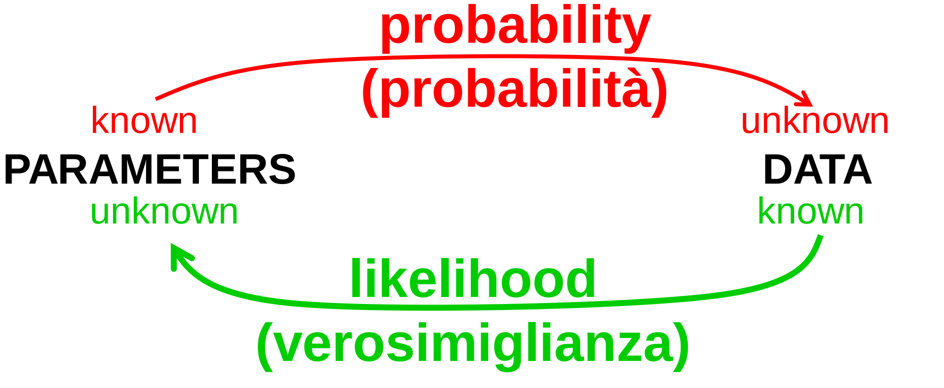
Exercise on the concept of Likelihood

"There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy."
- William Shakespeare, Hamlet (1.5.167-8)



A model is never true, a model can be useful.

A model is a mathematical interpretation of observed phenomena.

BINOMIAL DISTRIBUTION

$$p(x) = \binom{n}{x} \pi^x (1-\pi)^{n-x}$$

known π,
x probability can be computed

known x,
π likelihood can be computed

Number of SONS on 4 deliveries

$\pi = 0.514$

$p(0) = \binom{4}{0} 0.514^0 (1-0.514)^{4-0} = 0.0558$

$p(1) = \binom{4}{1} 0.514^1 (1-0.514)^{4-1} =$

$p(2) = \binom{4}{2} 0.514^2 (1-0.514)^{4-2} =$

$p(3) = \binom{4}{3} 0.514^3 (1-0.514)^{4-3} =$

$p(4) = \binom{4}{4} 0.514^0 (1-0.514)^{4-0} =$

$x = 3$

$p(3) = \binom{4}{3} 0.5^3 (1-0.5)^{4-3} = 0.25$

$p(3) = \binom{4}{3} 0.75^3 (1-0.75)^{4-3} = 0.422$

$p(3) = \binom{4}{3} 0.9^3 (1-0.9)^{4-3} = 0.292$

BINOMIAL DISTRIBUTION

$$p(x) = \left(\frac{n!}{x!(n-x)!}\right) \pi^x (1-\pi)^{n-x}$$

Probability to have a male = 0.514 according to the WHO

$\pi = 0.514$

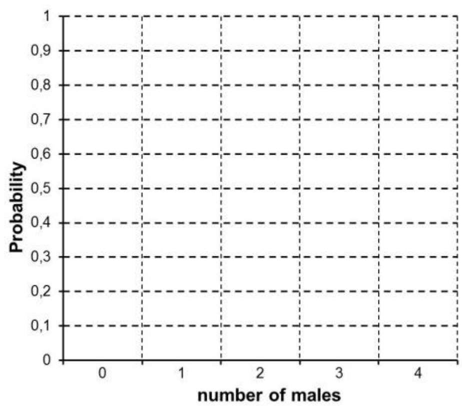
$p(0) = \binom{4}{0} 0.514^0 (1-0.514)^{4-0} = 0.0558$

