Transducive Systems

EECS 20

Lecture 6 (January 29, 2001)

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Continuous-Time Signals

sound
position

Discrete-Time Signals

sampledSound
movie

$$\text{ContinuousTime} = \{ x \in \text{Reals} \mid x \geq 0 \}$$

$$\text{DiscreteTime} = \{ 0, 1, 2, 3, 4, \ldots \}$$

$$\text{continuousTimeSignal} : \text{ContinuousTime} \rightarrow \text{Values}$$

$$\text{discreteTimeSignal} : \text{DiscreteTime} \rightarrow \text{Values}$$

continuousTimeSignal = evolution of values

discreteTimeSignal = stream of values

More Discrete-Time Signals

$\text{text} : \text{DiscreteTime} \to \text{Chars}$

$\text{file} : \text{DiscreteTime} \to \text{Bins}$

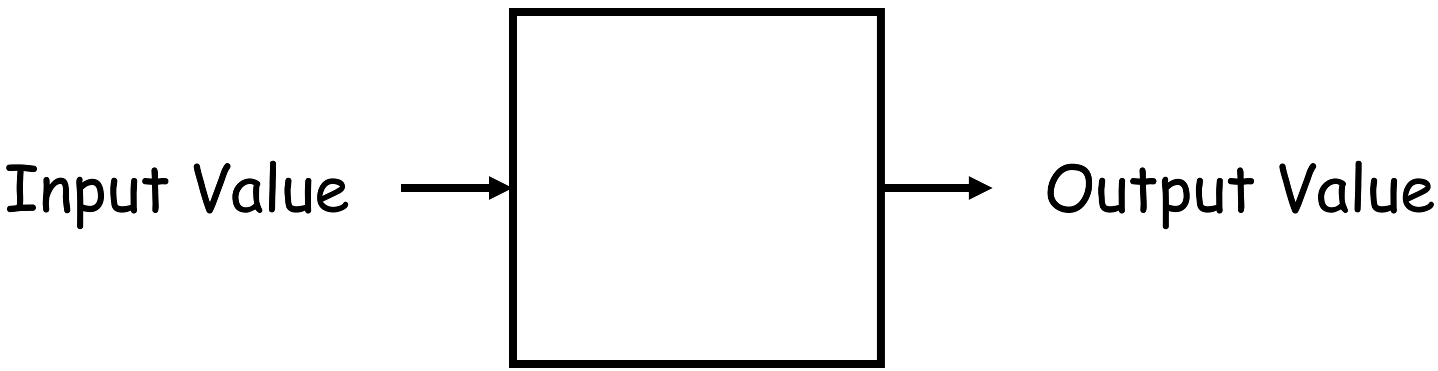
$\text{vendingMachine} : \text{DiscreteTime} \to \text{Events}$

$\text{Chars} = \{ a, b, c, \ldots \}$

$\text{Bins} = \{ 0, 1 \}$

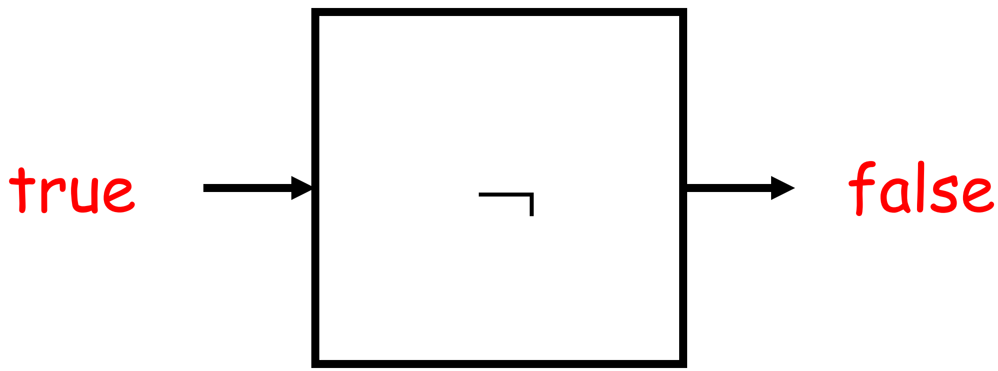
$\text{Events} = \{ \text{dropQuarter}, \text{requestCoke}, \text{deliverCoke}, \ldots \}$

