FUNZIONI PARZIALI

A, B $\quad f : A \to B$ FUNZIONE $\underbrace{\text{TOTALE}}$

$f \subseteq A \times B$

$\forall a \in A \ \exists b \in B$
$b = f(a)$

$(a,b) \in f \ \& \ (a, \bar{b}) \in f$
$\Rightarrow$
$b = \bar{b}$

A, B

f è una funzione parziale da A a B

$$(f : A \rightharpoonup B)$$

SSE

$$f \subseteq A \times B \quad \& \quad \forall a, b, c \quad (a,b) \in f \ \& \ (a,c) \in f \Rightarrow b = c$$

$$\mathrm{dom}(f) = \{ a \mid a \in A \ \& \ \exists b \in B \ \underbrace{f(a) = b}_{(a,b) \in f} \}$$

$$f : A \rightharpoonup B \Rightarrow \tilde{f} : \mathrm{dom}(f) \to B$$

f una funzione di var. reale

$$f : \mathbb{R} \to \mathbb{R}$$

$$tg : \mathbb{R} \to \mathbb{R}$$

ma

$$tg : \cancel{\mathbb{R} \to \mathbb{R}}$$

$f : A \to B \qquad g : B \to C$

$g \circ f : A \to C$

$f : A \to B \quad \& \quad g : B \to C$

$g \circ f$?

da pensarci a casa.

$$f : \mathbb{R} \to \mathbb{R} \qquad x \mapsto \frac{1}{1+x^2}$$

$$g : \mathbb{R} \to \mathbb{R} \qquad x \mapsto 2^x$$

$$f \circ g = ? \quad (f \circ g)(z) = f(g(z)) =$$

$$= f(2^z) = \frac{1}{1+(2^z)^2}$$

$f \circ g$ t.c. $x \mapsto \dfrac{1}{1+(2^z)^2}$

$$A \xrightarrow{\varphi} B \qquad B \xrightarrow{\psi} C \qquad A \xrightarrow{\psi \circ \varphi} C$$

1) φ & ψ INIETTIVE

$\psi \circ \varphi$ È INIETTIVA? $\boxed{\text{SI}}$

$\forall a_1, a_2 \in A \quad (\psi \circ \varphi)(a_1) = (\psi \circ \varphi)(a_2) \Rightarrow a_1 = a_2$

$(\psi \circ \varphi)(a_1) = (\psi \circ \varphi)(a_2) \Rightarrow \psi(\varphi(a_1)) = \psi(\varphi(a_2))$

POICHÉ ψ È INIET. $\quad \varphi(a_1) = \varphi(a_2)$ POICHÉ φ È INI.

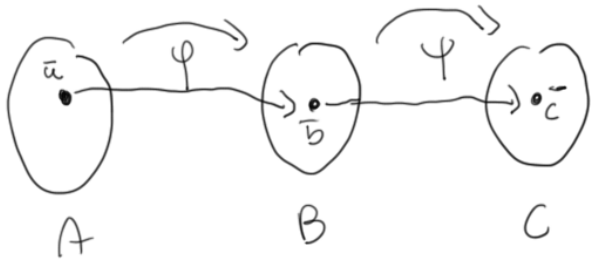
$a_1 = a_2$

$$A \xrightarrow{\varphi} B \qquad B \xrightarrow{\psi} C \qquad A \xrightarrow{\psi \circ \varphi} C$$

φ & ψ SONO SURIETT.

DIM. CHE $\psi \circ \varphi$ E' SUR.

$$\forall \bar{c} \in C \ \exists\, a \in A \ \ \psi(\varphi(a)) = c$$



$$\psi(\varphi(\bar{a})) =$$
$$\psi(\bar{b}) = \bar{c}$$

$$\overset{\varphi}{A \to B} \qquad \overset{\psi}{B \to C} \qquad \overset{\psi \circ \varphi}{A \to C}$$

φ & ψ SONO SURIETT.

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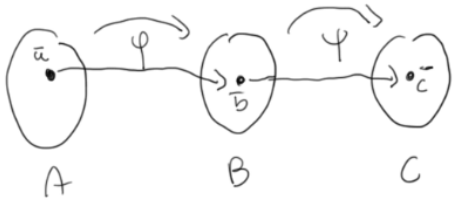


$\psi(\varphi(\bar{a})) =$

$\psi(\bar{b}) = \bar{c}$

$\bar{c} \in C$

$\exists\, \bar{b} \in B$

$\psi(\bar{b}) = \bar{c}$

$\exists\, \bar{a} \ \ \varphi(\bar{a}) = \bar{b}$

$\psi(\varphi(\bar{a})) = \bar{c}$

$$\begin{tikzcd} A \arrow[r, "\varphi"] \arrow[rr, bend right, "\psi \circ \varphi"'] & B \arrow[r, "\psi"] & C \end{tikzcd}$$

1) $\psi \circ \varphi$ INIETTIVA $\Rightarrow \varphi$ È INIETT.

$$A \Rightarrow B \qquad \sim B \Rightarrow \sim A$$

φ NON È INIETT $\Rightarrow \psi \circ \varphi$ NON È INIET.

