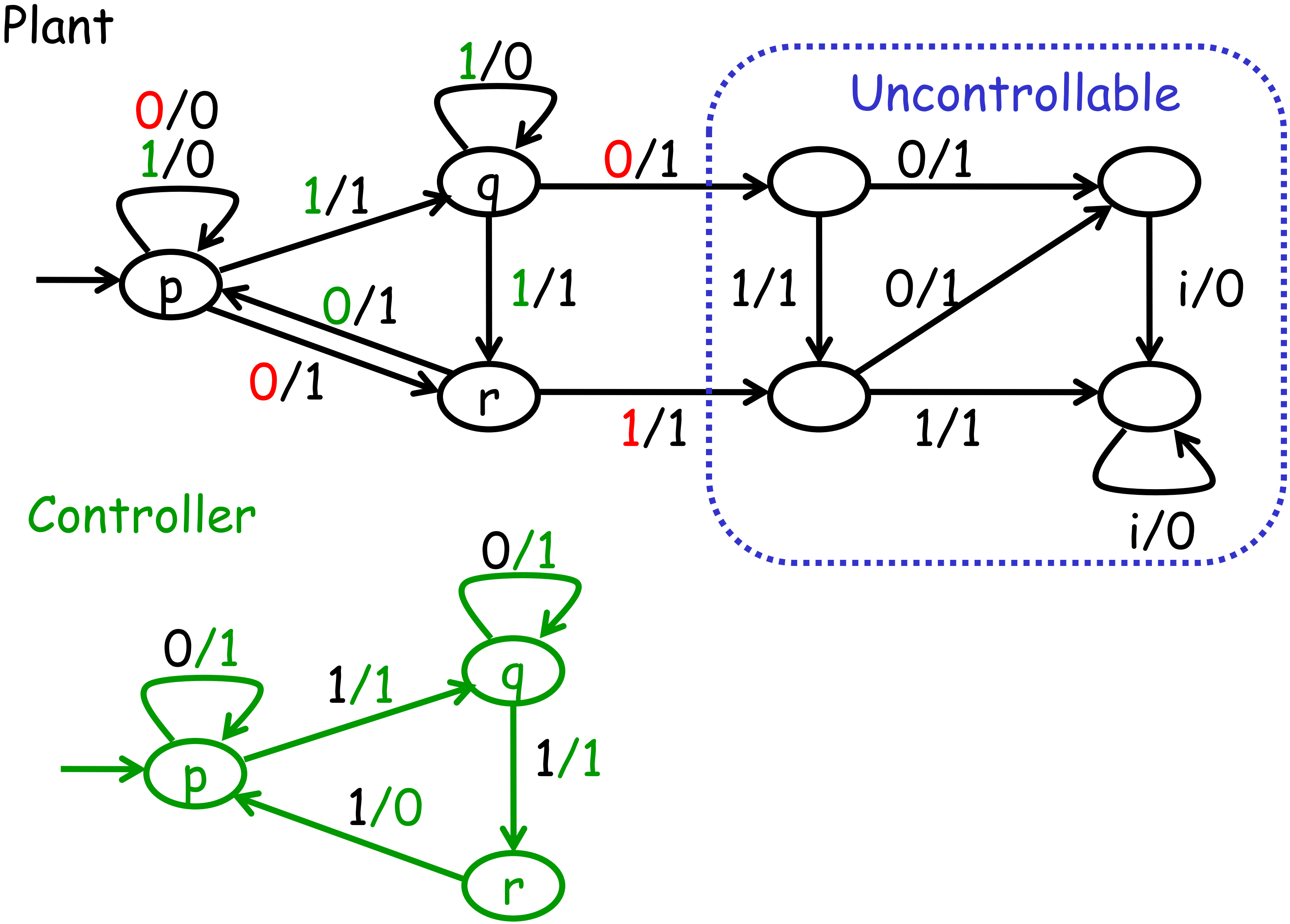
Step 2:

Design the Controller

1. For each controllable state s of the plant, choose one input i so that possibleUpdates (s,i) contains only controllable states.

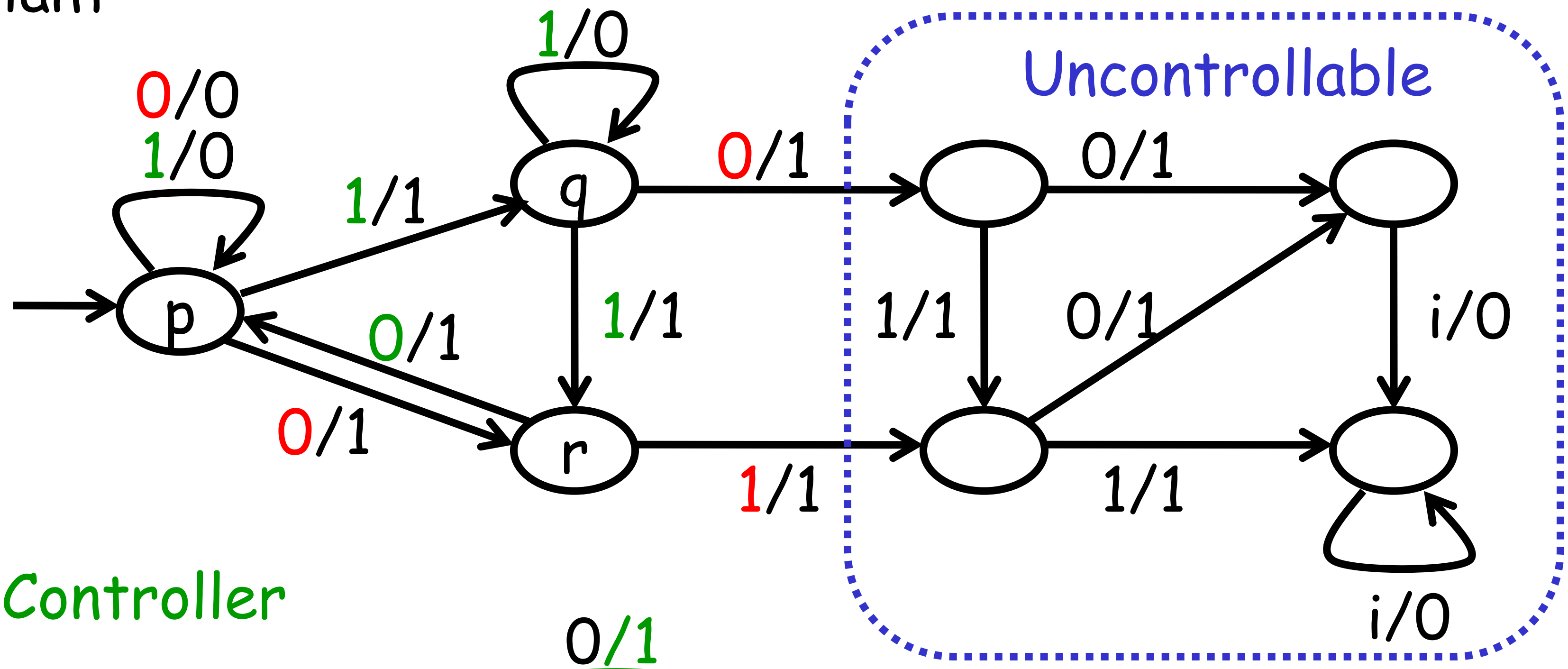
2. Have the Controller keep track of the state of the Plant:

   If Plant is output-deterministic,
   then Controller looks exactly like the controllable part of Plant, with inputs and outputs swapped.

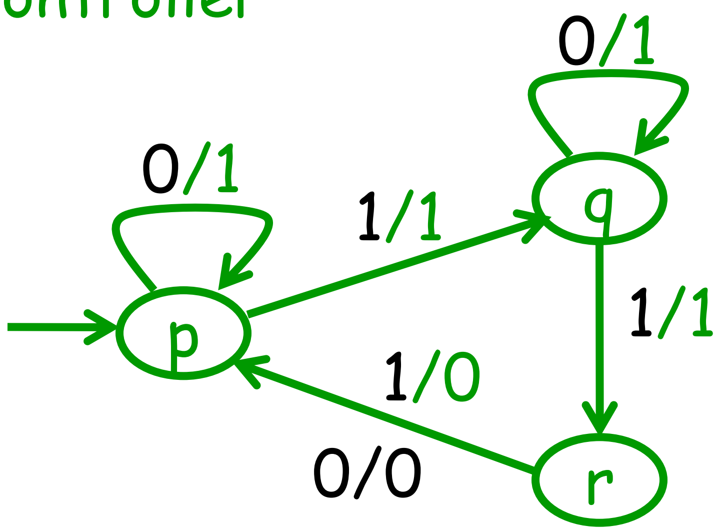


Plant
Uncontrollable
0/0
1/0
1/1
1/0
0/1
1/1
0/1
0/1
0/1
1/1
1/1
0/1
i/0
1/1
1/1
i/0
Controller
0/1
0/1
1/1
1/1
1/0
p
q
r
p
q
r