Transducive or Combinational System



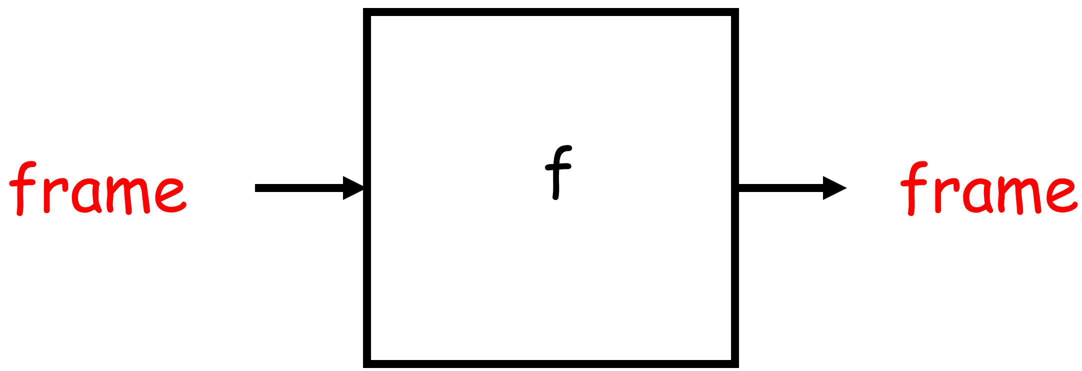
$$\text{transduciveSystem} : \text{Values} \rightarrow \text{Values}$$

Transducive or Combinational System



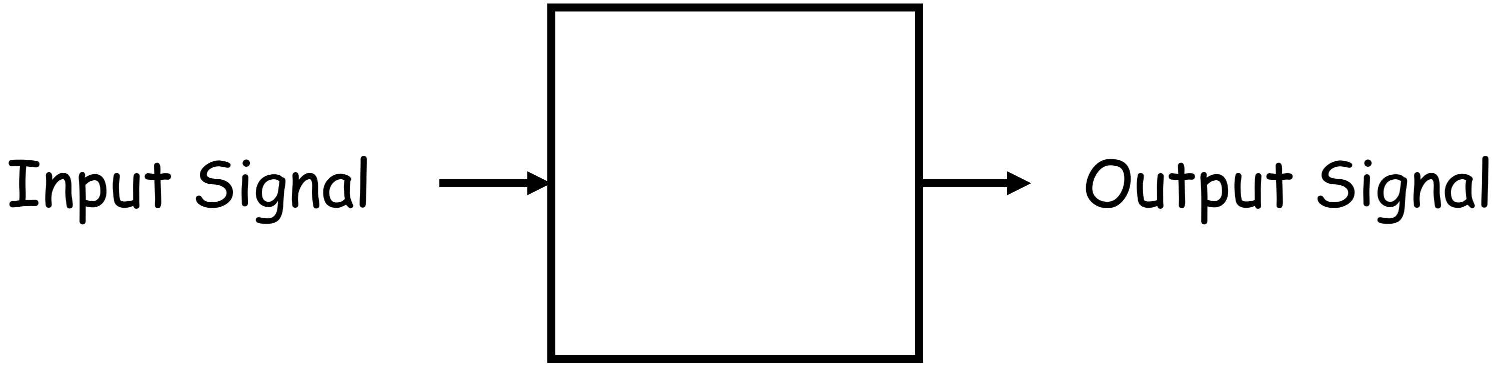
$$\neg : \text{Bools} \to \text{Bools}$$