$\exists\, a', a''$ $a' \neq a''$ & $\varphi(a') = \varphi(a'')$

$\Rightarrow$

$\exists\, a'$ & a'' $a' \neq a''$ & $\psi(\varphi(a')) = \psi(\varphi(a''))$

$$\begin{tikzcd} A \arrow[r, "\varphi"] \arrow[rr, bend right, "\psi \circ \varphi"'] & B \arrow[r, "\psi"] & C \end{tikzcd}$$

1) $\psi \circ \varphi$ INIETTIVA $\Rightarrow$ φ È INIETT.

$$\boxed{\forall\, a', a'' \quad \varphi(a') = \varphi(a'') \Rightarrow a' = a''}$$

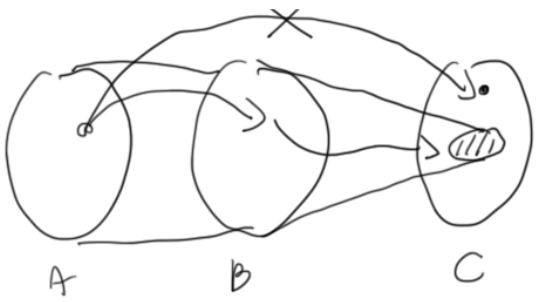
$$\underline{\varphi(a') = \varphi(a'')} \Rightarrow \psi(\varphi(a')) = \psi(\varphi(a''))$$

$$\Rightarrow (\psi \circ \varphi)(a') = (\psi \circ \varphi)(a'') \Rightarrow \underline{a' = a''}$$

$$\begin{tikzcd} A \arrow[r, "\varphi"] \arrow[rr, bend right, "\psi \circ \varphi"'] & B \arrow[r, "\psi"] & C \end{tikzcd}$$

1) $\psi \circ \varphi$ INIETTIVA $\Rightarrow \varphi$ È INIETT.

2) $\psi \circ \varphi$ SURIETT $\Rightarrow \psi$ È SURIET.



$\forall c \in C \ \exists b \in B$

$\psi(b) = c$

$\bar{c} \in C \quad \exists \bar{a} \in A$

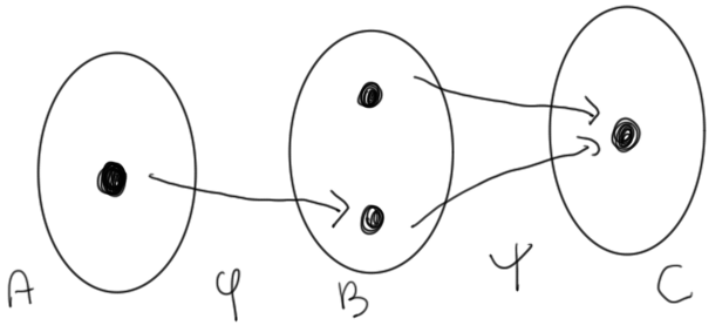
$\psi(\varphi(\bar{a})) = \bar{c}$

$\bar{b} = \varphi(\bar{a}) \quad \psi(\bar{b}) = \bar{c}$

$$\begin{tikzcd} A \arrow[r, "\varphi"] \arrow[rr, bend right, "\psi \circ \varphi"'] & B \arrow[r, "\psi"] & C \end{tikzcd}$$

1) $\psi \circ \varphi$ INIETTIVA $\Rightarrow$ φ, È INIETT.

2) $\psi \circ \varphi$ SURIETT $\Rightarrow$ ψ È SURIET.



