TOPOLOGIA E GEOMETRIA DIFFERENZIALE
Prof. M. Spera a.a. 2009/10

Lezione **XXV**

in $\mathbb{R}^3$ | Sia

$$\omega = dx \wedge dy \wedge dz$$

forma di volume

$$X = \underbrace{\alpha}_{X_1}\frac{\partial}{\partial x} + \underbrace{\beta}_{X_2}\frac{\partial}{\partial y} + \underbrace{\gamma}_{X_3}\frac{\partial}{\partial z}$$

$$\mathcal{L}_X \omega = d\, i_X \omega + \underbrace{i_X \overset{0}{\overset{\|}{d\omega}}}_{\overset{\|}{0}}$$

$$i_{X_1}\omega = \alpha\, dy \wedge dz$$

$$i_{X_2}\omega = -\beta\, dx \wedge dz$$

$$i_{X_3}\omega = \gamma\, dx \wedge dy$$

$$= d\left[\alpha\, dy \wedge dz - \beta\, dx \wedge dz + \gamma\, dx \wedge dy\right]$$

$$= \frac{\partial \alpha}{\partial x} dx \wedge dy \wedge dz - \frac{\partial \beta}{\partial y} dy \wedge dx \wedge dz + \frac{\partial \gamma}{\partial z} dz \wedge dx \wedge dy$$

$$= \left(\frac{\partial \alpha}{\partial x} + \frac{\partial \beta}{\partial y} + \frac{\partial \gamma}{\partial z}\right) dx \wedge dy \wedge dz$$

div X

⚠ attenzione non è vero che in gen.. $\mathcal{L}_{\alpha X} = \alpha \mathcal{L}_X$

$$(\mathcal{L}_{\alpha X} Y)(f) = [\alpha X, Y](f)$$
$$= \alpha X Y(f) - Y(\alpha X(f))$$
$$= \alpha (XY - YX)(f) - Y(\alpha) X(f)$$
$$= \alpha [X, Y](f) - Y(\alpha) X(f)$$

[⚠ è sottintesa la metrica euclidea]

tuttavia $\mathcal{L}_{\alpha X} f = \alpha X(f) = \alpha \mathcal{L}_X f$

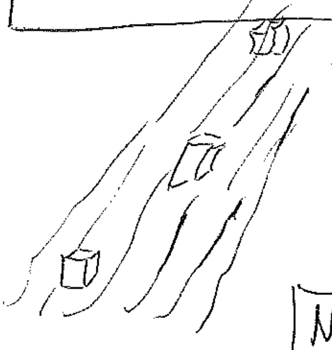
Per una varietà riemanniana

$$\mathcal{L}_X \mathrm{vol}_g = \mathrm{div}\, X\, \mathrm{vol}_g$$

X conserva $\mathrm{vol}_g \iff \mathrm{div}\, X = 0$

cruciale in fluidodinamica

div = 0 : incomprimibilità del fluido



X = velocità

XXV - 1

NUOVA