Transducive or Combinational System



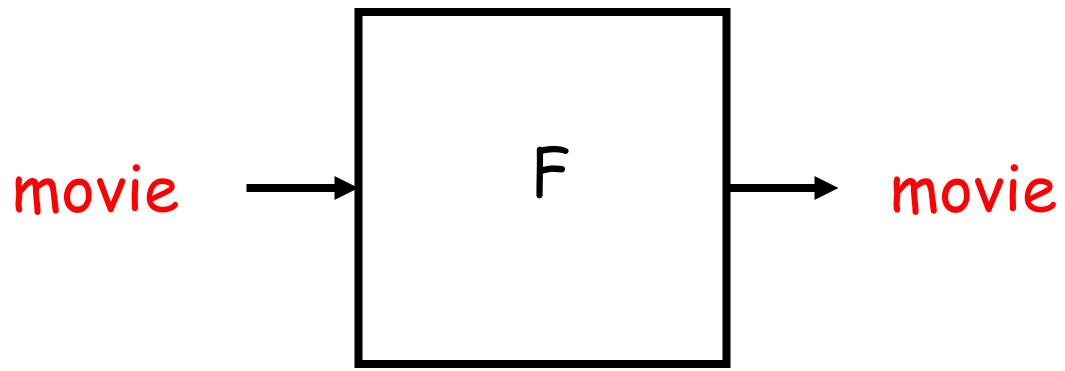
$$f : \text{Frames} \rightarrow \text{Frames}$$

Reactive or Sequential System



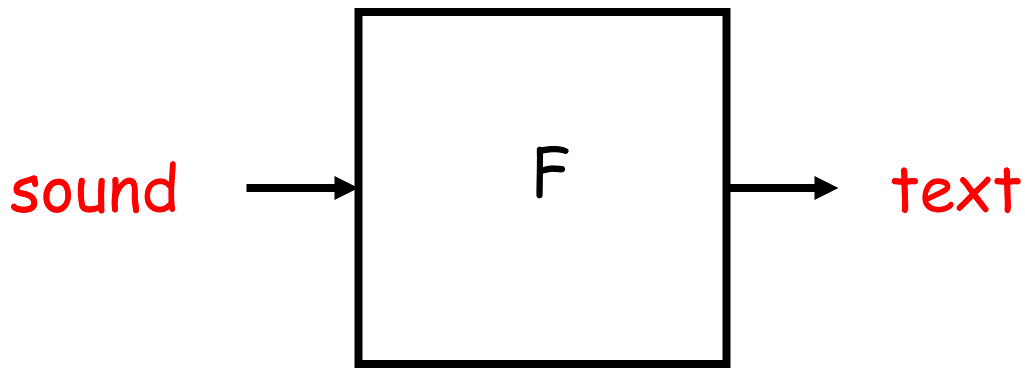
reactiveSystem : [Time → Values] → [Time → Values]

Reactive or Sequential System



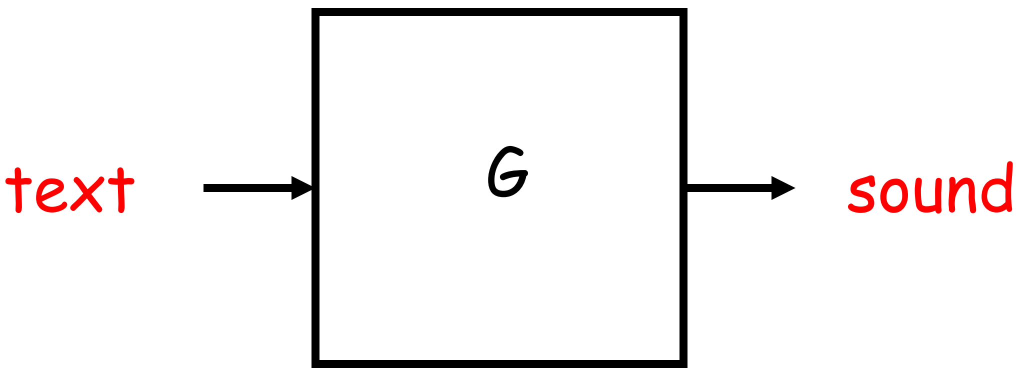
$$F : [\ \text{Time} \rightarrow \text{Frames}\ ] \rightarrow [\ \text{Time} \rightarrow \text{Frames}\ ]$$