Plant



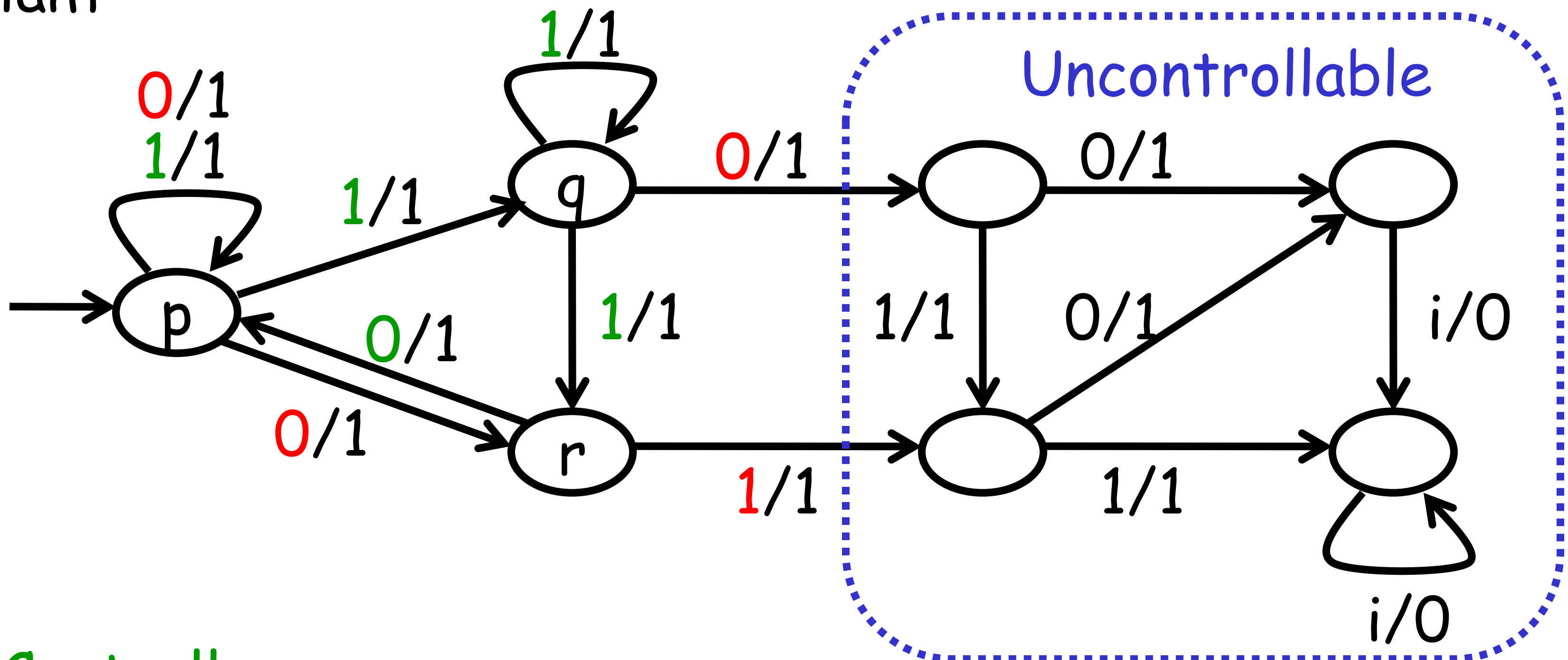
Controller



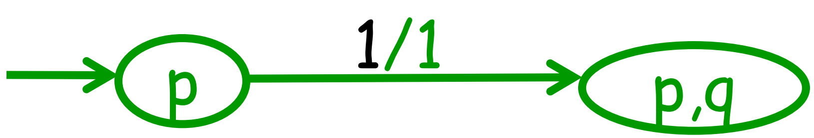
(the Controller can be made receptive in any way)

What if the Plant is not output-deterministic?

