Simulation

EECS 20

Lecture 15 (February 21, 2001)

Tom Henzinger

Revised by Matteo Zavatteri and Tiziano Villa, Fall 2020

Deterministic Reactive System:

for every input signal, there is exactly one output signal.

Function:

$$\text{DetSys} : \; [\, \text{Time} \to \text{Inputs} \,] \to [\, \text{Time} \to \text{Outputs} \,]$$

Nondeterministic Reactive System:

for every input signal, there is one or more output signals.

Binary relation:

$$\text{NondetSys} \subseteq [\ \text{Time} \to \text{Inputs}\ ] \times [\ \text{Time} \to \text{Outputs}\ ]$$

such that $\forall\ x \in [\ \text{Time} \to \text{Inputs}\ ],$

$\exists\ y \in [\ \text{Time} \to \text{Outputs}\ ],\ (x,y) \in \text{NondetSys}$

Every pair $(x,y) \in \text{NondetSys}$ is called a behavior.

S1 is a more detailed description of S2;

S2 is an abstraction or property of S1.

System S1 refines system S2

iff

1. Time [S1] = Time [S2] ,
2. Inputs [S1] = Inputs [S2] ,
3. Outputs [S1] = Outputs [S2] ,
4. Behaviors [S1] $\subseteq$ Behaviors [S2] .

Systems S1 and S2 are equivalent

iff

1. Time [S1] = Time [S2] ,
2. Inputs [S1] = Inputs [S2] ,
3. Outputs [S1] = Outputs [S2] ,
4. Behaviors [S1] = Behaviors [S2] .

Nondeterministic causal discrete-time reactive systems
can be implemented by
nondeterministic state machines.

Nondeterministic State Machine

Inputs

Outputs

States

possibleInitialStates $\subseteq$ States

possibleUpdates :

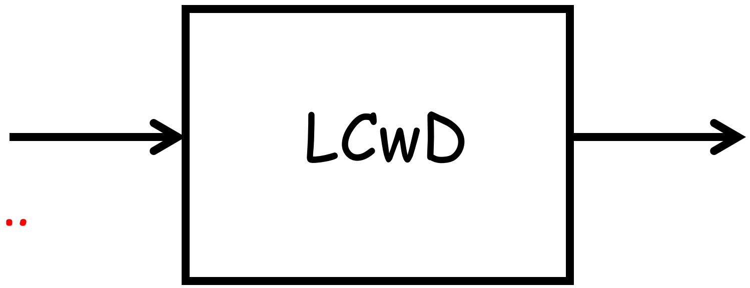
$$\text{States} \times \text{Inputs} \to P(\text{States} \times \text{Outputs}) \setminus \emptyset$$

receptiveness (i.e., machine must be prepared to accept every input)

Lossy Channel without Delay

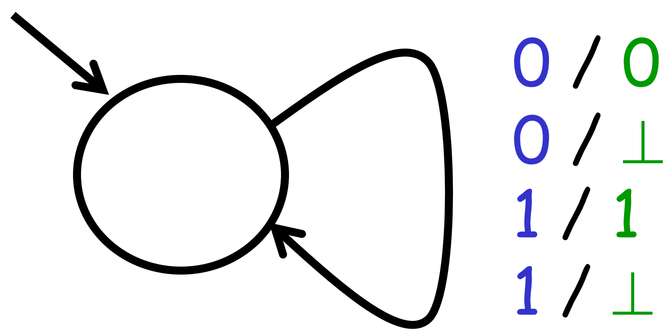
$Nats_0 \rightarrow Bins$

0 1 1 0 0 0 1 1 ...

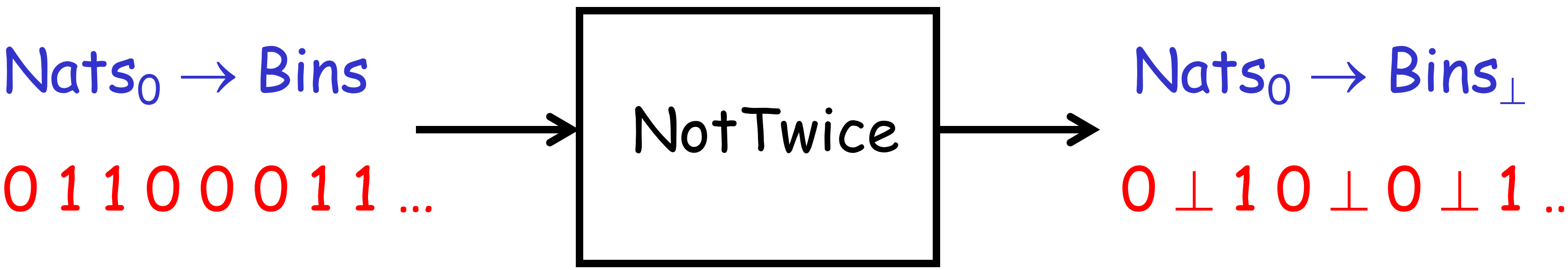


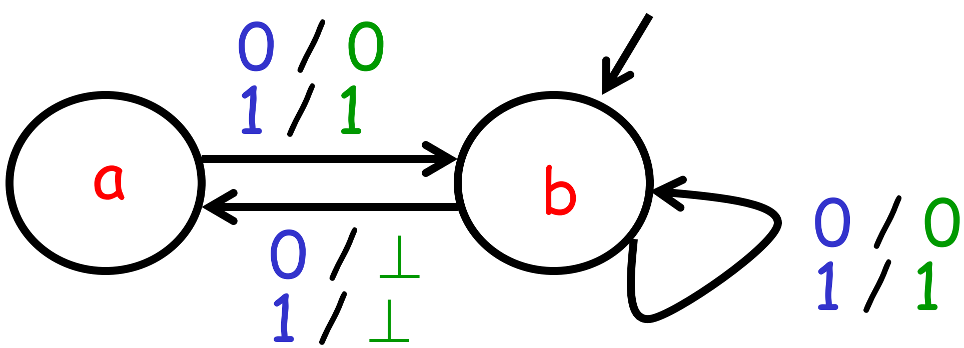
$Nats_0 \rightarrow Bins_\perp$

0 $\perp$ 1 0 $\perp$ $\perp$ 1 1 ...



Channel that never drops two in a row





Typical Exam Questions

A. Convert between three (possibly nondeterministic) system representations:

1. Input-output definition

   Deterministic case:
   define $S: [Nats_0 \to Inputs] \to [Nats_0 \to Outputs]$ such that for all $x \in [Nats_0 \to Inputs]$ and all $y \in Nats_0$, $(S(x))(y) = \ldots$

   Nondeterministic case:
   define $S \subseteq [Nats_0 \to Inputs] \times [Nats_0 \to Outputs]$ such that for all $x \in [Nats_0 \to Inputs]$ and all $y \in [Nats_0 \to Outputs]$, $(x,y) \in S$ iff …

2. Transition diagram

3. Block diagram

B. Equivalence, bisimulation, minimization of deterministic state machines

Refinement, simulation, and minimization of nondeterministic state machines

condition on behaviors

Theorem :

The nondeterministic state machines M1 refines the nondeterministic state machine M2

if

there exists a simulation of M1 by M2 .

relation between states

Theorem :

The nondeterministic state machines M1 refines the nondeterministic state machine M2

if

there exists a simulation of M1 by M2 .

Not "if-and-only-if" !

Not symmetric !

(We say that "M2 simulates M1".)

In the following, assume

Inputs [M1] = Inputs [M2]

Outputs [M1] = Outputs [M2]

A binary relation $S \subseteq States[M1] \times States[M2]$ is a simulation of M1 by M2

iff

1. $\forall\, p \in possibleInitialStates[M1]$,

   $\exists\, q \in possibleInitialStates[M2],\ (p, q) \in S$ and

2. $\forall\, p \in States[M1],\ \forall\, q \in States[M2]$,

   if $(p, q) \in S$,

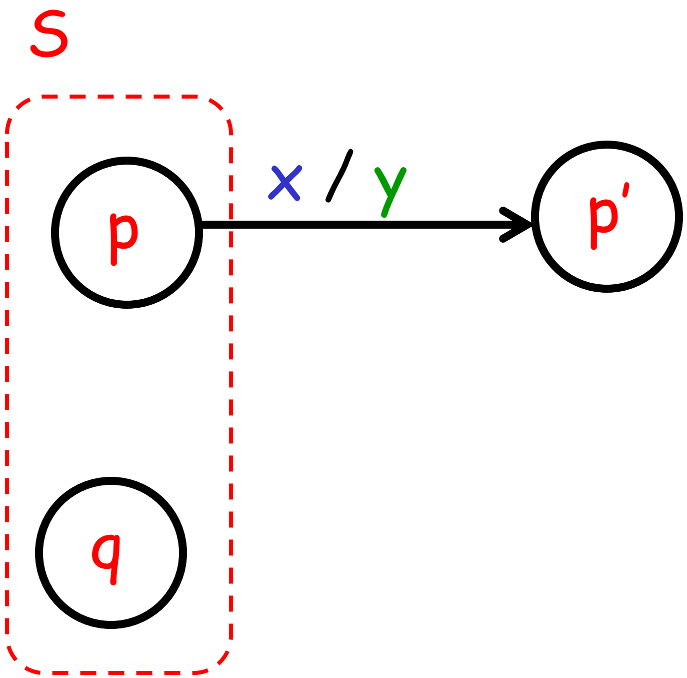
   then $\forall\, x \in Inputs,\ \forall\, y \in Outputs,\ \forall\, p' \in States[M1]$,

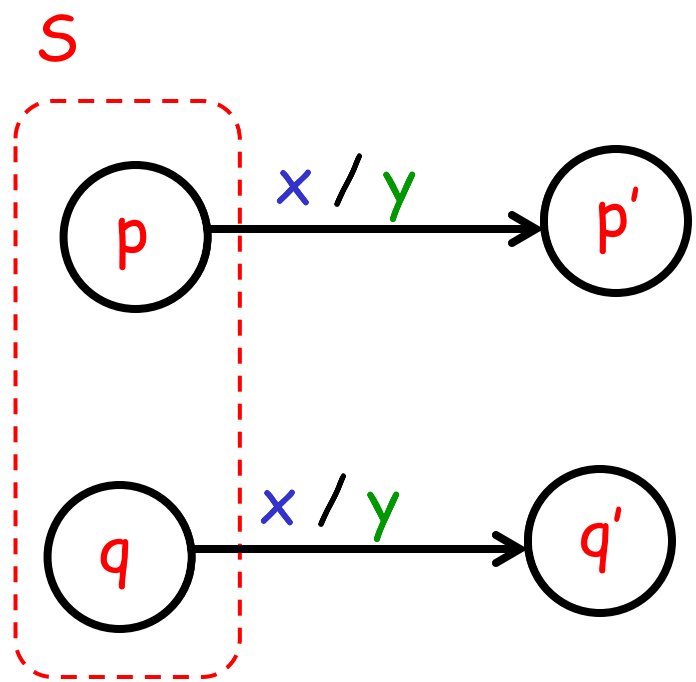
   if $(p', y) \in possibleUpdates[M1](p, x)$

   then $\exists\, q' \in States[M2]$,

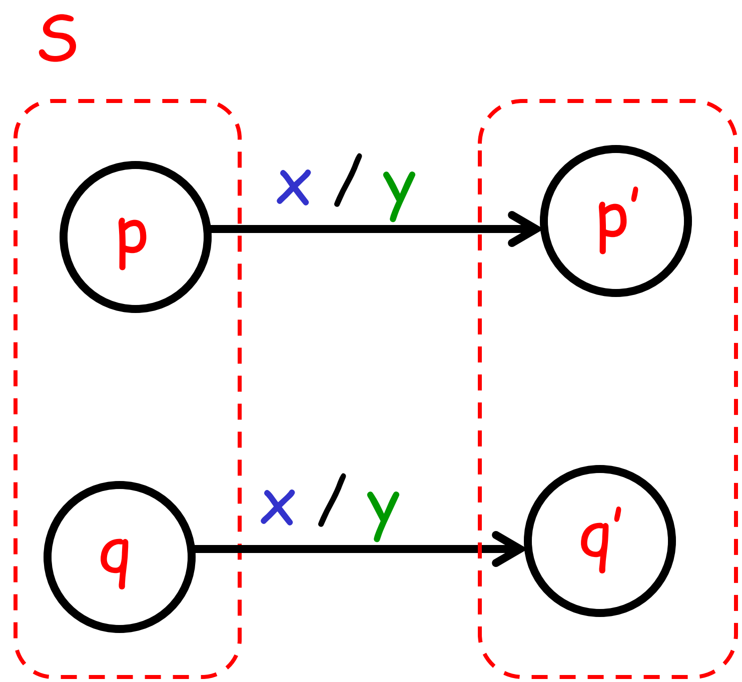
   $(q', y) \in possibleUpdates[M2](q, x)$ and

   $(p', q') \in S$.