Reactive or Sequential System



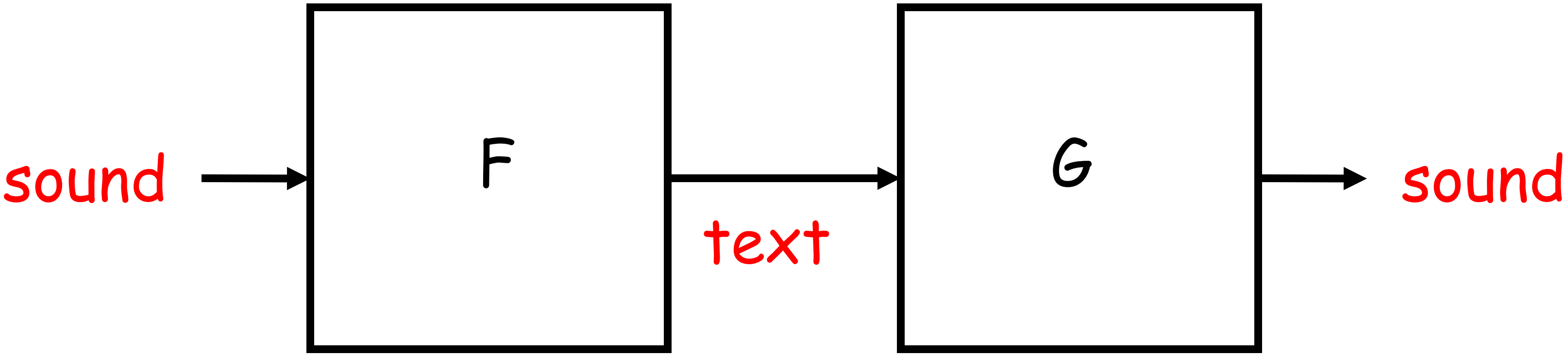
F: [ContinuousTime → Pressure] → [DiscreteTime → Chars]

Reactive or Sequential System

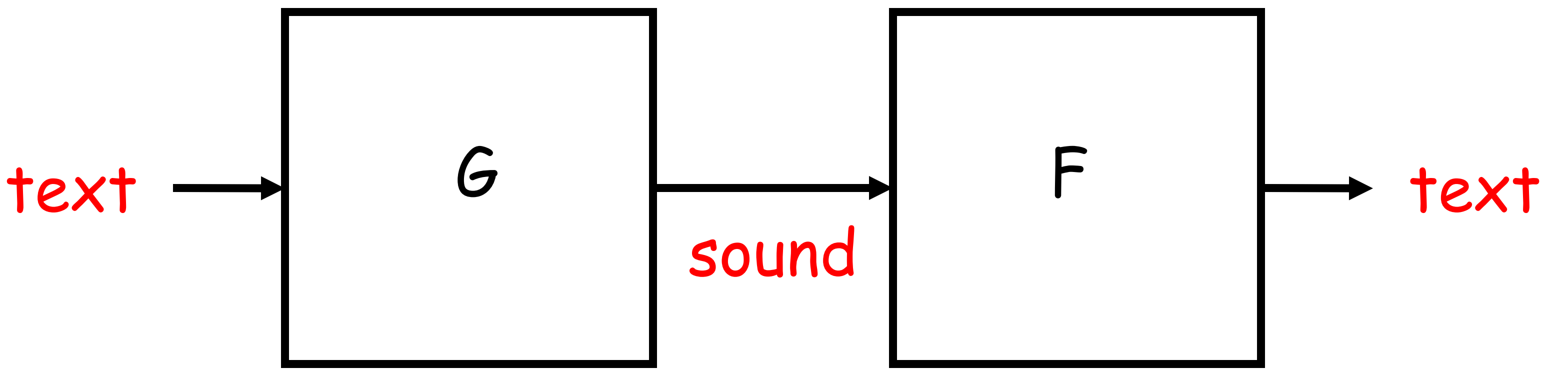


G: [DiscreteTime $\to$ Chars] $\to$ [ContinuousTime $\to$ Pressure]

Identity ?



Identity ?



Reactive Systems

1 Memory-free systems

2 Delays

3 Causality

4 Finite-memory systems

5 Infinite-memory systems

Every transducive system is a reactive system

$$f : \text{Values} \rightarrow \text{Values}$$

$$\downarrow$$

$$F : [\, \text{Time} \rightarrow \text{Values} \,] \rightarrow [\, \text{Time} \rightarrow \text{Values} \,]$$

such that $\forall\, x \in [\, \text{Time} \rightarrow \text{Values} \,],\ \forall\, y \in \text{Time},$