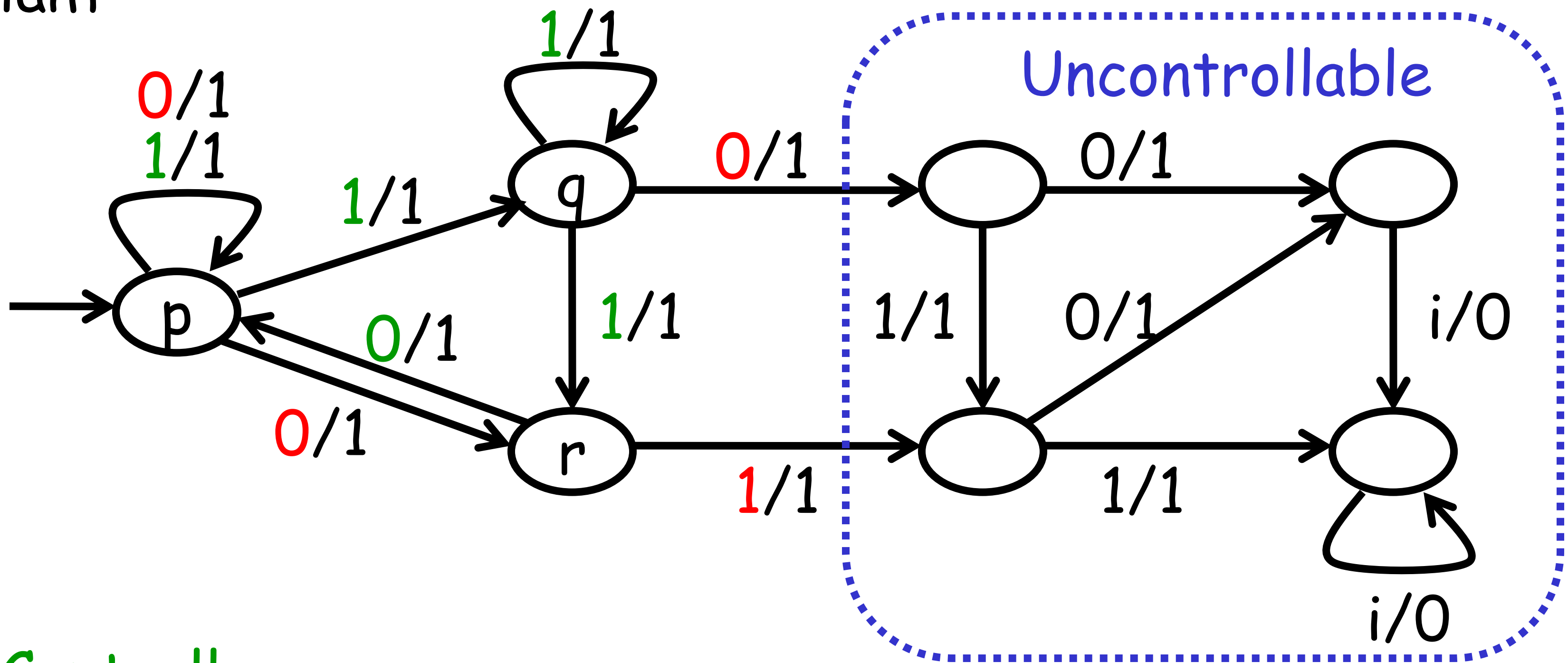
Plant



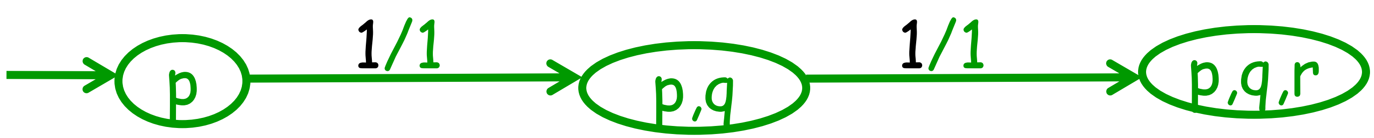
Controller



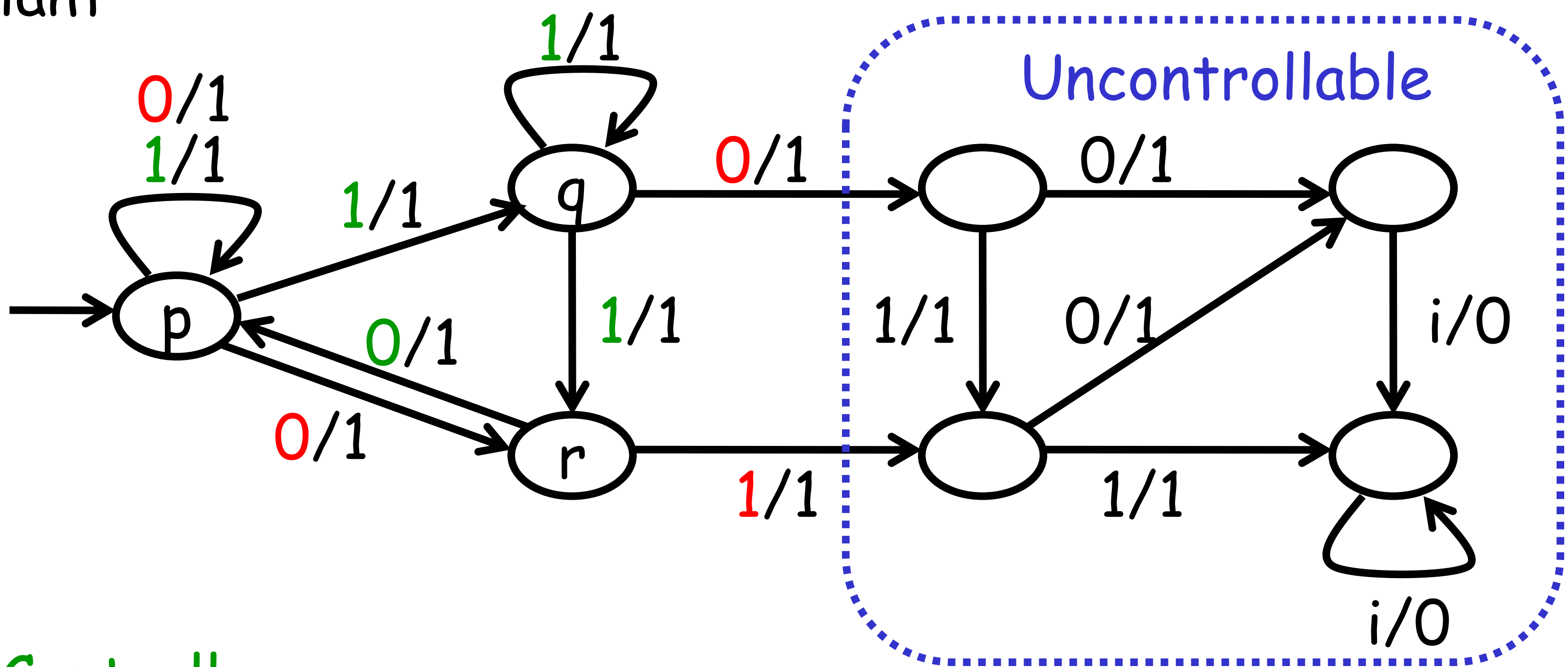
Plant



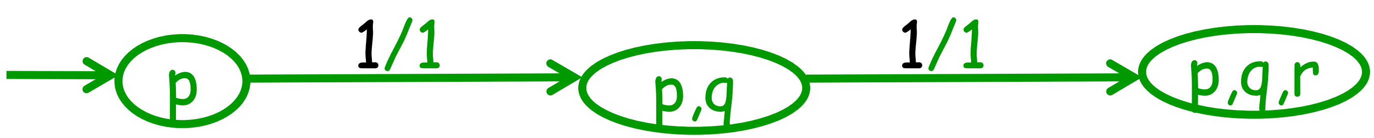
Controller



Plant

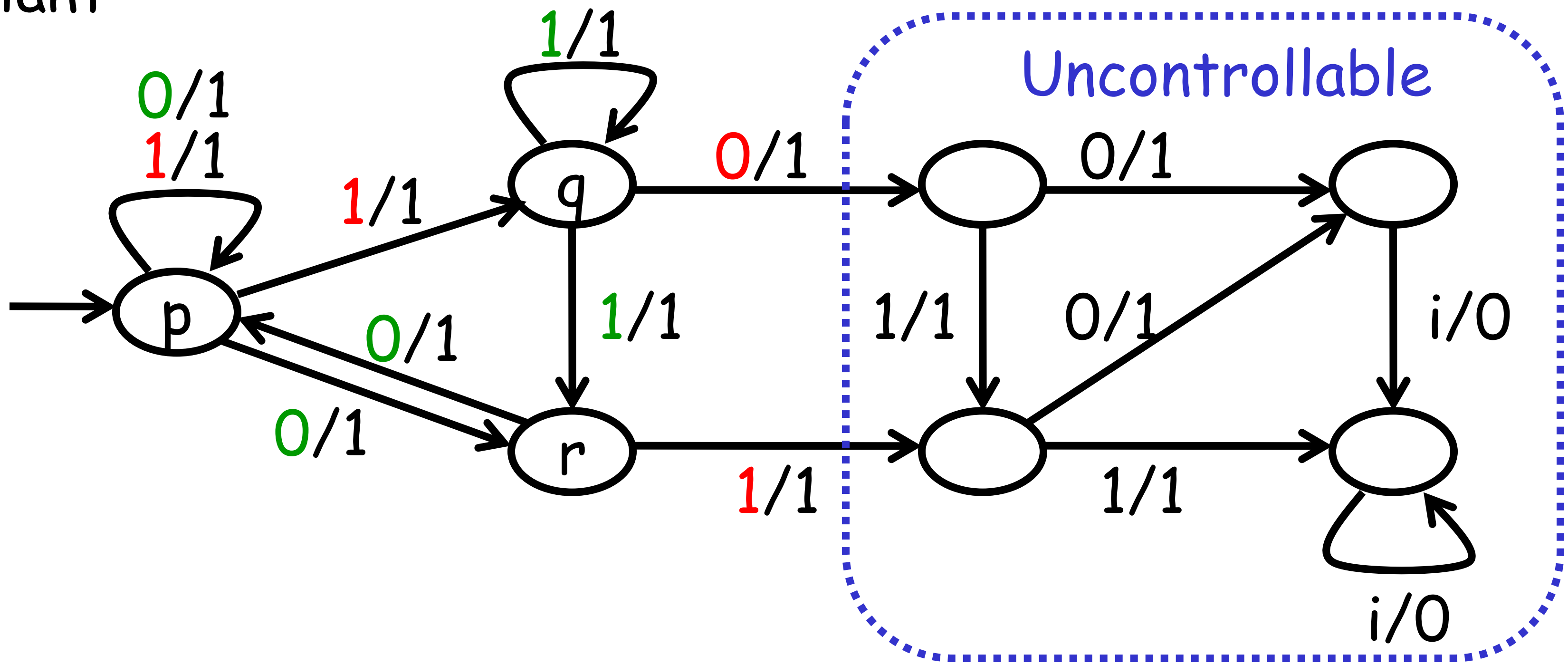


Controller

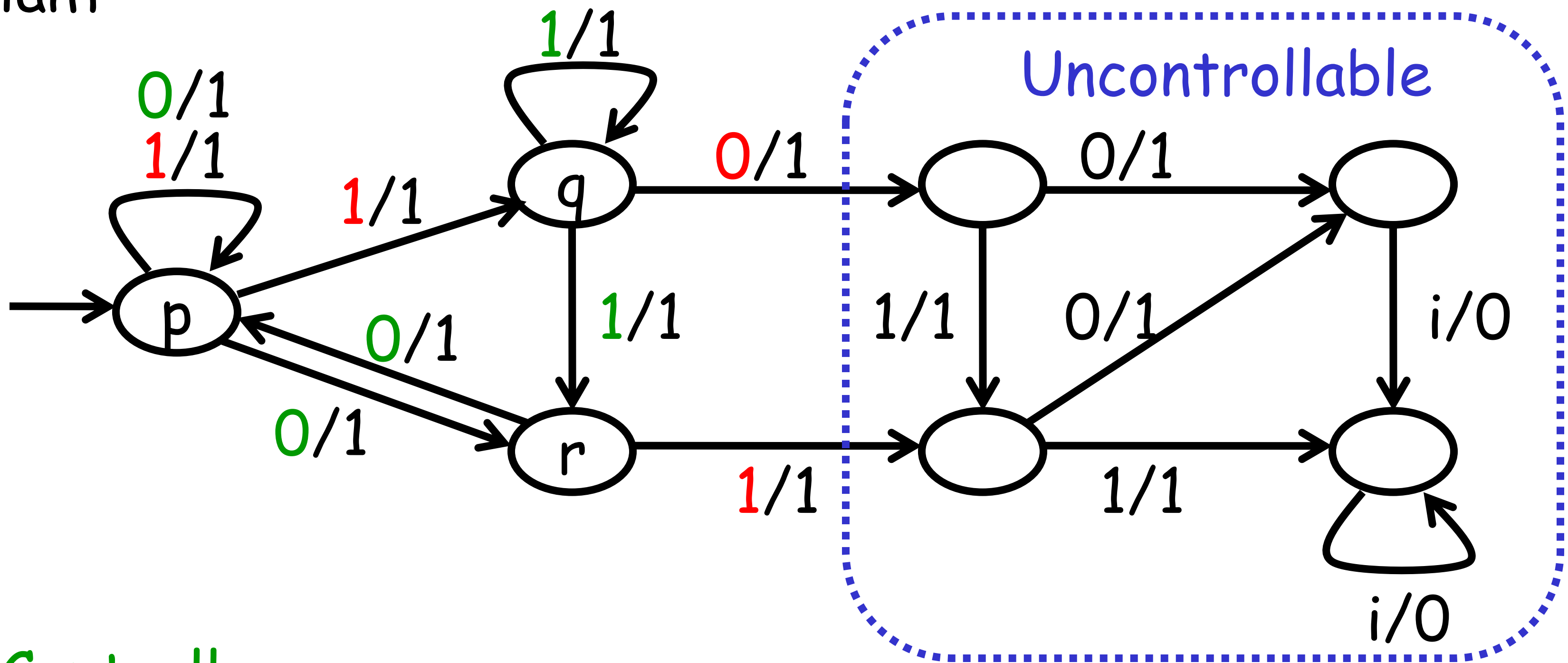


Neither 0 nor 1 is safe !