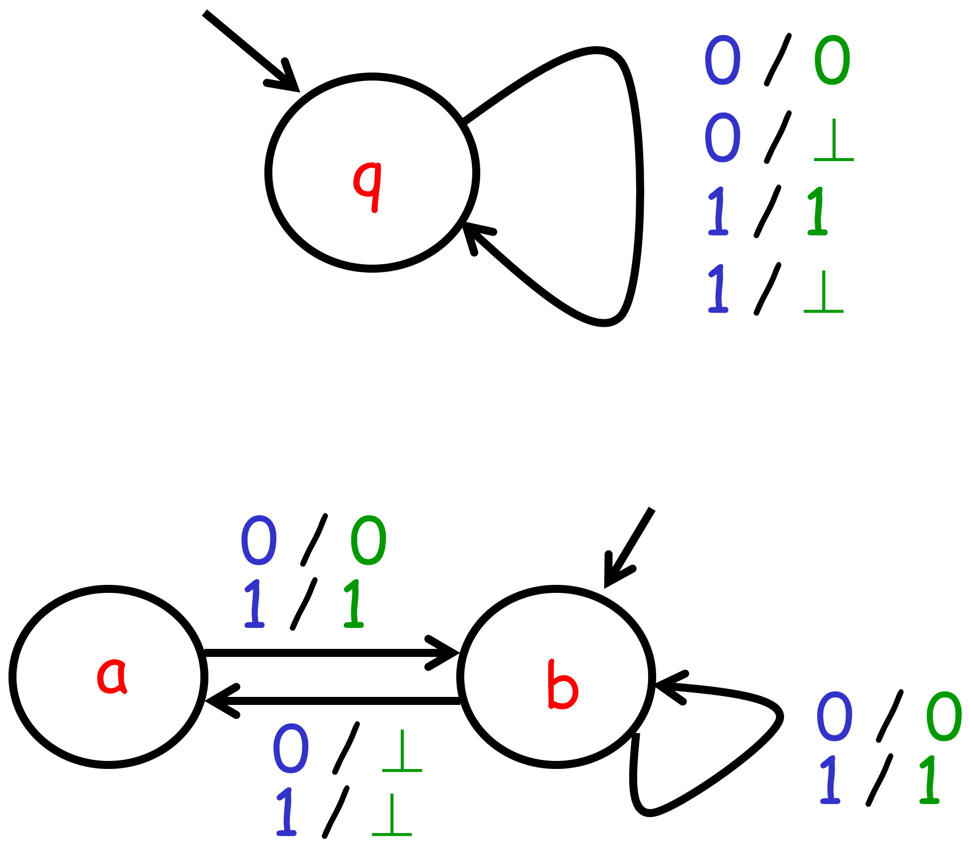

S
p
x / y
p'
q



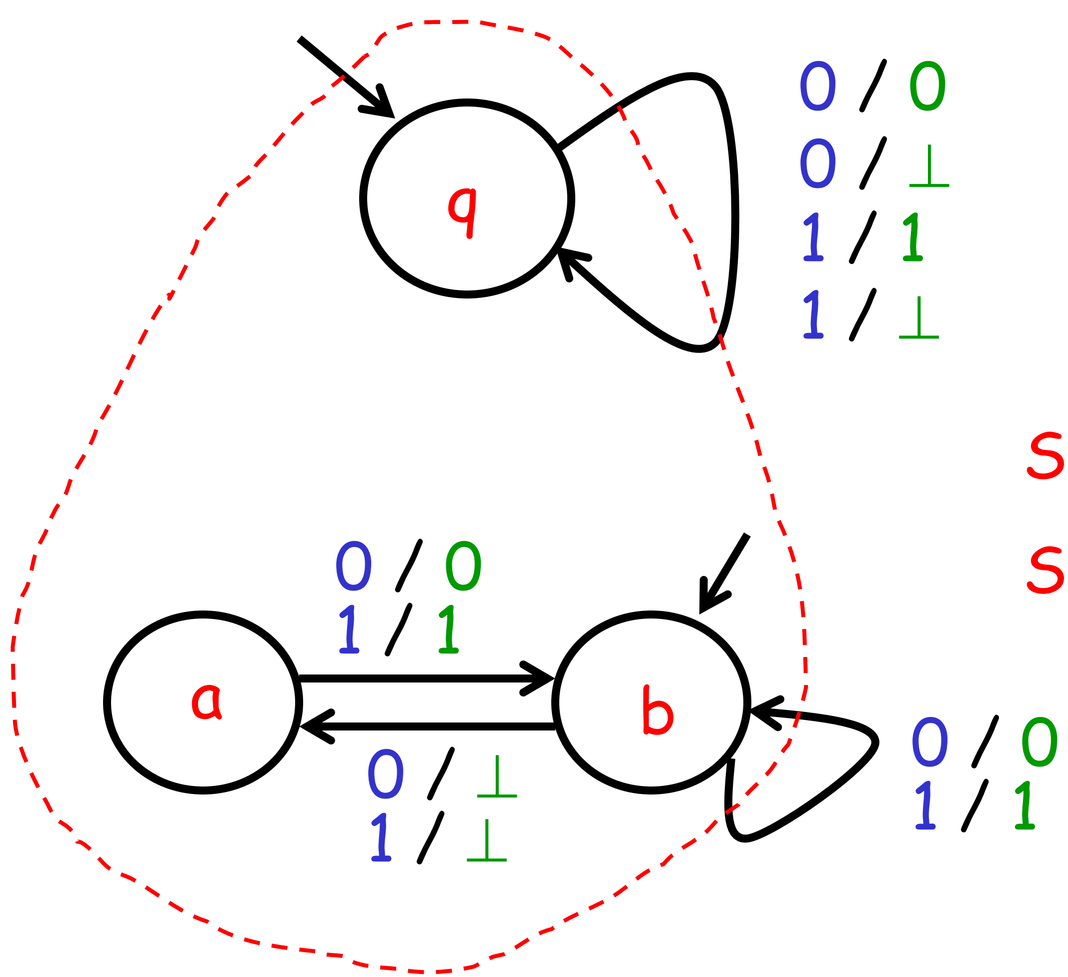
S
p
x / y
p'
q
x / y
q'


S
p
x / y
p'
q
x / y
q'

NotTwice refines Lossy Channel without Delay



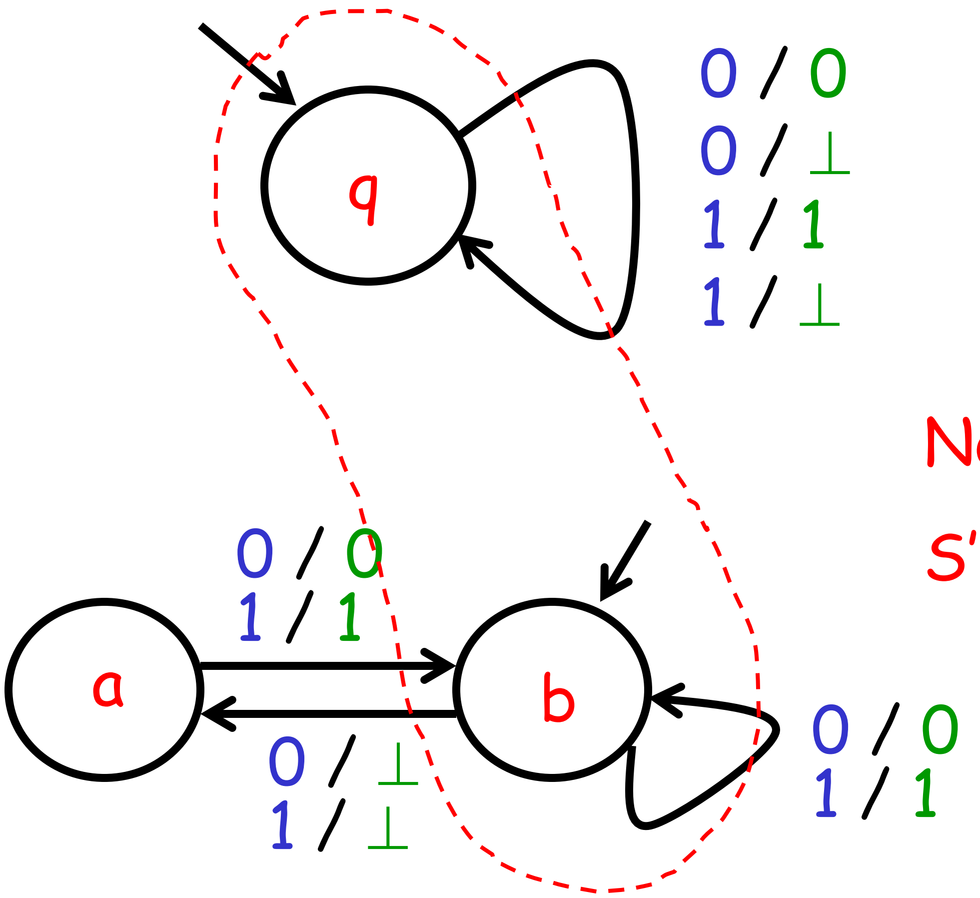
NotTwice is simulated by Lossy Channel without Delay



