

EX 2 folia B Analisi Matematica 2

Studiare la funzione $f(x) = \frac{1-x^3}{x^2}$

Ris

$$A = (-\infty, 0) \cup (0, +\infty)$$

segno: $f(x) > 0$ se $1-x^3 > 0$ in A se $x^3 < 1$ se $x < 1$ in A .

$$f(x) = 0 \text{ se } x = 1$$

(1,0) intersezione con

$$f(x) < 0 \text{ se } x > 1$$

$$\lim_{x \rightarrow \pm\infty} \frac{1-x^3}{x^2} = \lim_{x \rightarrow \pm\infty} \frac{x^3(\frac{1}{x^3}-1)}{x^2} = \mp\infty \Rightarrow \text{non ci sono asintoti orizzontali}$$

$$\lim_{x \rightarrow 0^\pm} \frac{1-x^3}{x^2} = +\infty \Rightarrow x=0 \text{ asintoto verticale}$$

$$\lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} = \lim_{x \rightarrow \pm\infty} \frac{1-x^3}{x^3} = -1 = m$$

$$\lim_{x \rightarrow \pm\infty} (f(x) - mx) = \lim_{x \rightarrow \pm\infty} \frac{1-x^3}{x^2} + x = \lim_{x \rightarrow \pm\infty} \frac{1-x^3+x^3}{x^2} = \lim_{x \rightarrow \pm\infty} \frac{1}{x^2} = 0 = q$$

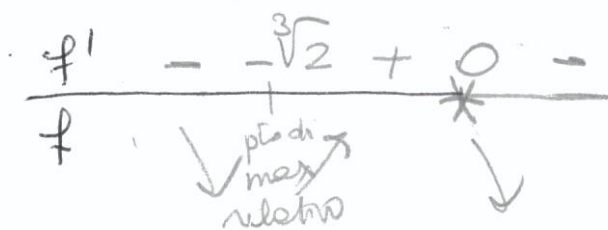
$\Rightarrow y=x$ asintoto obliquo sx e dx

$$f'(x) = \frac{-3x^2 - 2x(1-x^3)}{x^4} = \frac{-3x^4 - 2x + 2x^4}{x^4} = \frac{-x^4 - 2x}{x^4} = -\frac{x^3+2}{x^3}$$

$$f'(x) > 0?$$

$$-(x^3+2) > 0 \quad x^3+2 < 0 \quad x^3 < -2 \quad x < \sqrt[3]{-2}$$

$-(x^3+2)$	+	0	-	*	-
x^3	-	-	*	+	
$f(x)$	-	+	*	-	



$$f(\sqrt[3]{-2}) = \frac{-1+2}{\sqrt[3]{-2}^2} = +\frac{1}{\sqrt[3]{2}^2} = \frac{\sqrt[3]{2}}{2}$$

$$f'(x) = -\frac{x^3+2}{x^3} = -1 - \frac{2}{x^3} = -1 - 2x^{-3}$$

$$f''(x) = +6x^{-4} > 0 \quad \forall x \in A$$

$\Rightarrow f$ è concava in $(-\infty, 0) \cup (0, +\infty)$