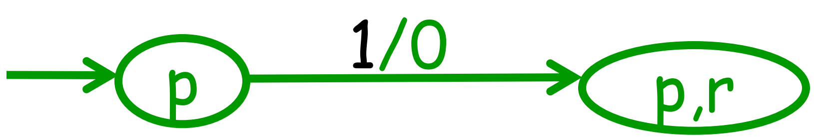
Plant



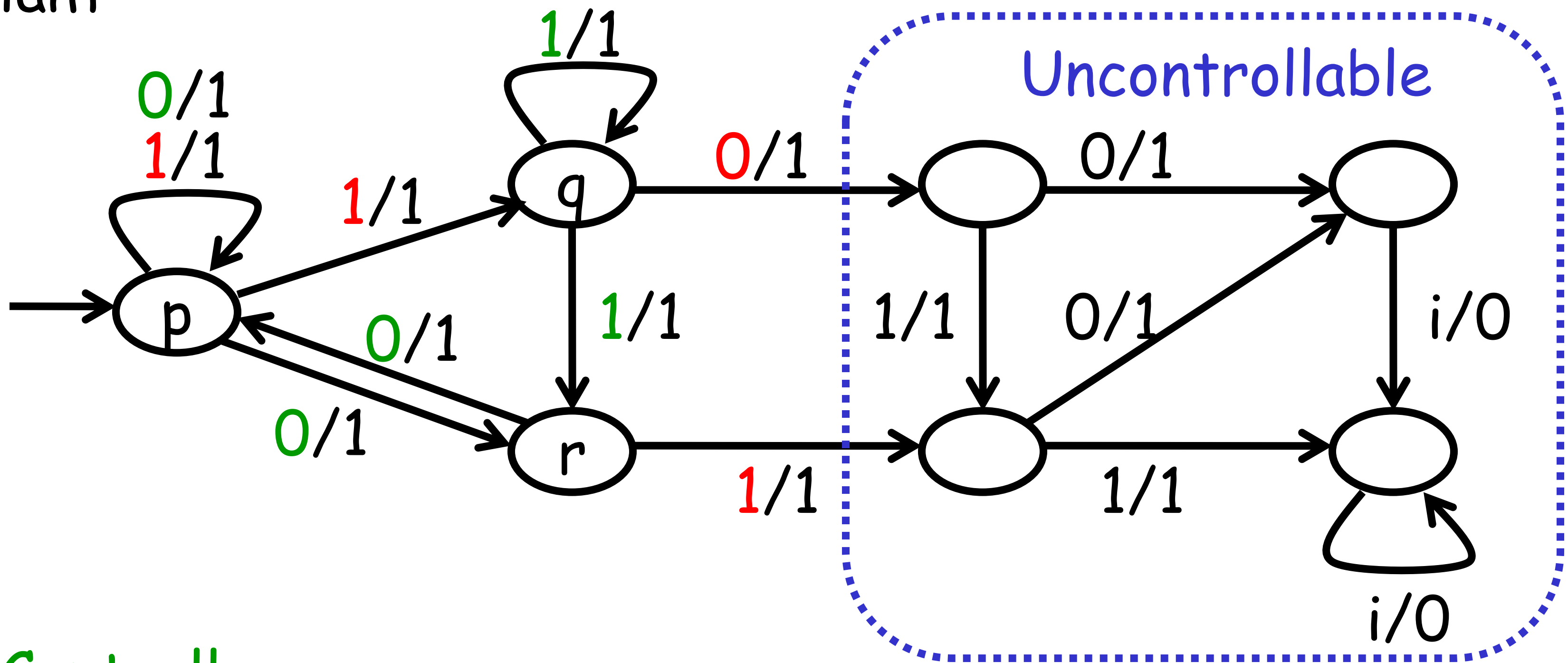
Plant



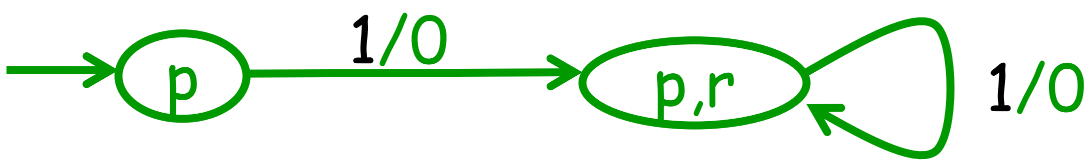
Controller



Plant



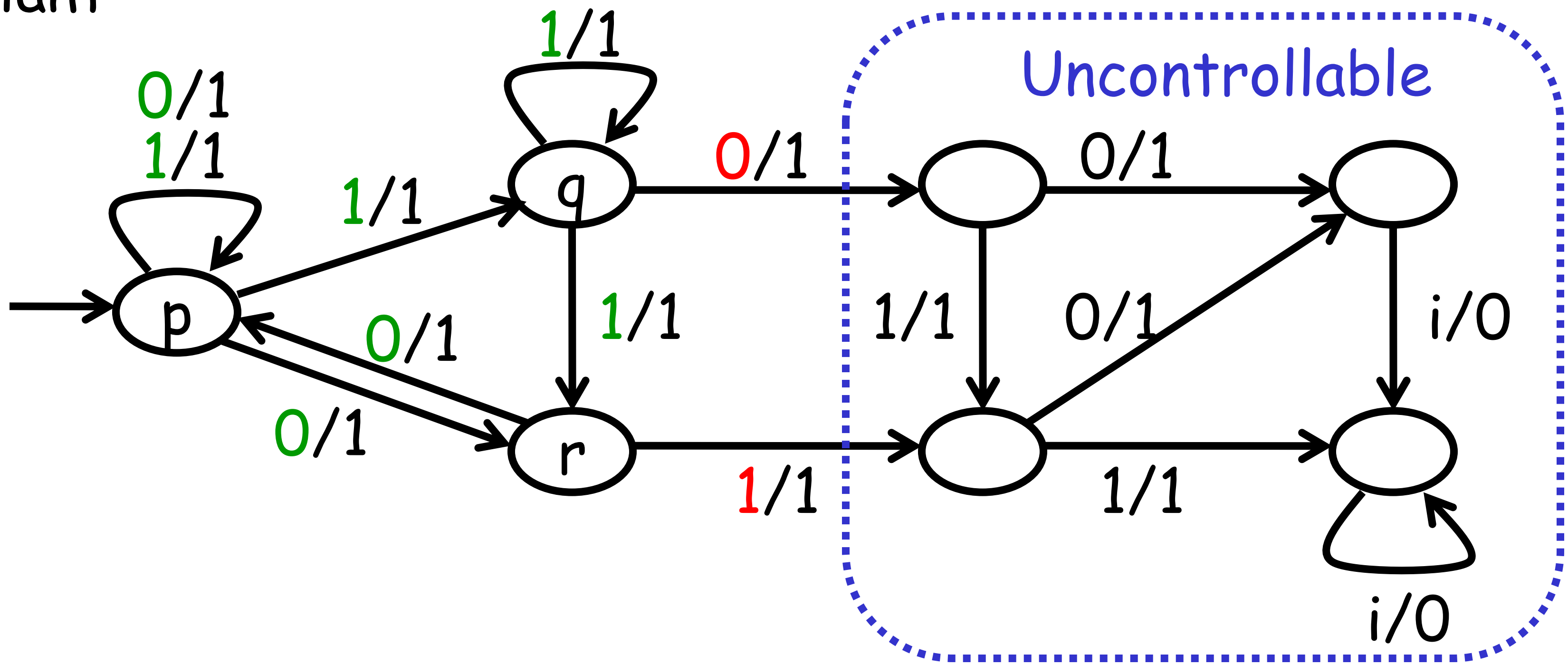
Controller



Step 2: Design the Controller

1. Let Controllable be the controllable states of the Plant. A subset $S \subseteq$ Controllable is consistent if there is an input i such that for all states $s \in S$, all states in possibleUpdates (s,i) are controllable.
2. Let M be the state machine whose states are the consistent subsets of Controllable. Prune from M the states that have no successor, until no more states can be pruned.
3. If the result contains possibleInitialStates (of the plant) as a state, then it is the desired Controller. Otherwise, no controller exists.