Simulation

$S = \{ (a, q), (b, q) \}$

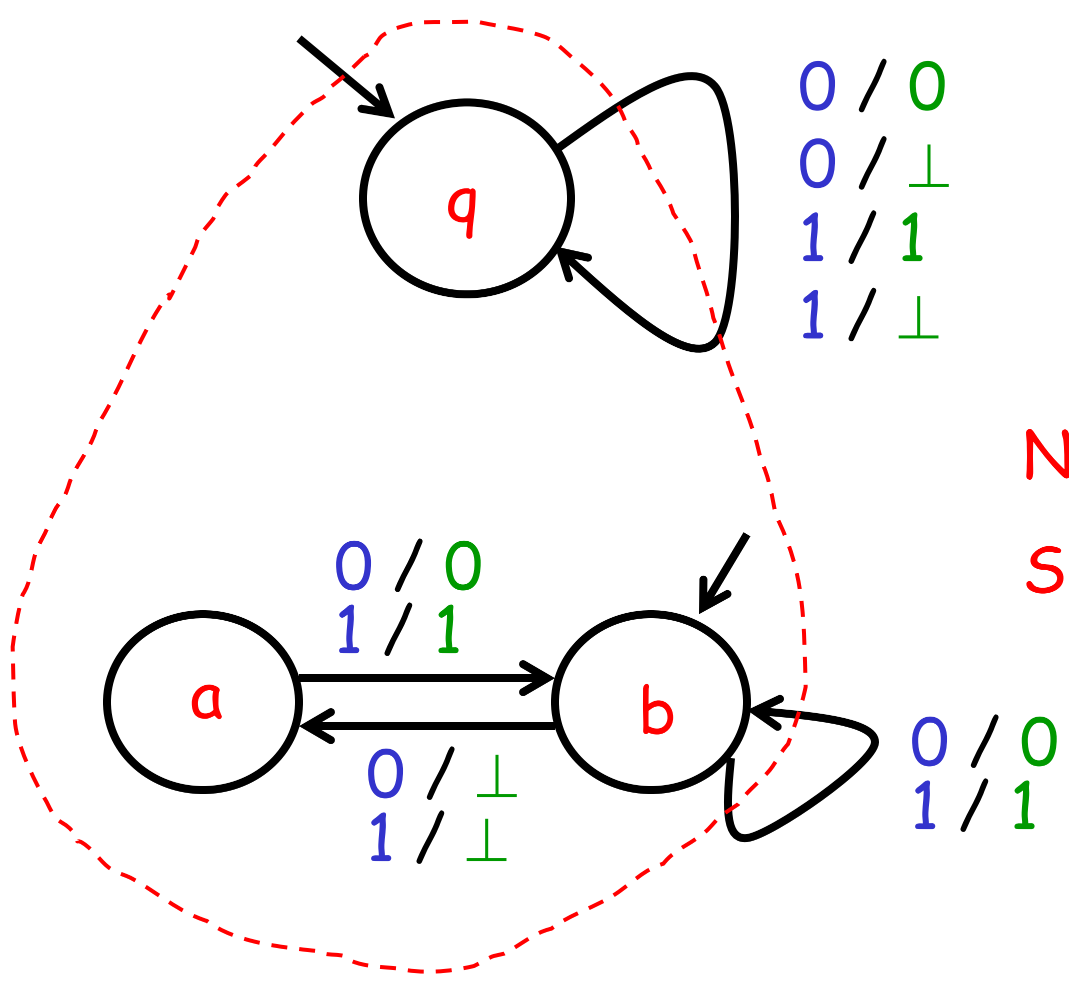
Lossy Channel without Delay is not simulated by NotTwice



Not a simulation

S' = { (q, b) }

Lossy Channel without Delay is not simulated by NotTwice



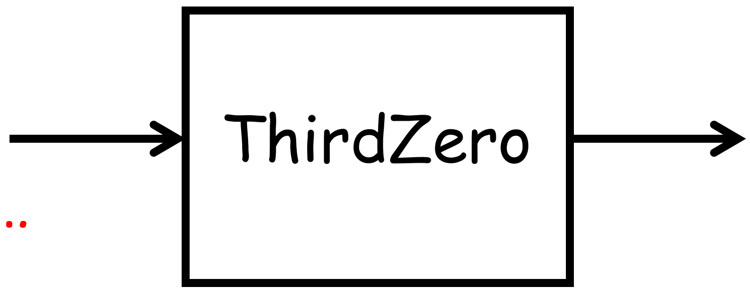
Not a simulation

S″ = { (q, a), (q, b) }

Channel that drops every third zero

$Nats_0 \rightarrow Bins$

0 1 1 0 0 0 1 1 …



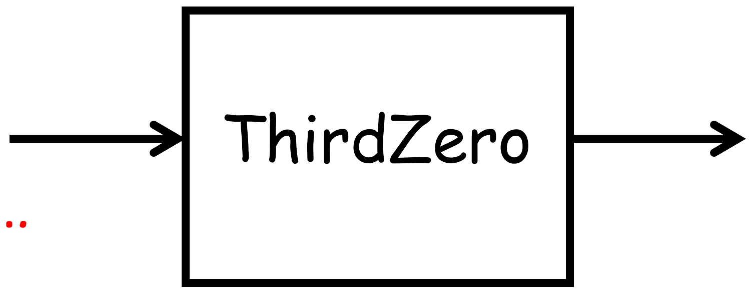
$Nats_0 \rightarrow Bins_\perp$

0 1 1 0 $\perp$ 0 1 1 …

Channel that drops every third zero

$Nats_0 \rightarrow Bins$

$0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ \ldots$



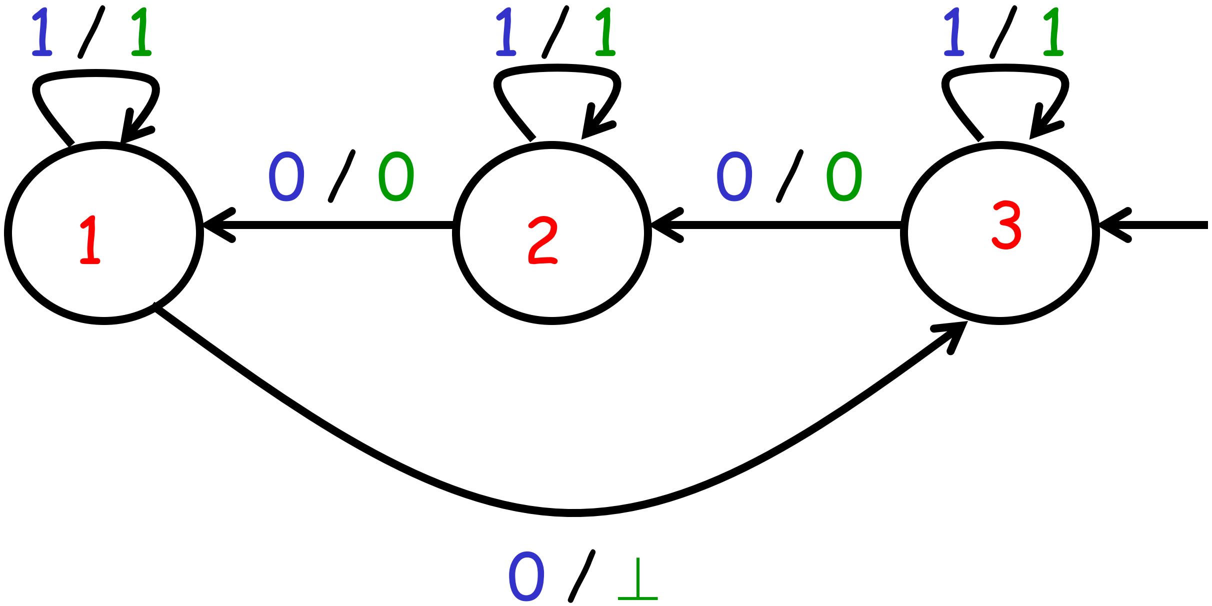
$Nats_0 \rightarrow Bins_\perp$

$0\ 1\ 1\ 0\ \perp\ 0\ 1\ 1\ \ldots$

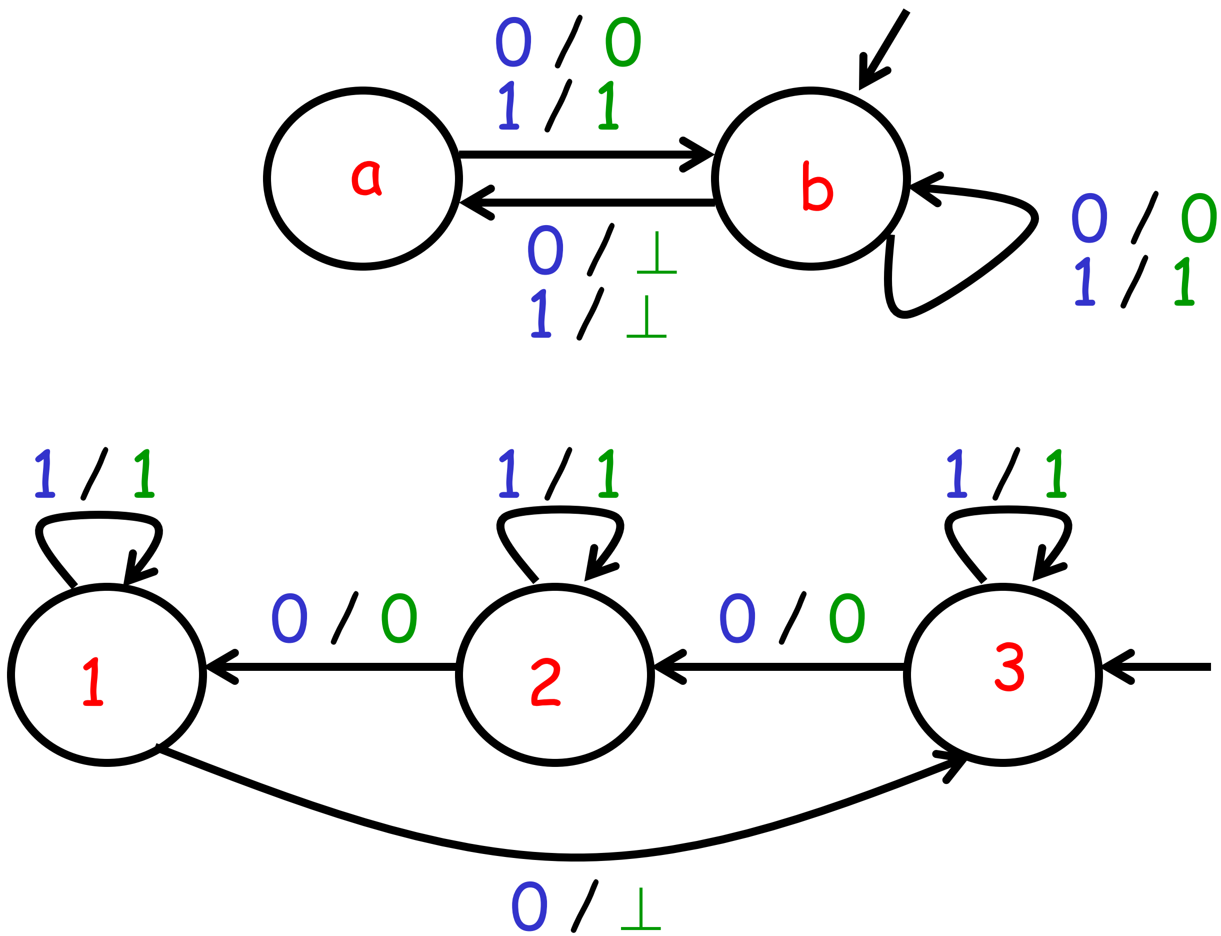
State between time t-1 and time t:

- 3 third zero from now will be dropped
- 2 second zero from now will be dropped
- 1 next zero will be dropped

