EX 2 fila B   Analisi Matematica 1

Studiare la funzione $f(x)=\dfrac{1-x^3}{x^2}$

<u>Ris</u>

$A=(-\infty,0)\cup(0,+\infty)$

segno: $f(x)\geq 0$ sse $1-x^3\geq 0$ in A sse $x^3<1$ sse $x<1$ in A.

$f(x)=0$ sse $x=1$ $(1,0)$ intersezione assi

$f(x)<0$ sse $x<1$

$$\lim_{x\to\pm\infty}\frac{1-x^3}{x^2}=\lim_{x\to\pm\infty}\frac{x^{\cancel{3}}\left(\frac{1}{x^3}-1\right)}{\cancel{x^2}}=\mp\infty$$ $\Rightarrow$ non ci sono asintoti orizzontali

$$\lim_{x\to 0^\pm}\frac{1-x^3}{x^2}=+\infty$$ $\Rightarrow$ $x=0$ asintoto verticale

$$\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{1-x^3}{x^3}=-1=m$$

$$\lim_{x\to\pm\infty}(f(x)-mx)=\lim_{x\to\pm\infty}\frac{1-x^3}{x^2}+x=\lim_{x\to\pm\infty}\frac{1-x^3+x^3}{x^2}=\lim_{x\to\pm\infty}\frac{1}{x^2}=0=q$$

$\Rightarrow$ $y=x$ asintoto obliquo sx e dx

$$f'(x)=\frac{-3x^2\cancel{x^2}-2x(1-x^3)}{x^4}=\frac{-3x^4-2x+2x^4}{x^4}=\frac{-x^4-2x}{x^4}=-\frac{x^3+2}{x^3}$$

$f'(x)>0$?

$-(x^3+2)>0 \quad x^3+2<0 \quad x^3<-2 \quad x<-\sqrt[3]{2}$

		$-\sqrt[3]{2}$		0	
$-(x^3+2)$	$+$	0	$-$	$*$	$-$
x^3	$-$		$-$	$*$	$+$
$f(x)$	$-$		$+$	$*$	$-$

		$-\sqrt[3]{2}$		0	
f'	$-$		$+$	$*$	$-$
f	$\searrow$	pto di max relativo	$\nearrow$	$*$	$\searrow$

$$f(-\sqrt[3]{2})=\frac{1+2}{\sqrt[3]{2^2}}=+\frac{3}{\sqrt[3]{2^2}}=\frac{3\sqrt[3]{2}}{2}$$

$$f'(x)=-\frac{x^3+2}{x^3}=-1-\frac{2}{x^3}=-1-2x^{-3}$$

$f''(x)=+6x^{-4}>0 \quad \forall x\in A$

$\Rightarrow$ f è concava in $(-\infty,0)\cup(0,+\infty)$