Plant

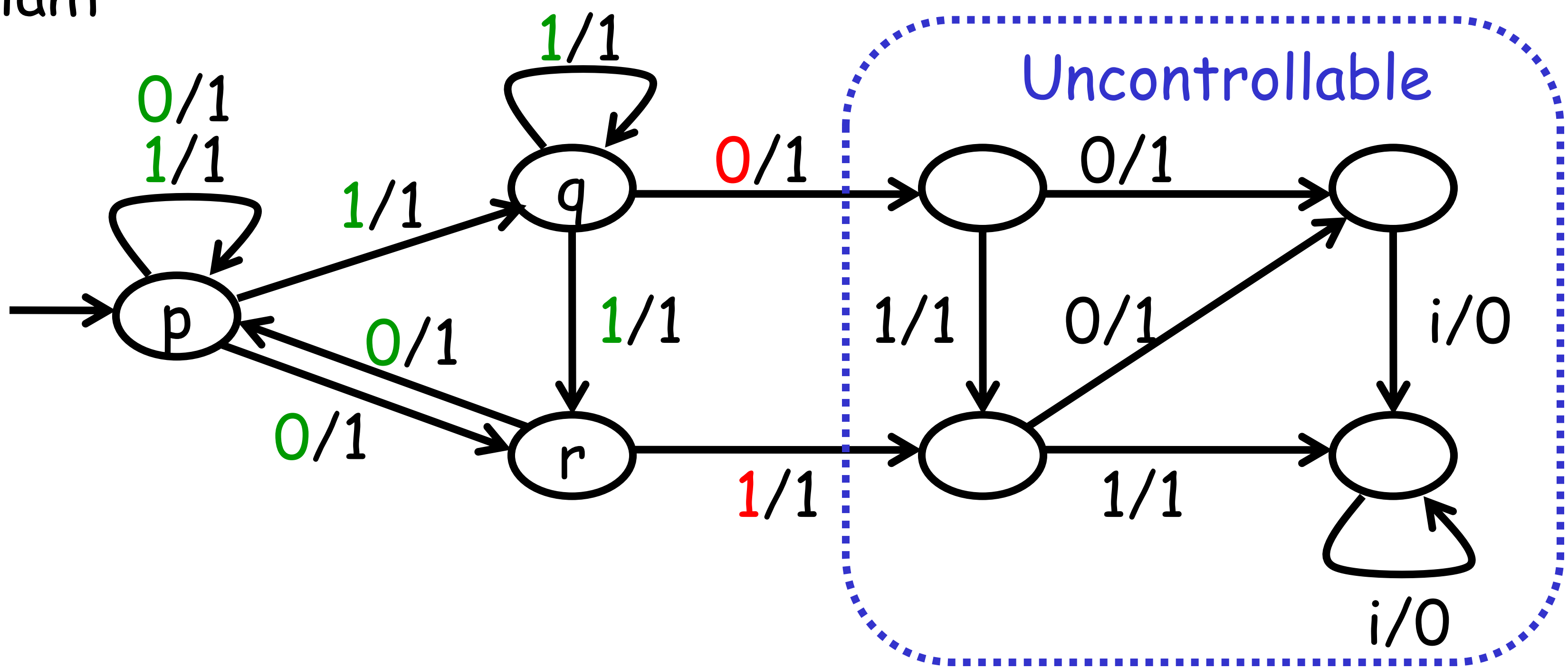


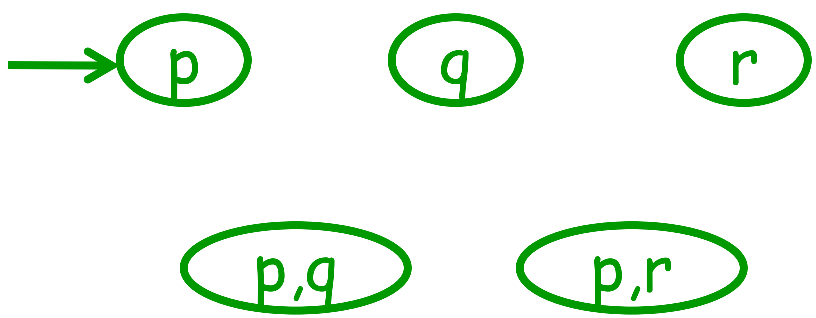
Consistent subsets

{p} : 0, 1
{q} : 1
{r} : 0

{p,q} : 1
{p,r} : 0
{q,r}, {p,q,r} not consistent

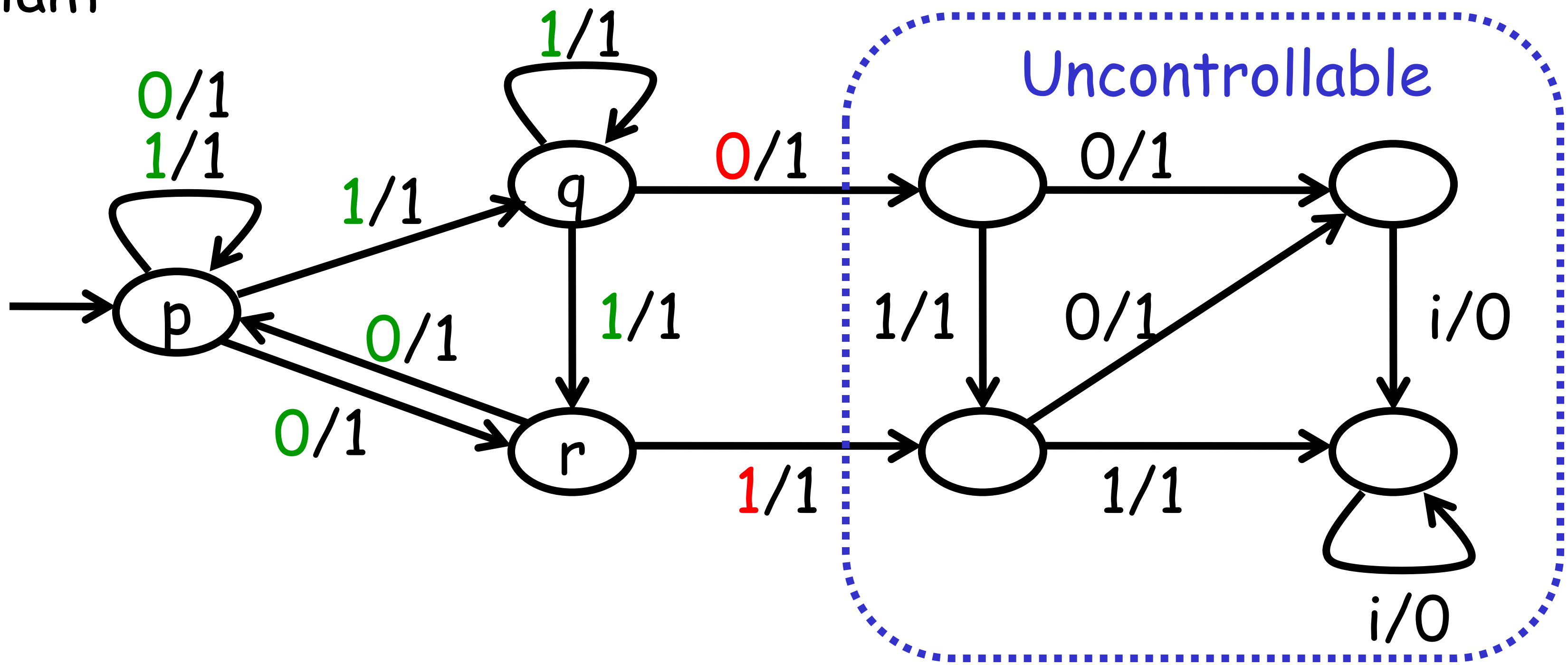
Plant

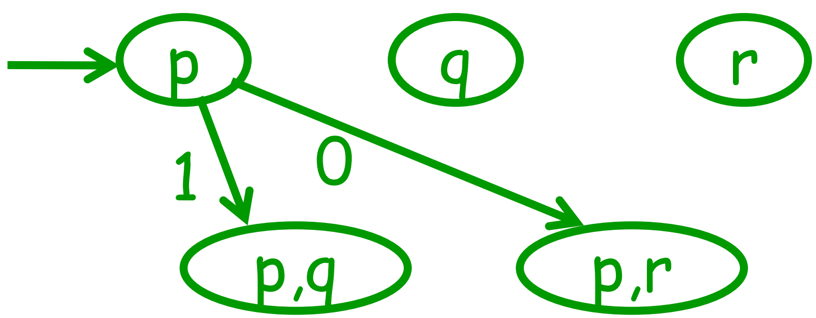