Channel that drops every third zero



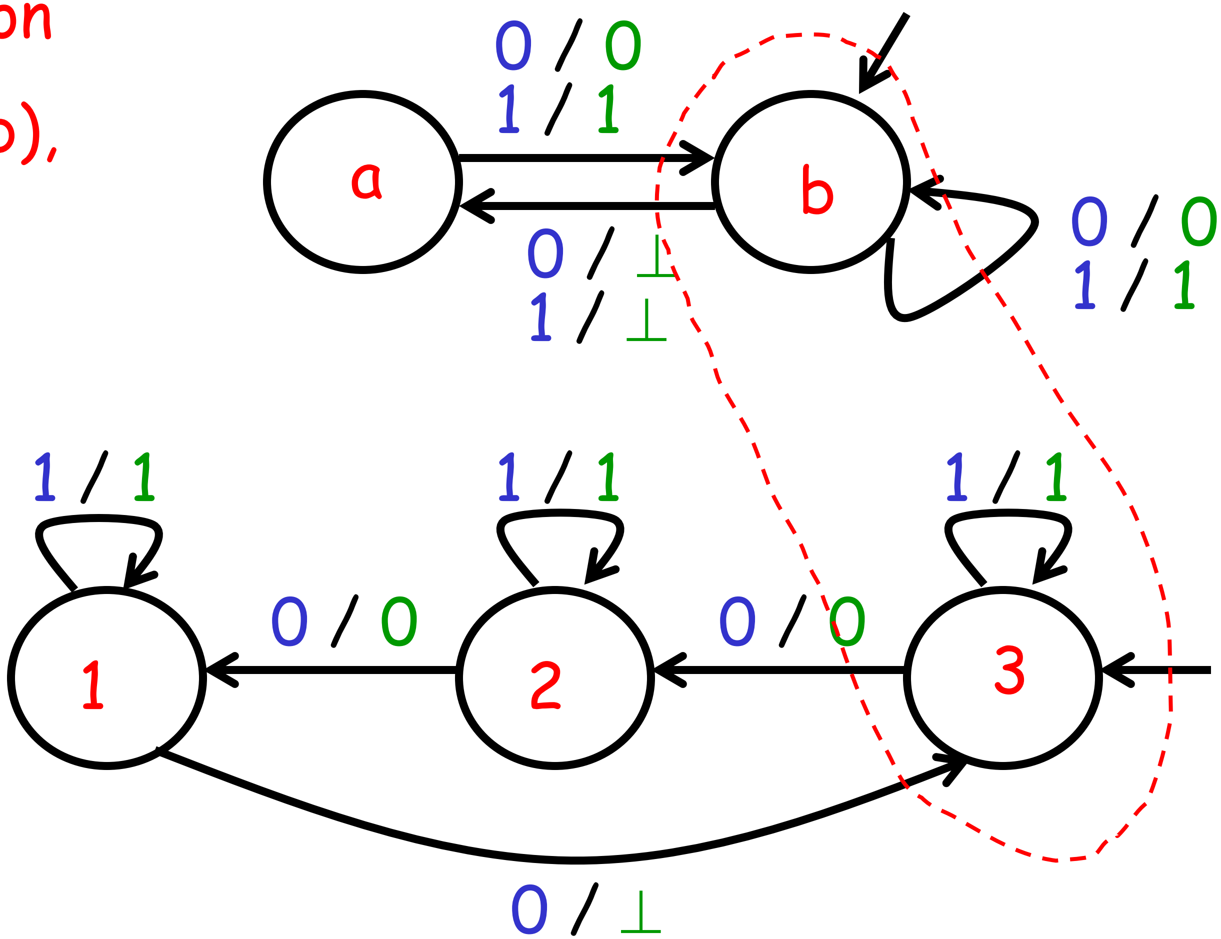
ThirdZero refines NotTwice



ThirdZero is simulated by NotTwice

Simulation

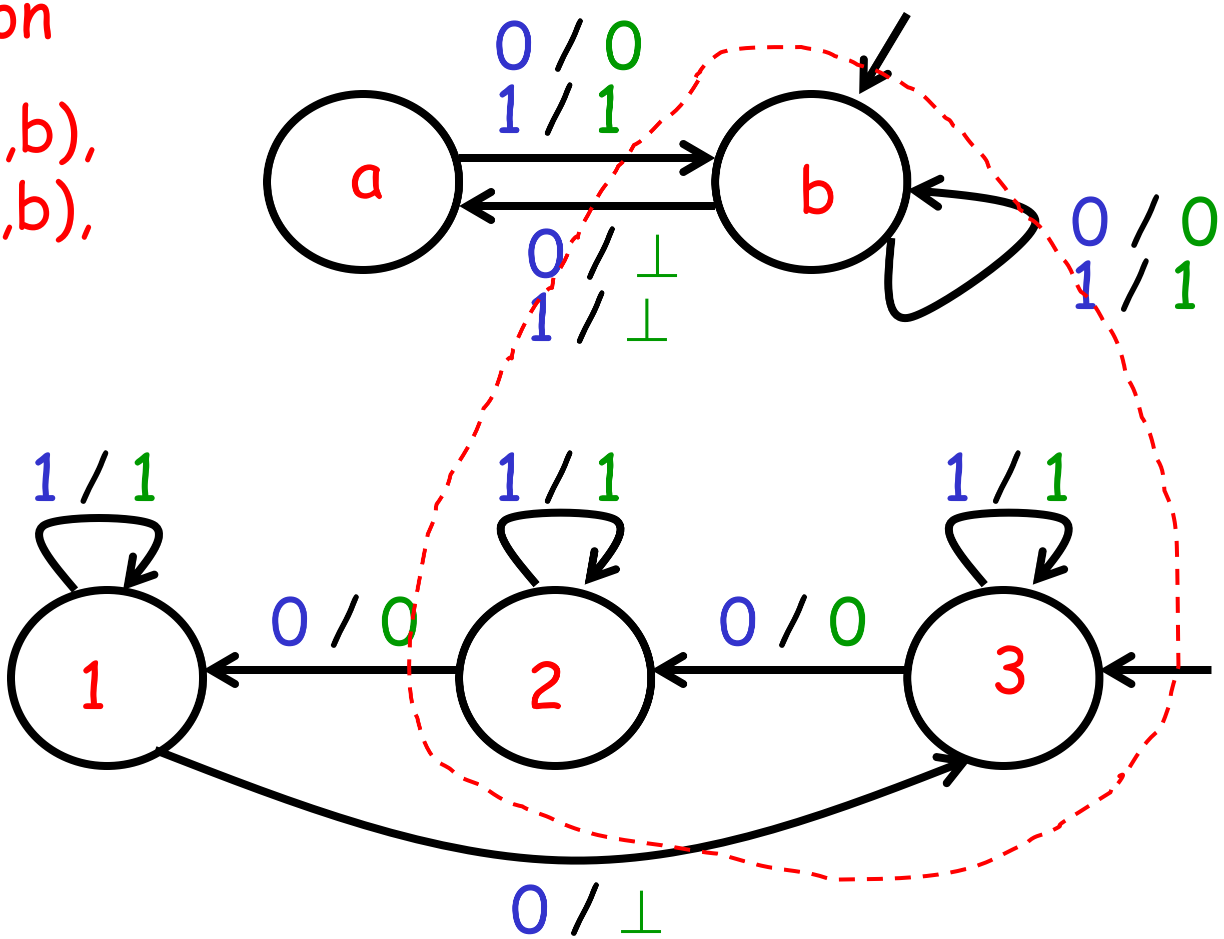
S = { (3,b),



ThirdZero is simulated by NotTwice

Simulation

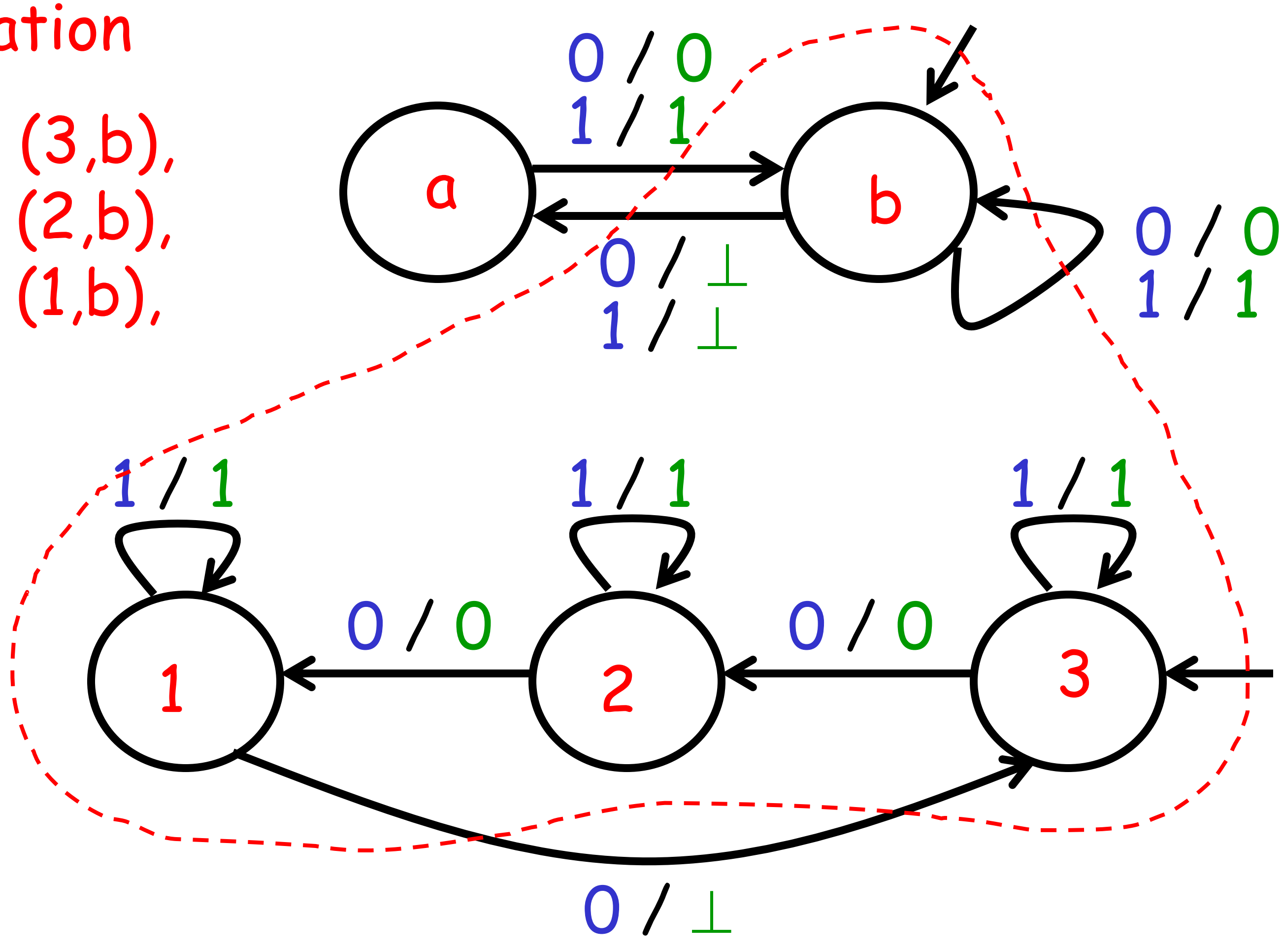
S = { (3,b),
(2,b),



ThirdZero is simulated by NotTwice

Simulation

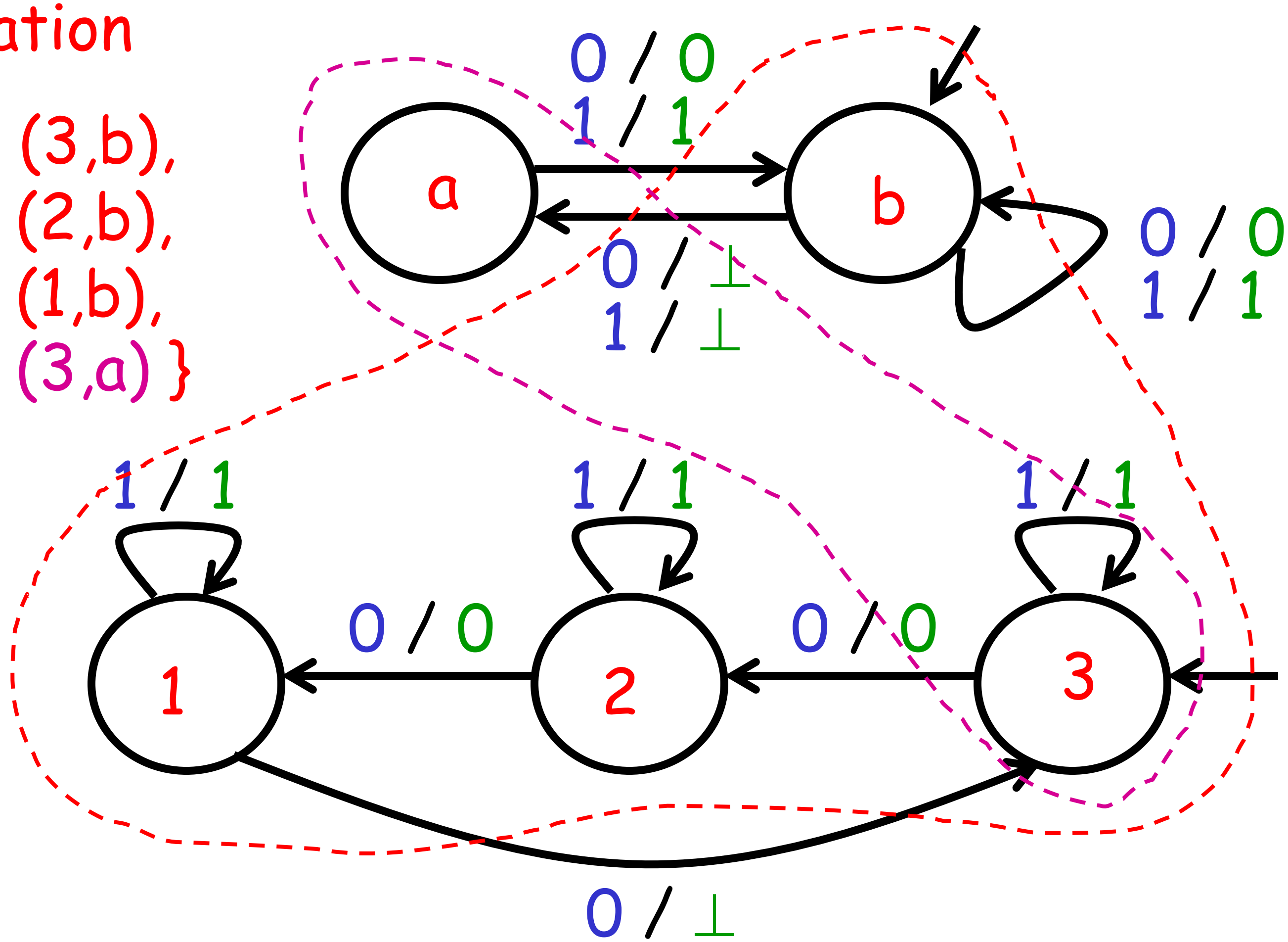
S = { (3,b),
(2,b),
(1,b),



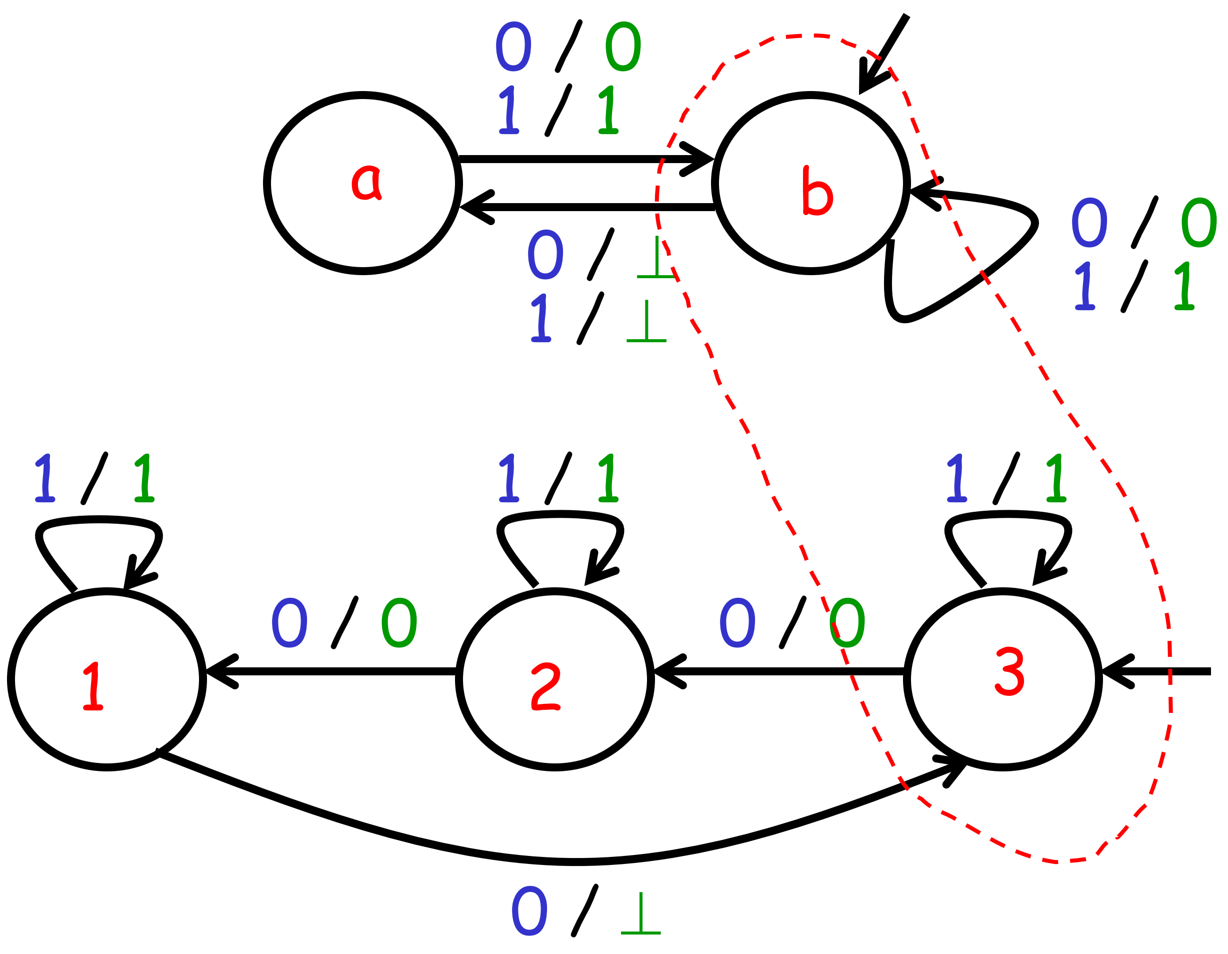
ThirdZero is simulated by NotTwice

Simulation

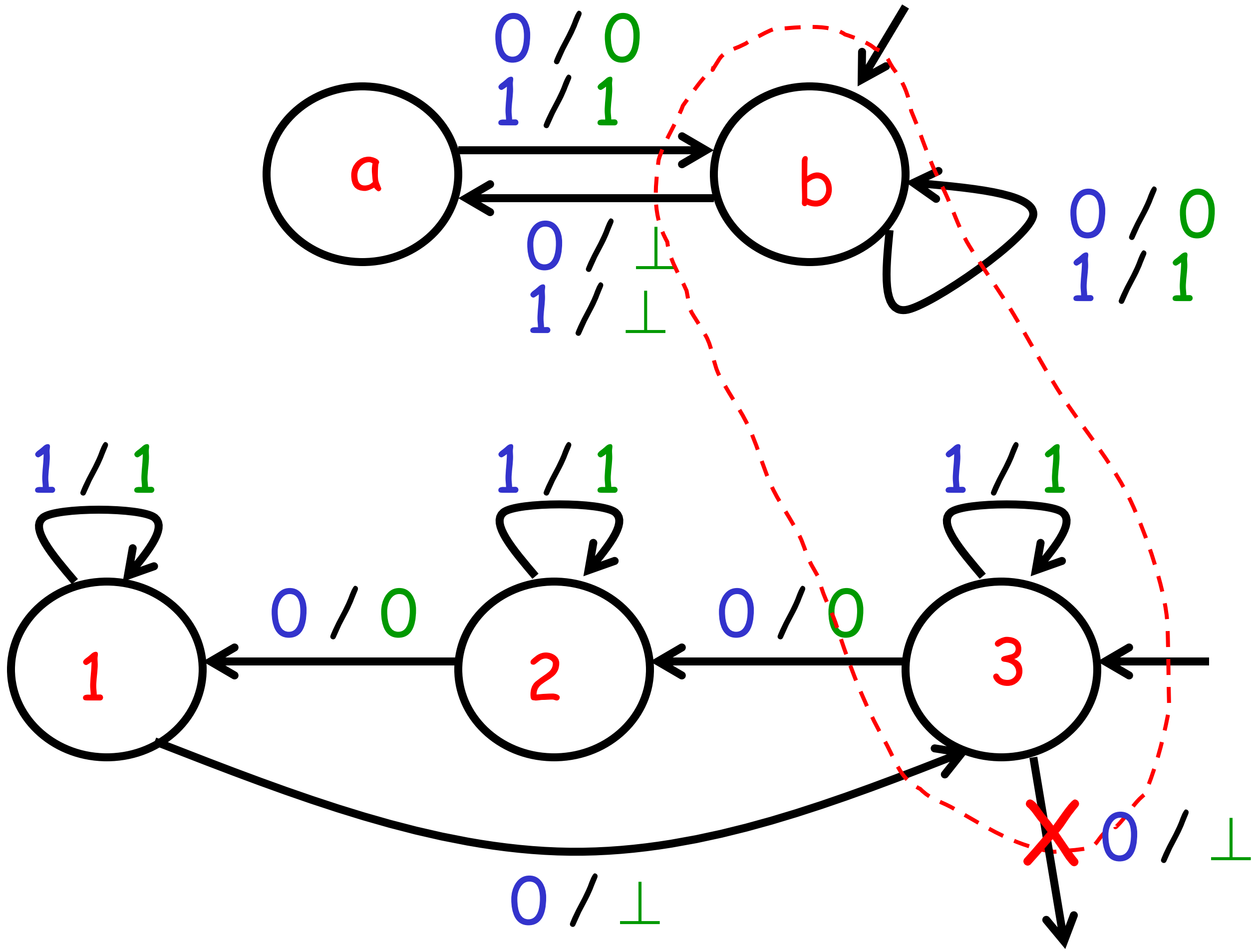
S = { (3,b),
(2,b),
(1,b),
(3,a) }



NotTwice is not simulated by ThirdZero



NotTwice is not simulated by ThirdZero



Special cases of nondeterministic state machines

1. Deterministic
2. Output-deterministic (“almost deterministic”)

For these machines, simulations can be found easily (provided they exist).

A state machine is output-deterministic

iff

1. there is only one initial state, and
2. for every state and every input-output pair, there is only one successor state.

Deterministic implies output-deterministic, but not vice versa.

For example, LCwD and NotTwice are output-deterministic; ThirdZero is deterministic.

Deterministic: for every input signal x, there is exactly one run of the state machine.

Output-deterministic: for every behavior (x,y), there is exactly one run.