$p(1) = \binom{4}{1} 0.514^1 (1-0.514)^{4-1} =$

$p(2) = \binom{4}{2} 0.514^2 (1-0.514)^{4-2} =$

$p(3) = \binom{4}{3} 0.514^3 (1-0.514)^{4-3} =$

$p(4) = \binom{4}{4} 0.514^0 (1-0.514)^{4-0} =$



BINOMIAL DISTRIBUTION

$$p(x) = \left(\frac{n!}{x!(n-x)!}\right) \pi^x (1-\pi)^{n-x}$$

NUMBER OF MALES ON 4 DELIVERIES = 3

$\pi=0.5$ $p(3) = \binom{4}{3} 0.5^3 (1-0.5)^{4-3} = 0.25$

$\pi=0.55$ $p(3) = \binom{4}{3} 0.55^3 (1-0.55)^{4-3} =$

$\pi=0.6$ $p(3) = \binom{4}{3} 0.6^3 (1-0.6)^{4-3} =$

$\pi=0.65$ $p(3) = \binom{4}{3} 0.65^3 (1-0.65)^{4-3} =$

$\pi=0.7$ $p(3) = \binom{4}{3} 0.7^3 (1-0.7)^{4-3} =$

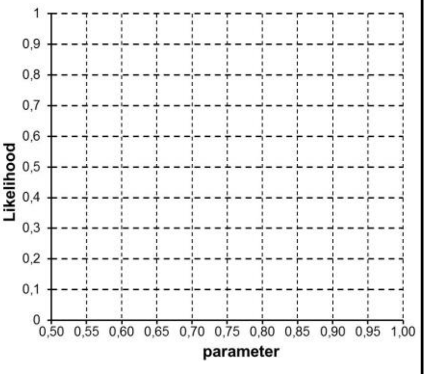
$\pi=0.75$ $p(3) = \binom{4}{3} 0.75^3 (1-0.75)^{4-3} = 0.422$

$\pi=0.8$ $p(3) = \binom{4}{3} 0.8^3 (1-0.8)^{4-3} =$

$\pi=0.85$ $p(3) = \binom{4}{3} 0.85^3 (1-0.85)^{4-3} =$

$\pi=0.9$ $p(3) = \binom{4}{3} 0.9^3 (1-0.9)^{4-3} = 0.292$

$\pi=0.95$ $p(3) = \binom{4}{3} 0.95^3 (1-0.95)^{4-3} =$



BINOMIAL DISTRIBUTION

$$p(x) = \left(\frac{n!}{x!(n-x)!}\right) \pi^x (1-\pi)^{n-x}$$

NUMBER OF MALES ON 4 DELIVERIES = 3

$\pi = 0.514$

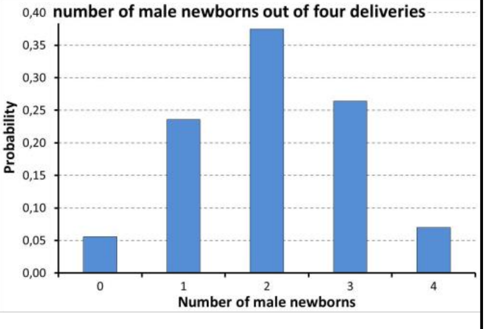
$p(0) = \binom{4}{0} 0.514^0 (1-0.514)^{4-0} = 0.0558$

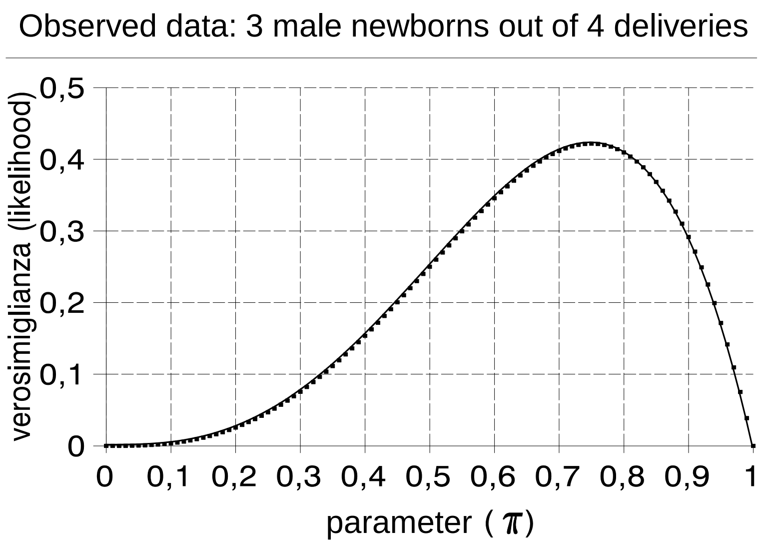
$p(1) = \binom{4}{1} 0.514^1 (1-0.514)^{4-1} = 0.2360$