{p} : 0, 1
{q} : 1
{r} : 0
{p,q} : 1
{p,r} : 0

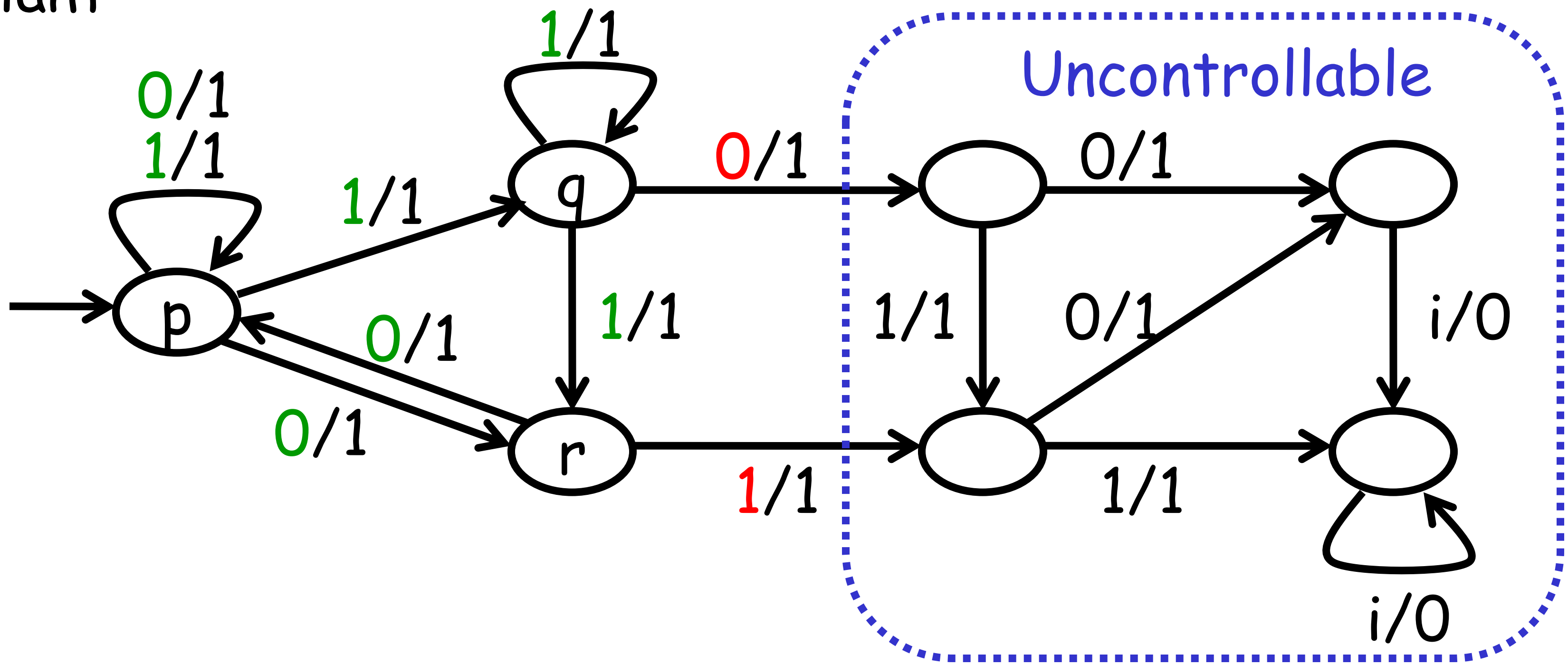
Plant

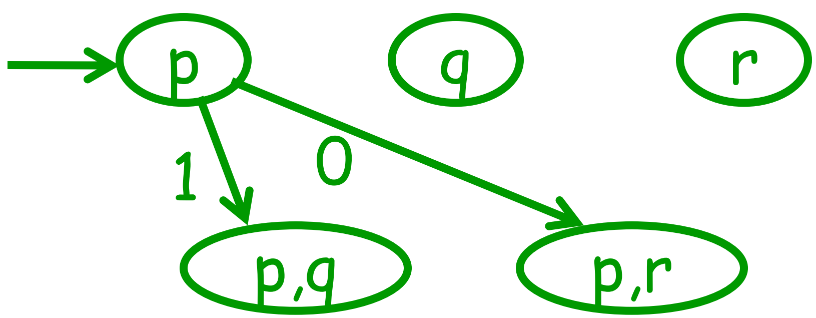




{q} : 1
{r} : 0
{p,q} : 1
{p,r} : 0

Plant



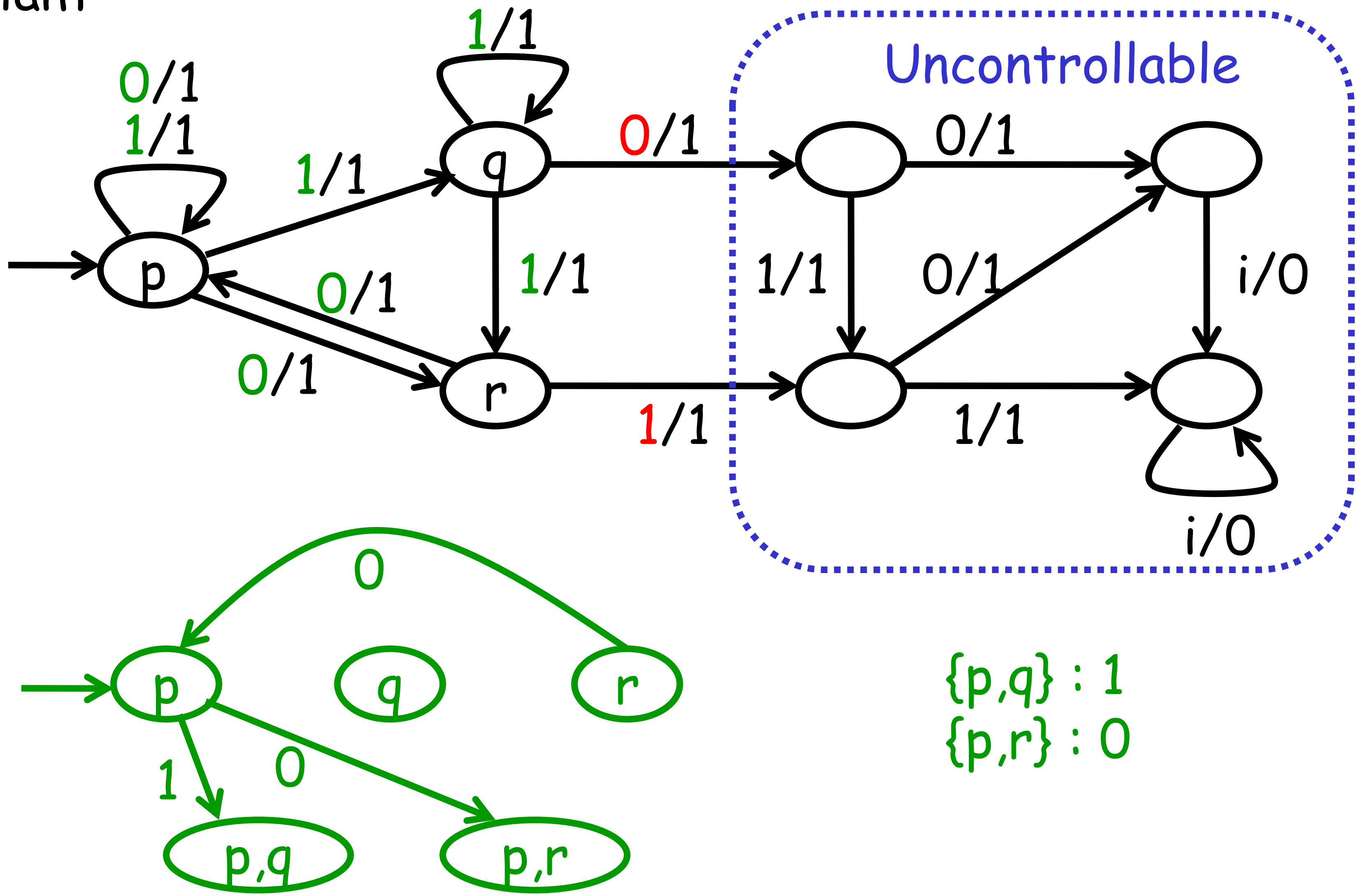


{r} : 0

{p,q} : 1