$$(\, F\,(x)\,)\,(y) = f\,(\, x\,(y)\,)$$

Every transducive system is a reactive system

$$f : \text{Values} \rightarrow \text{Values}$$

$$\downarrow$$

$$F : [\,\text{Time} \rightarrow \text{Values}\,] \times \text{Time} \rightarrow \text{Values}$$

such that $\forall\, x \in [\,\text{Time} \rightarrow \text{Values}\,],\ \forall\, y \in \text{Time},$

$$F(x,y) = f(x(y))$$

Every transducive system is a continuous-time reactive system

$$f : \text{Values} \rightarrow \text{Values}$$

$$\downarrow$$

$$F : [\,\text{ContTime} \rightarrow \text{Values}\,] \rightarrow [\,\text{ContTime} \rightarrow \text{Values}\,]$$

such that $\forall\, x \in [\,\text{ContTime} \rightarrow \text{Values}\,],\ \forall\, y \in \text{ContTime},$

$$(\,F\,(x)\,)\,(y) = f\,(\,x\,(y)\,)$$

$$\text{normalize} : \text{Reals} \to \text{Reals}$$

such that $\forall\, x \in \text{Reals}, \quad \text{normalize}(x) = x - 50$

$$\Big\downarrow$$

$$\text{Normalize} : [\,\text{Reals}_+ \to \text{Reals}\,] \to [\,\text{Reals}_+ \to \text{Reals}\,]$$

such that $\forall\, x \in [\,\text{Reals}_+ \to \text{Reals}\,],\ \forall\, y \in \text{Reals}_+,$

$$(\,\text{Normalize}(x)\,)(y) = \text{normalize}(\,x(y)\,)$$

$$= x(y) - 50$$

$\text{Normalize}(\sin) : \text{Reals}_+ \to \text{Reals}$

such that $\forall\, y \in \text{Reals}_+$, $\text{Normalize}(\sin)(y) = \sin(y) - 50$.

$\text{Normalize}(\text{id}) : \text{Reals}_+ \to \text{Reals}$

such that $\forall\, y \in \text{Reals}_+$, $\text{Normalize}(\text{id})(y) = y - 50$.

$\text{Normalize}(53) : \text{Reals}_+ \to \text{Reals}$

such that $\forall\, y \in \text{Reals}_+$, $\text{Normalize}(53)(y) = 3$.

$\text{Normalize}(\sin) : \text{Reals}_+ \to \text{Reals}$

such that $\forall\, y \in \text{Reals}_+ ,\quad \text{Normalize}(\sin)(y) = \sin(y) - 50 .$

$\text{Normalize}(\text{id}) : \text{Reals}_+ \to \text{Reals}$

such that $\forall\, y \in \text{Reals}_+ ,\quad \text{Normalize}(\text{id})(y) = y - 50 .$

$\text{Normalize}(53) : \text{Reals}_+ \to \text{Reals}$

such that $\text{Normalize}(53) = 3 .$

$$\mathrm{trunc} : \mathrm{Reals} \to \mathrm{Reals}$$

$$\text{such that } \forall x \in \mathrm{Reals}, \quad \mathrm{trunc}(x) = \begin{cases} 256 & \text{if } x > 256 \\ x & \text{if } -256 \leq x \leq 256 \\ -256 & \text{if } x < -256 \end{cases}$$

$$\downarrow$$

$$\mathrm{Trunc} : [\mathrm{Reals}_+ \to \mathrm{Reals}] \to [\mathrm{Reals}_+ \to \mathrm{Reals}]$$

$$\text{such that } \forall x \in [\mathrm{Reals}_+ \to \mathrm{Reals}], \ \forall y \in \mathrm{Reals}_+,$$

$$(\mathrm{Trunc}(x))(y) = \mathrm{trunc}(x(y))$$

$\text{Trunc}(\sin) : \text{Reals}_+ \to \text{Reals}$

such that $\forall\, y \in \text{Reals}_+ ,\quad \text{Trunc}(\sin)(y) = \sin(y)$.

$\text{Trunc}(\text{id}) : \text{Reals}_+ \to \text{Reals}$

such that $\forall\, y \in \text{Reals}_+ ,\quad \text{Trunc}(\text{id})(y) = \begin{cases} y & \text{if } y < 256 \\ 256 & \text{if } y \geq 256 \end{cases}$

$\text{Trunc}(500) : \text{Reals}_+ \to \text{Reals}$

such that $\text{Trunc}(500) = 256$.