If M2 is an output-deterministic state machine, then a simulation S of M1 by M2 can be found as follows:

1. If $p \in$ possibleInitialStates [M1] and

   possibleInitialStates [M2] $= \{ q \}$,

   then $(p,q) \in S$.

2. If $(p,q) \in S$ and

   $(p',y) \in$ possibleUpdates [M1] (p,x) and

   possibleUpdates [M2] $(q,x) = \{ (q',y) \}$,

   then $(p',q') \in S$.

If M2 is a deterministic state machine, then

M1 is simulated by M2

iff

M1 is equivalent to M2.

If M2 is a deterministic state machine, then

M1 is simulated by M2

iff

M1 is equivalent to M2.

"if and only if"

M1 refines M2, and M2 refines M1

If M2 is a nondeterministic state machine, then

M1 is simulated by M2

implies

M1 refines M2,

but M2 may not refine M1.

(Recall M1 = ThirdZero and M2 = NotTwice .)

If M2 is a nondeterministic state machine, then

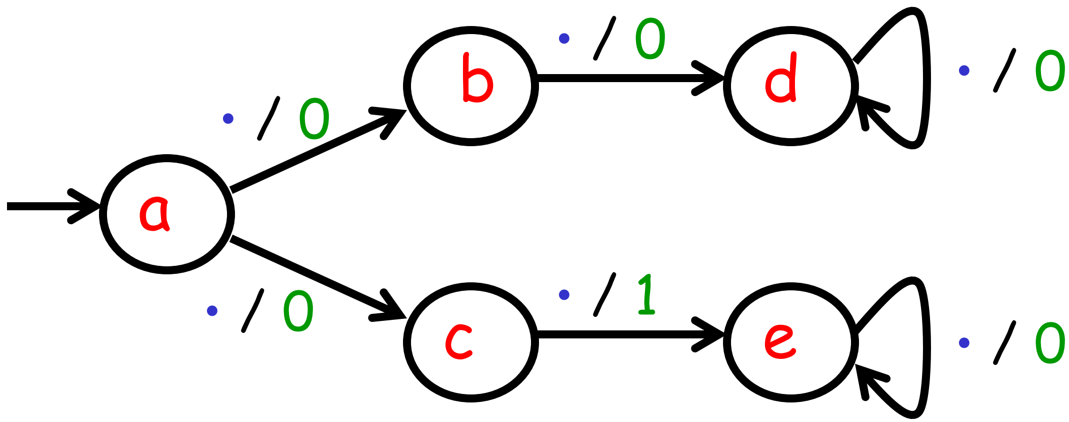
M1 is simulated by M2

implies

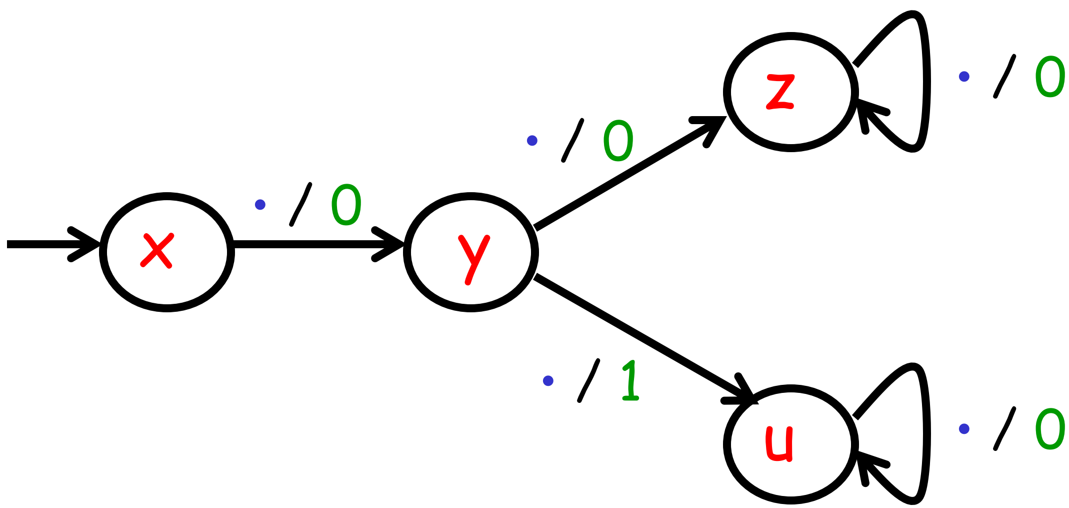
M1 refines M2,

but M1 refine M2 even if M1 is not simulated by M2.

M1

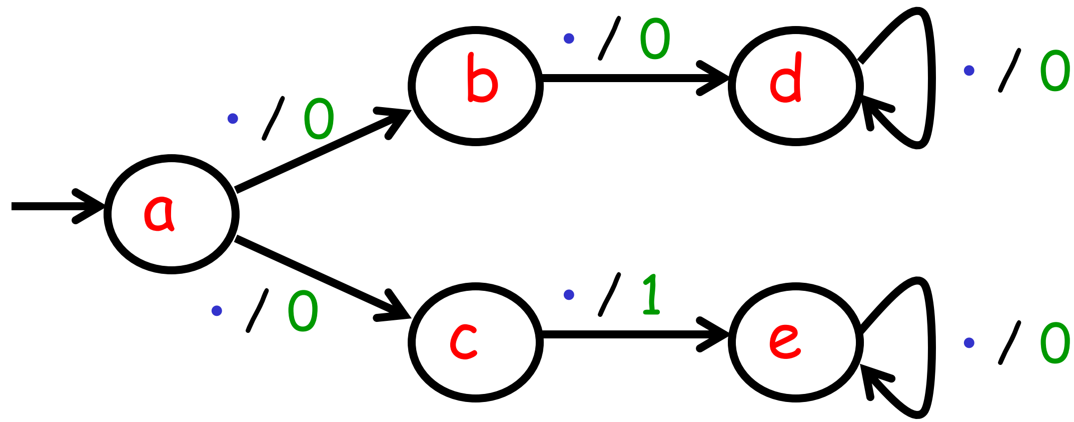


M2



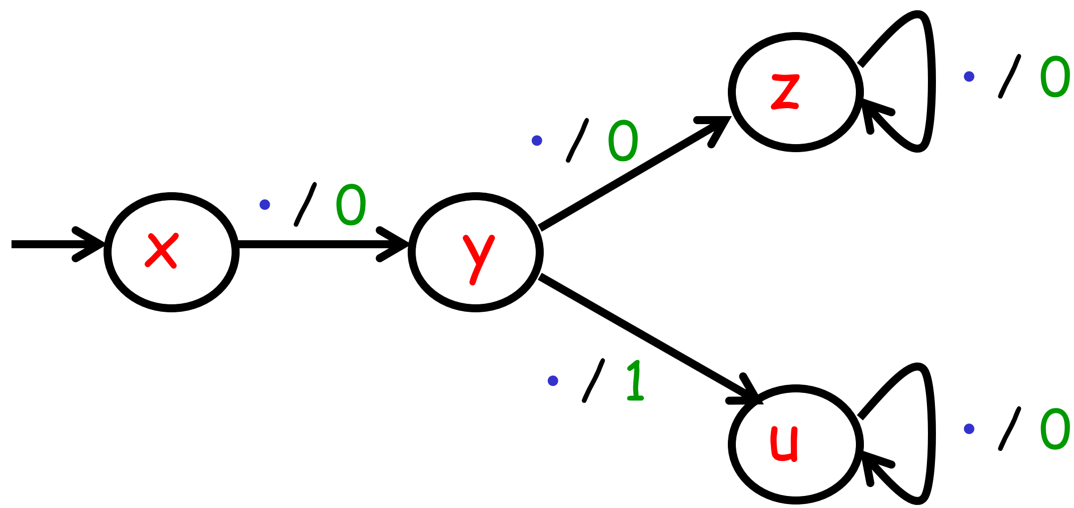
Two behaviors: 00000..., 01000...