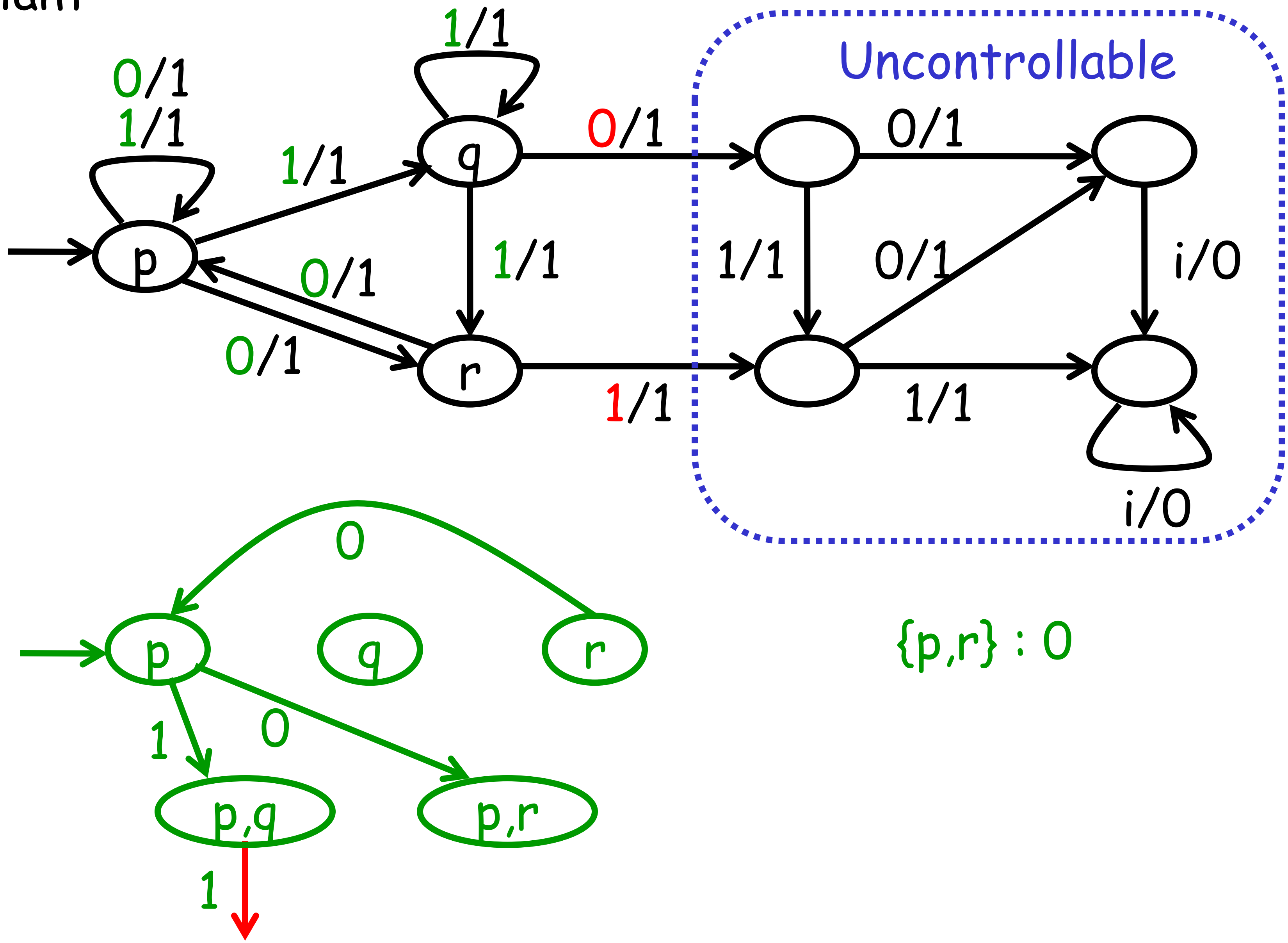
{p,r} : 0

Plant

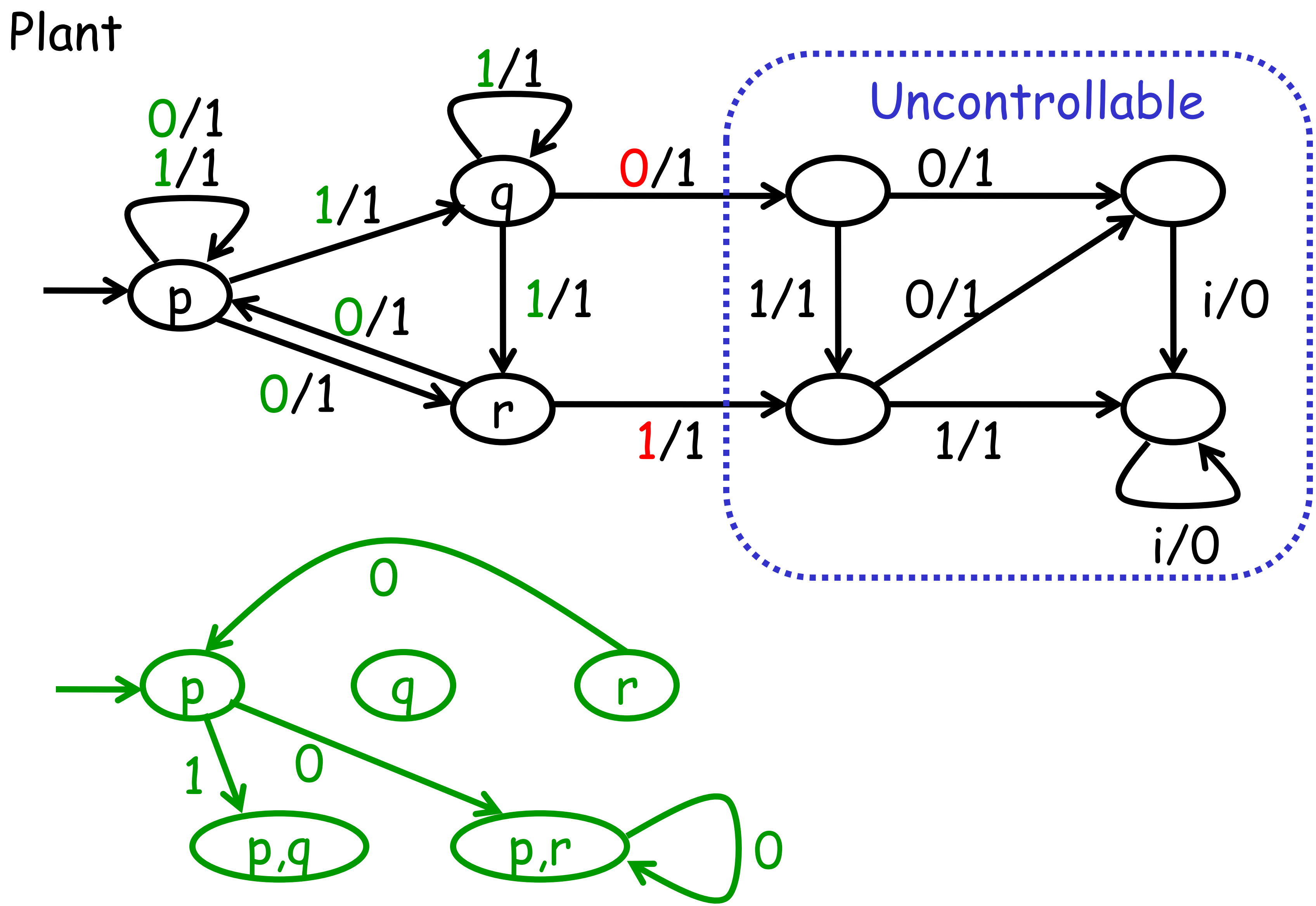


{p,q} : 1
{p,r} : 0

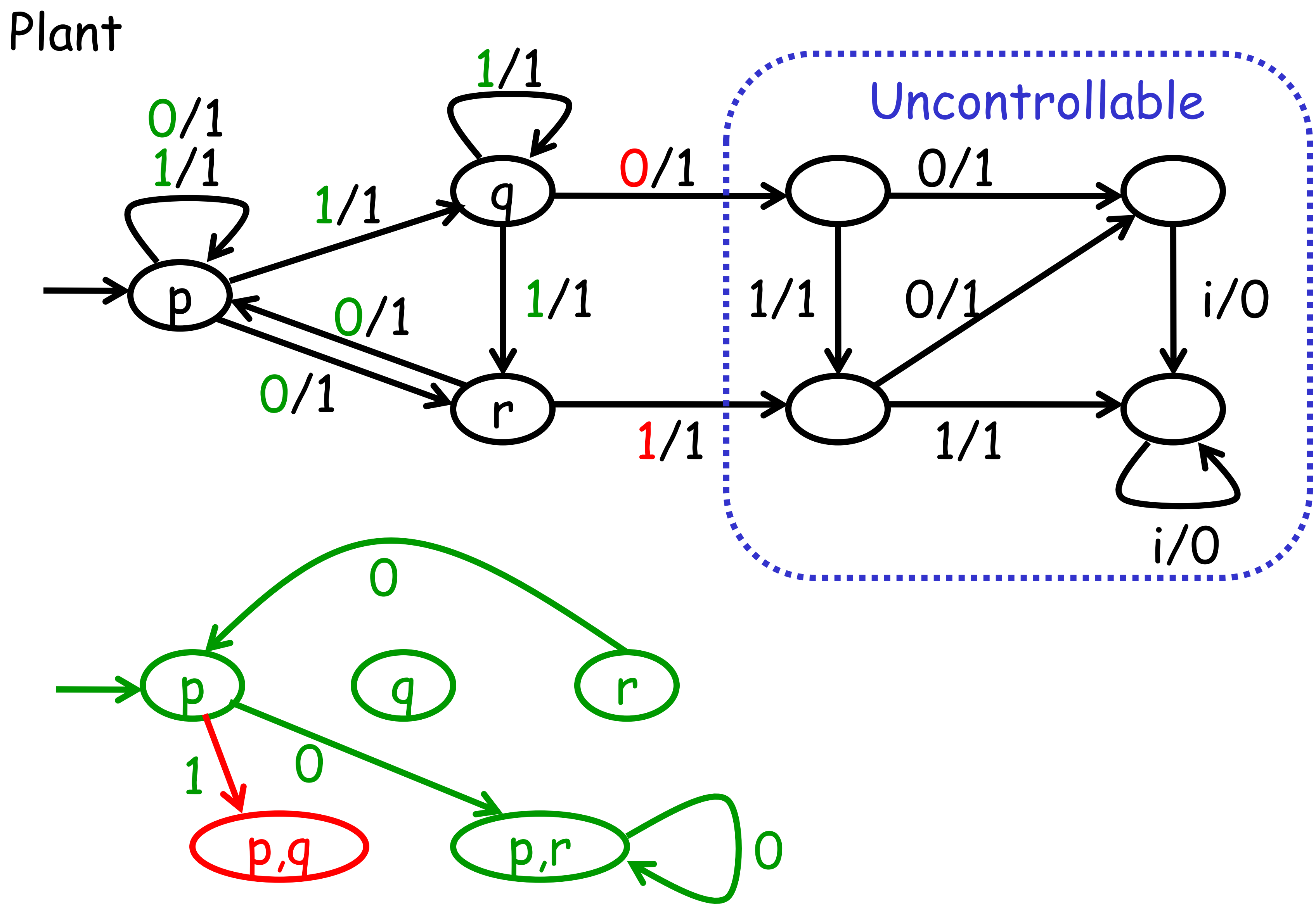
Plant



Uncontrollable
p
q
r
0/1
1/1
1/1
1/1
0/1
0/1
1/1
0/1
1/1
0/1
1/1
0/1
i/0
1/1
i/0
0
1
0
p,q
p,r
1
{p,r} : 0