Every transducive system is a discrete-time reactive system

$$f : \text{Values} \to \text{Values}$$

$$\Big\downarrow$$

$$F : [\,\text{DiscTime} \to \text{Values}\,] \to [\,\text{DiscTime} \to \text{Values}\,]$$

such that $\forall\, x \in [\,\text{DiscTime} \to \text{Values}\,],\ \forall\, y \in \text{DiscTime},$

$$(\,F\,(x)\,)\,(y) = f\,(\,x\,(y)\,)$$

$$\text{quantize} : \text{Reals} \rightarrow \text{ComputerInts}$$

$$\text{such that } \forall x \in \text{Reals}, \quad \text{quantize}(x) = \text{trunc}(\lfloor x \rfloor)$$

$$\downarrow$$

$$\text{Quantize} : [\,\text{Nats}_0 \rightarrow \text{Reals}\,] \rightarrow [\,\text{Nats}_0 \rightarrow \text{ComputerInts}\,]$$

$$\begin{aligned} \text{such that } & \forall x \in [\,\text{Nats}_0 \rightarrow \text{Reals}\,], \ \forall y \in \text{Nats}_0, \\ & (\text{Quantize}(x))(y) = \text{quantize}(x(y)) \\ & \qquad\qquad\qquad\;\; = \text{trunc}(\lfloor x(y) \rfloor) \end{aligned}$$

Quantize (squareroot) : $\text{Nats}_0 \rightarrow$ ComputerInts

such that Quantize (squareroot) =

{ (0,0), (1,1), (2,1), (3,1), (4,2), (5,2), ... , (1000000,256), ... } .

Quantize (id) : $\text{Nats}_0 \rightarrow$ ComputerInts

such that Quantize (id) =

{ (0,0), (1,1), (2,2), (3,3), ... , (256,256), (257,256), ... } .

Quantize (pi) : $\text{Nats}_0 \rightarrow$ ComputerInts

such that Quantize (pi) = 3 .

$$\text{negate} : \text{Bools} \to \text{Bools}$$

such that $\forall\, x \in \text{Bools},\ \ \text{negate}(x) = \neg x$

$$\downarrow$$

$$\text{Negate} : [\,\text{Nats}_0 \to \text{Bools}\,] \to [\,\text{Nats}_0 \to \text{Bools}\,]$$

such that $\forall\, x \in [\,\text{Nats}_0 \to \text{Bools}\,],\ \forall\, y \in \text{Nats}_0,$

$$\begin{aligned}(\,\text{Negate}(x)\,)(y) &= \text{negate}(\,x(y)\,)\\ &= \neg\, x(y)\end{aligned}$$

$$\mathrm{alt} : \mathrm{Nats}_0 \rightarrow \mathrm{Bools}$$

$$\text{such that } \forall y \in \mathrm{Nats}_0,\ \mathrm{alt}(y) = \begin{cases} \text{true} & \text{if } y \text{ odd} \\ \text{false} & \text{if } y \text{ even} \end{cases}$$

$$\mathrm{Negate}(\mathrm{alt}) : \mathrm{Nats}_0 \rightarrow \mathrm{Bools}$$

$$\text{such that } \mathrm{Negate}(\mathrm{alt}) =$$

$$\{ (0,\text{true}),\ (1, \text{false}), (2, \text{true}), (3, \text{false}), (4, \text{true}), \ldots \}.$$

$$\mathrm{Negate}(\text{true}) : \mathrm{Nats}_0 \rightarrow \mathrm{Bools}$$

$$\text{such that } \mathrm{Negate}(\text{true}) = \text{false}.$$

A reactive system

$F : [\ \mathrm{Time} \to \mathrm{Values}\ ] \to [\ \mathrm{Time} \to \mathrm{Values}\ ]$

is memory-free

iff

there exists a transducive system

$f : \mathrm{Values} \to \mathrm{Values}$

such that

$\forall\, x \in [\ \mathrm{Time} \to \mathrm{Values}\ ],\ \forall\, y \in \mathrm{Time},$

$(\, F\,(x)\,)\,(y) = f\,(\, x\,(y)\,)\,.$