M1



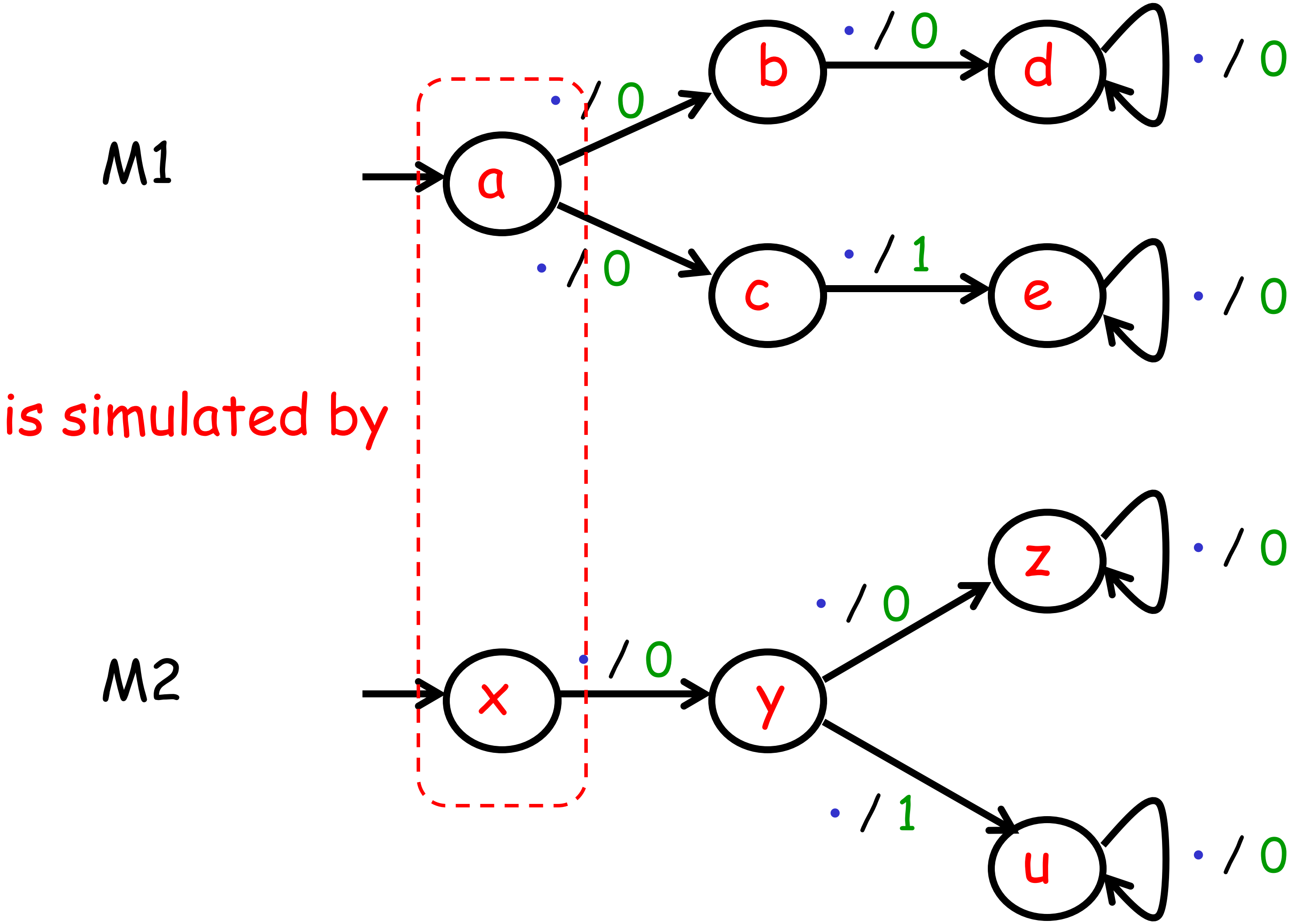
is equivalent to

M2

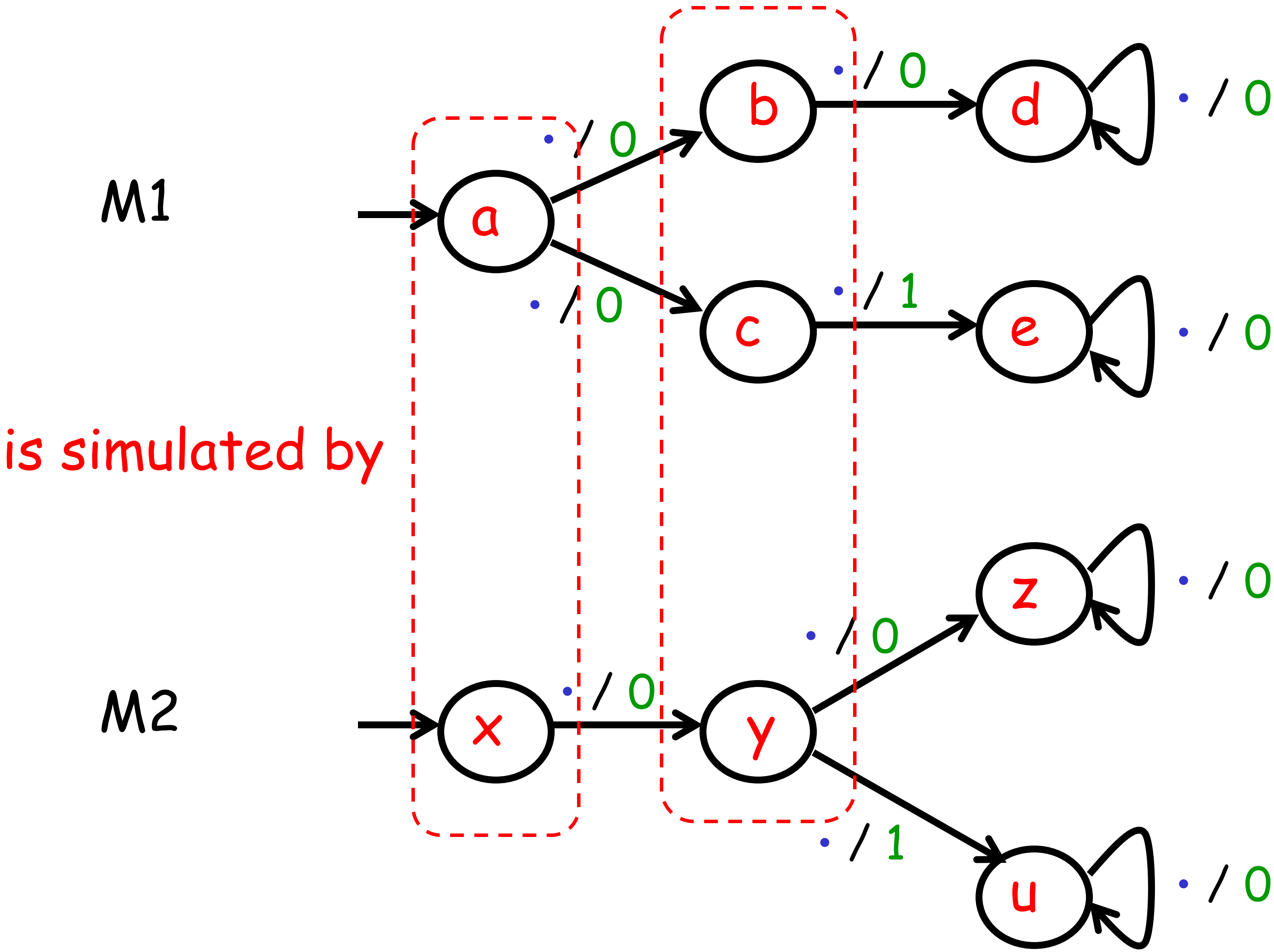


Same two behaviors: 00000..., 01000...

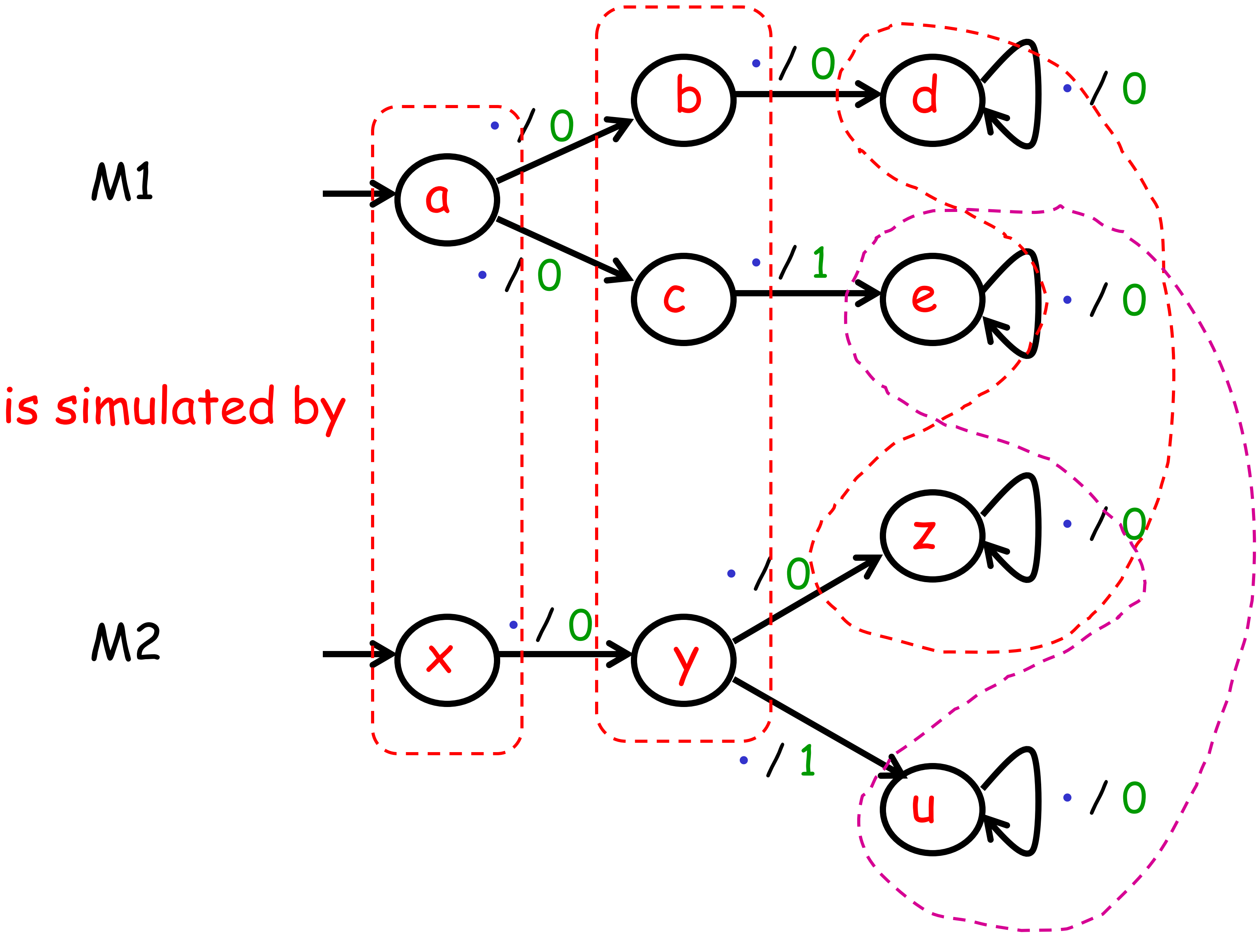
Simulation S = { (a, x),



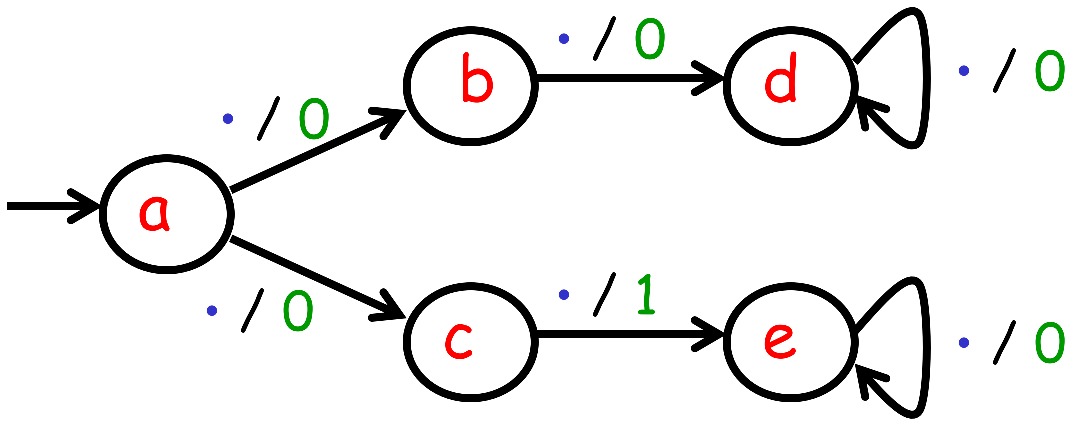
Simulation S = { (a, x), (b, y), (c, y),