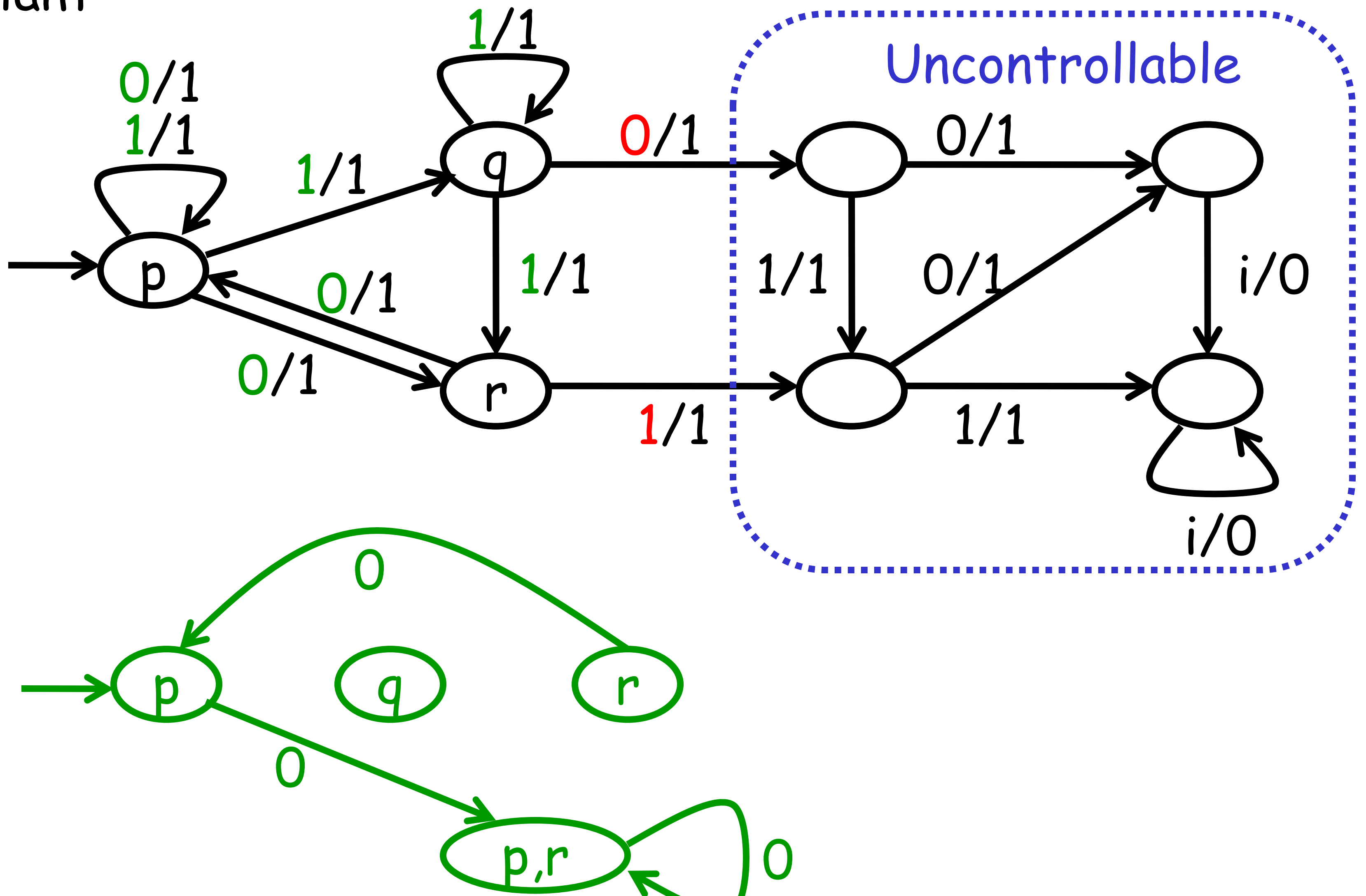


Plant
Uncontrollable
p
q
r
0/1
1/1
i/0
p,q
p,r
0
1

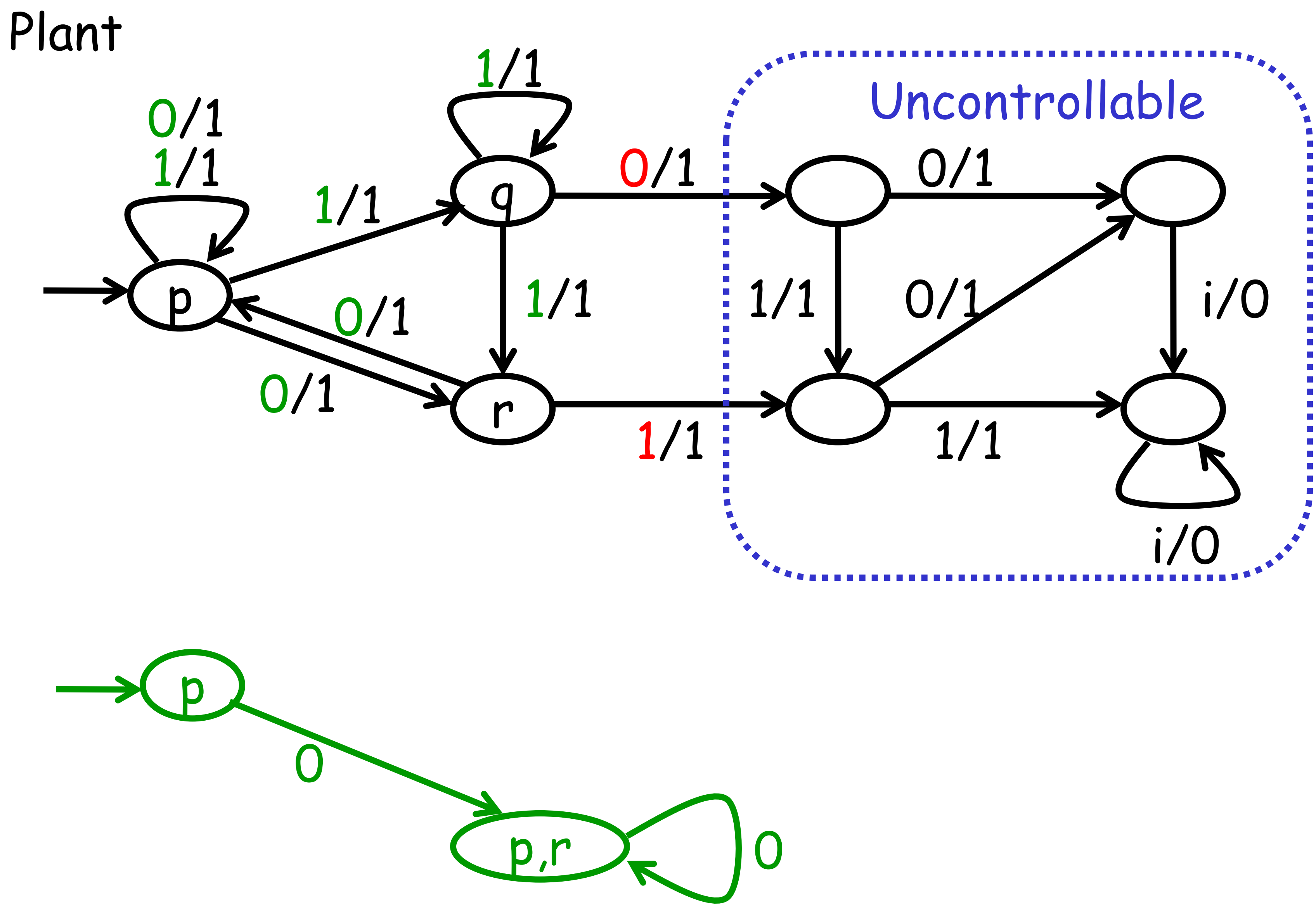


Plant
Uncontrollable
0/1
1/1
1/1
1/1
0/1
0/1
0/1
1/1
0/1
0/1
1/1
0/1
i/0
1/1
1/1
i/0
p
q
r
0
1
0
0
p
q
r
p,q
p,r

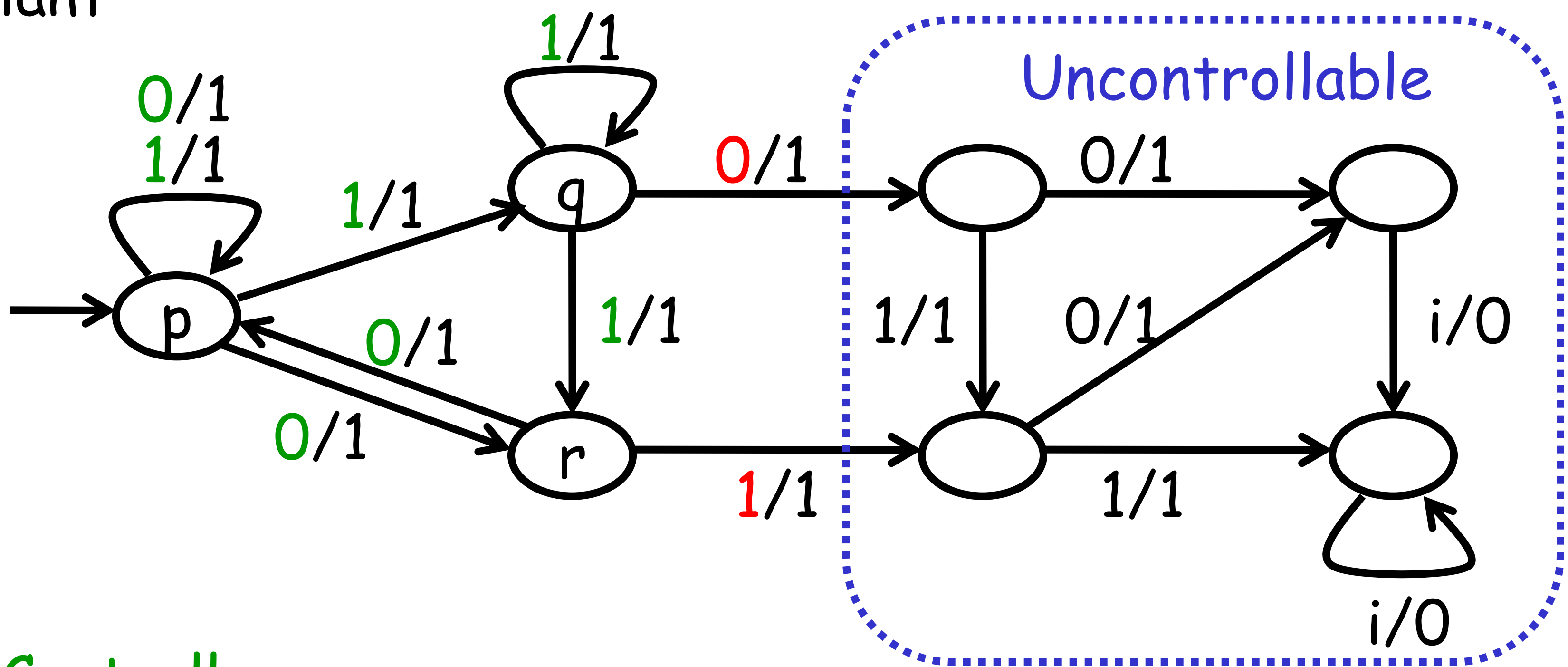
Plant



0/1
1/1
1/1
1/1
0/1
0/1
1/1
0/1
Uncontrollable
0/1
1/1
0/1
i/0
1/1
1/1
i/0
p
q
r
0
p
q
r
0
p,r
0


Plant
Uncontrollable
0/1
1/1
1/1
1/1
0/1
0/1
1/1
0/1
1/1
0/1
1/1
0/1
1/1
i/0
1/1
i/0
p
q
r
p
0
p,r
0

Plant



Controller