$\psi \circ \varphi$ È BIETTIVA

$$A \xrightarrow[\varphi]{} B \xrightarrow[\psi]{} C \xrightarrow[\omega]{} D$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$A \xrightarrow[\varphi]{} B \xrightarrow[\omega\circ\psi]{} D \qquad\qquad A \xrightarrow[\psi\circ\varphi]{} C \xrightarrow[\omega]{} D$$

$$(\omega\circ\psi)\circ\varphi \quad = \quad \omega\circ(\psi\circ\varphi)$$

$$\forall a\in A \quad [(\omega\circ\psi)\circ\varphi](a) = [\omega\circ(\psi\circ\varphi)](a)$$

$$A \xrightarrow[\varphi]{} B \xrightarrow[\psi]{} C \xrightarrow[\omega]{} D$$

$$\boxed{[\omega \circ (\psi \circ \varphi)](a)} \qquad [(\omega \circ \psi) \circ \varphi](a)$$

$$=$$

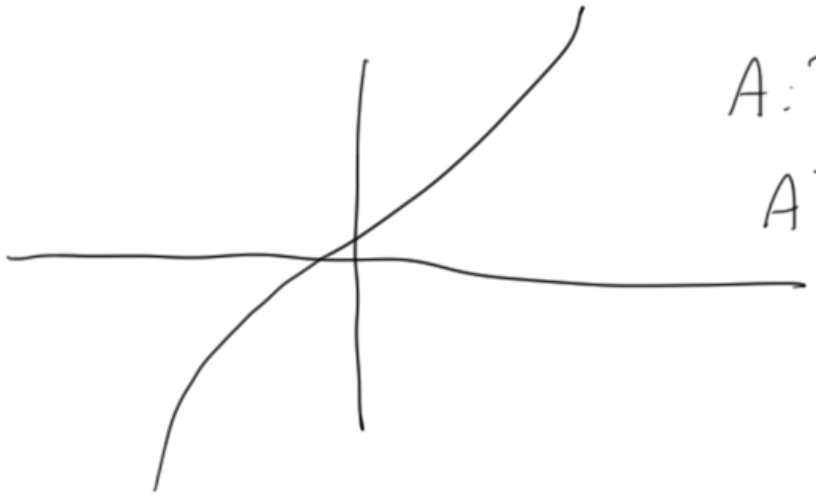
$$\omega((\psi \circ \varphi)(a))$$

$$=$$

$$\omega(\psi(\varphi(a))) = (\omega \circ \psi)(\varphi(a)) =$$

$$= \boxed{((\omega \circ \psi) \circ \varphi)(a)}$$

BIIETTIVE BIGETTIVE INVERTIBILI



$A : V \to V$

A^{-1} $\quad A \circ A^{-1} = A^{-1} \circ A$

$f : A \to B$ è BIIETT. $\iff$

$\exists\, g : B \to A$ $\quad g \circ f : A \to A$ t.c. $g \circ f = 1_A$

$g = f^{-1}$ $\quad f \circ g : B \to B$ t.c. $f \circ g = 1_B$

PROP

$\varphi : A \to B \qquad \psi_1, \psi_2 : B \to A$

$\psi_1 \circ \varphi : A \to A$

$$\begin{tikzcd} A \arrow[loop below, "\iota_A"] \arrow[r, "\varphi"'] & B \arrow[l, bend right, "\psi_1"'] \arrow[l, bend left, "\psi_2"] \arrow[loop below, "\iota_B"] \end{tikzcd}$$

$\varphi \circ \psi_2 : B \to B$

$\psi_1 \circ \varphi = \iota_A \qquad ? \qquad \varphi \circ \psi_2 = \iota_B$

$\Downarrow$

$\psi_1 = \psi_2$

$$\psi_1 = \psi_1 \circ \iota_B = \psi_1 \circ (\varphi \circ \psi_2) = (\psi_1 \circ \varphi) \circ \psi_2 = \iota_A \circ \psi_2 = \psi_2$$

$$\begin{tikzcd} A \arrow[r, bend left, "\varphi"] & B \arrow[l, bend left, "\varphi^{-1}"] \end{tikzcd}$$

se $\exists\, \psi_1, \psi_2$

$$\psi_1 \circ \varphi = 1_A$$

$$\varphi \circ \psi_2 = 1_B$$

$$\underbrace{\psi_1 = \psi_2}_{\varphi^{-1}}$$

φ^{-1} è detta inversa di φ

TEOREMA

$\varphi : A \to B \qquad \exists \psi$ t.c. $\psi : B \to A$

$$\varphi \circ \psi = \iota_B$$

$$\boxed{\psi \circ \varphi = \iota_A}$$

$f \circ g$ è iniett

g è iniett.

$f \circ g$ è sur.

f è sur.

$\Rightarrow$

φ è BIIETTIVA

$\psi \circ \varphi$ è iniett $\Rightarrow$ $\boxed{\varphi \text{ è iniett.}}$

$\varphi \circ \psi$ è SUR. $\Rightarrow$ φ è SUR.