Simulation $S = \{ (a, x), (b, y), (c, y), (d, z), (e, u) \}$

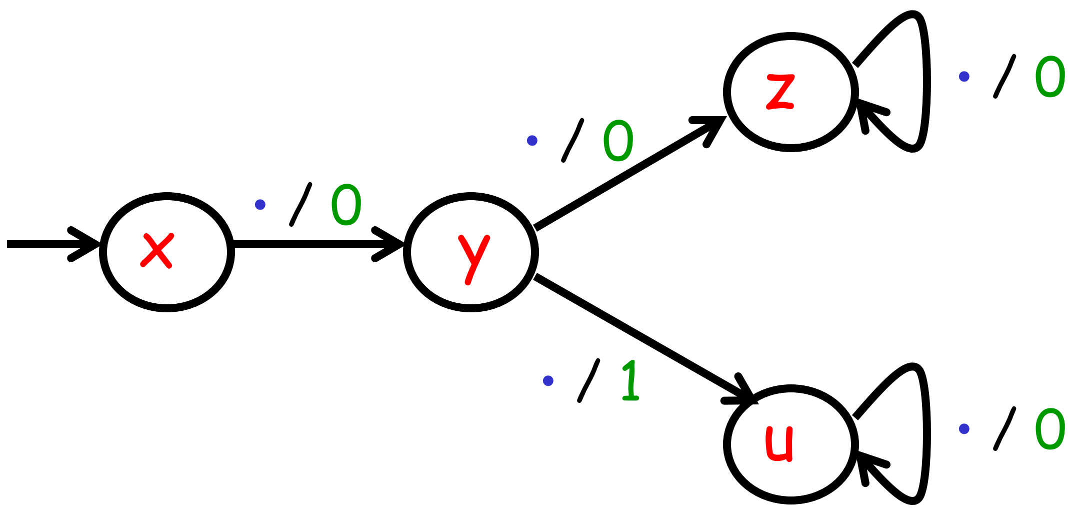


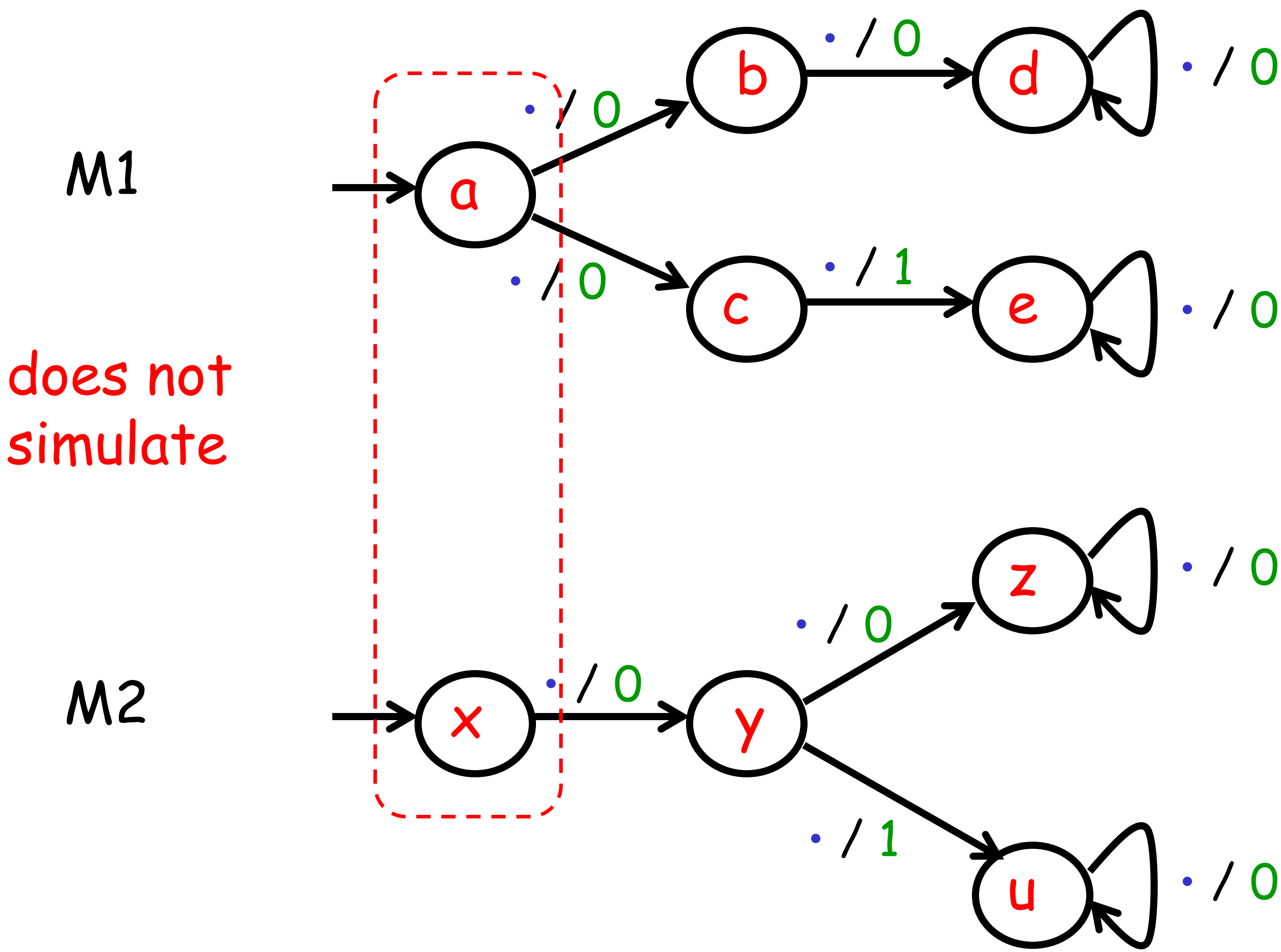
M1



does not simulate

M2




M1
M2
does not simulate
a
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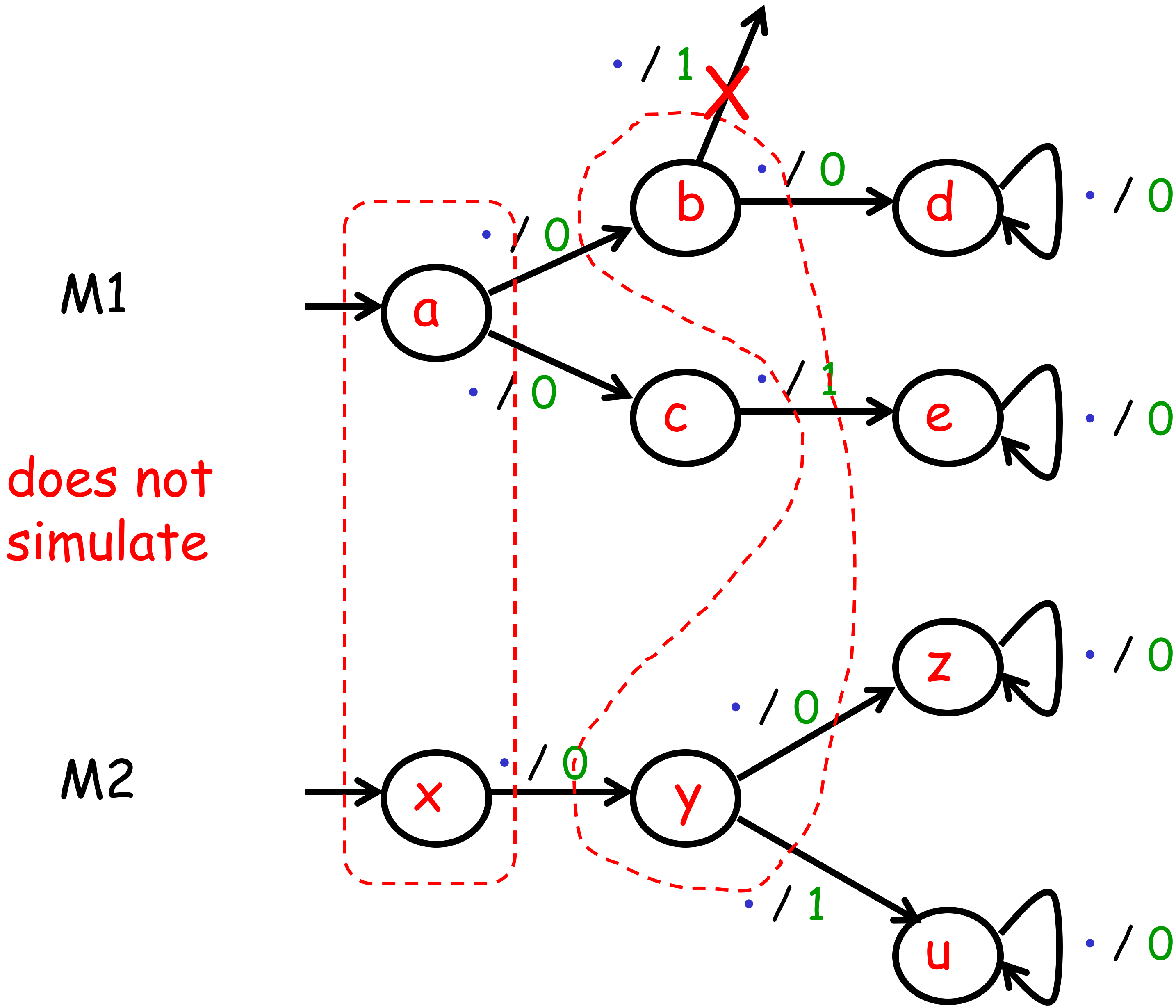

M1
M2
does not simulate
a
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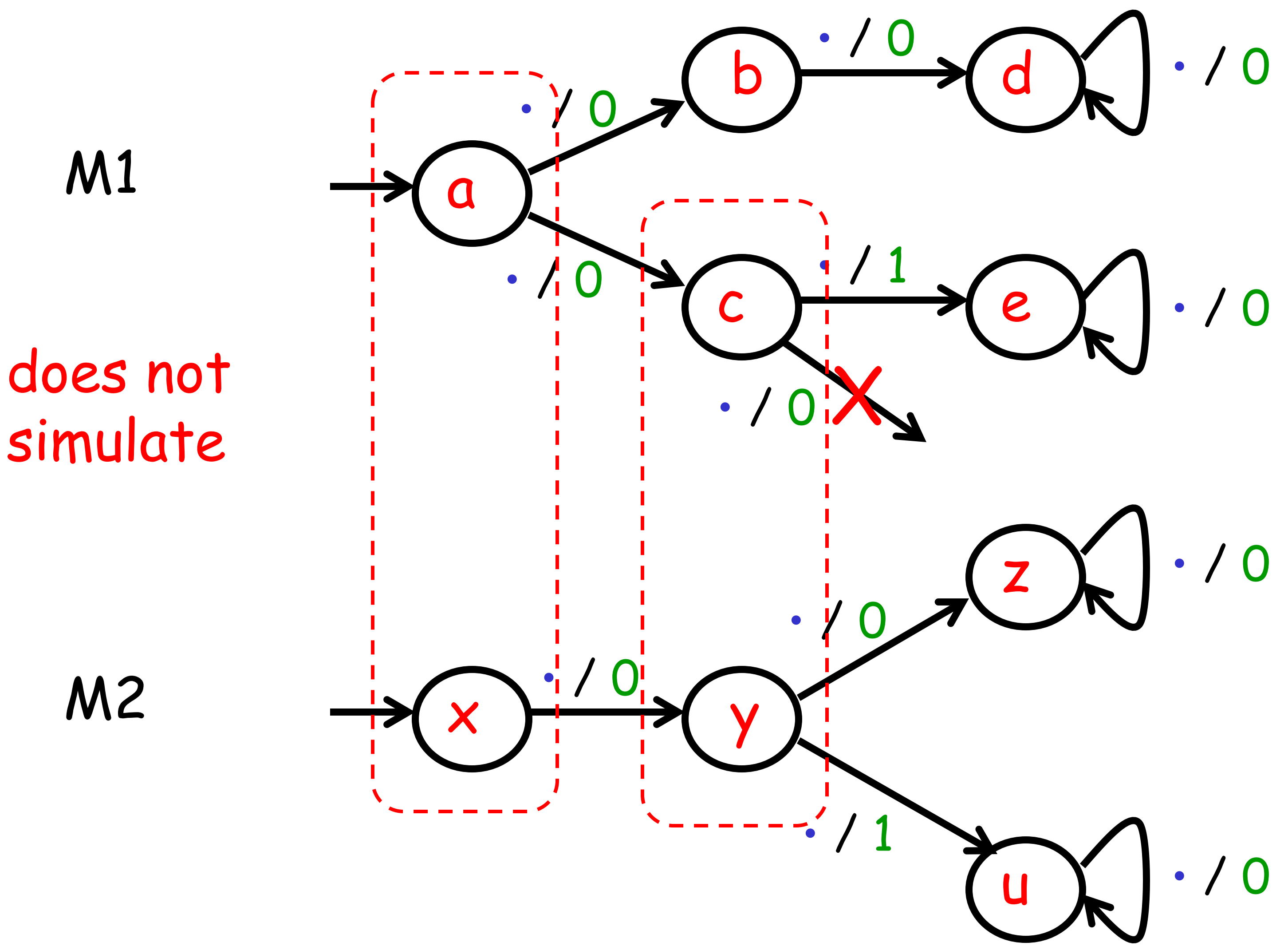
M1
M2
does not simulate
a
b
c
d
e
x
y
z
u
• / 0
• / 1