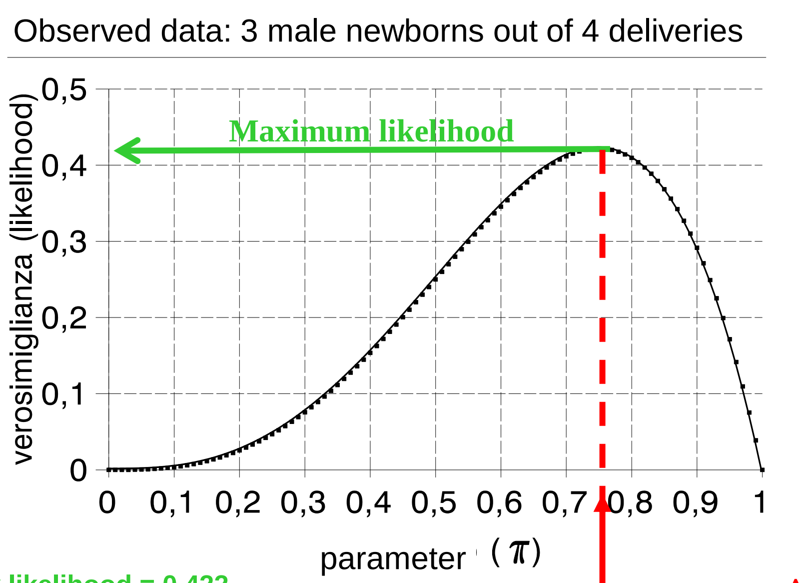
$p(2) = \binom{4}{2} 0.514^2 (1-0.514)^{4-2} = 0.3744$

$p(3) = \binom{4}{3} 0.514^3 (1-0.514)^{4-3} = 0.2640$

$p(4) = \binom{4}{4} 0.514^0 (1-0.514)^{4-0} = 0.0698$





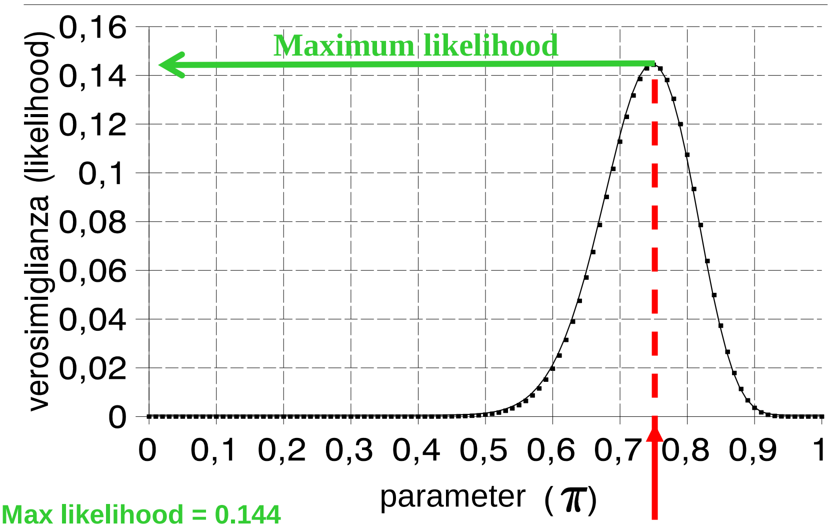


Max likelihood = 0.422

$\hat{\pi}$ = 0.75

Maximum Likelihood Estimate (MLE), $\hat{\pi}$
(Stima di massima verosimiglianza)

Observed data: 30 male newborns out of 40 deliveries



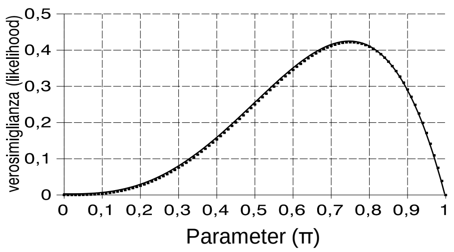
Max likelihood = 0.144

$\hat{\pi}$ = 0.75

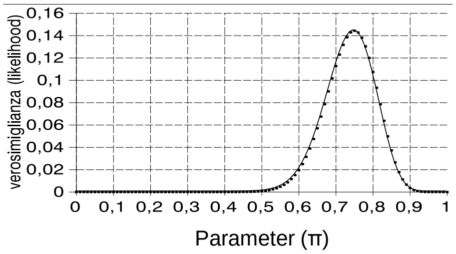
Maximum Likelihood Estimate (MLE), $\hat{\pi}$
(Stima di massima verosimiglianza)

Likelihood profile
(Profilo della verosimiglianza)

Observed data: 3 male newborns out of 4 deliveries



Observed data: 30 male newborns out of 40 deliveries



PROBABILITY	LIKELIHOOD
p(x/π)	l(π/x)
easy to understand	difficult to understand
less useful in Medical Statistics	more useful in Medical Statistics
Σ probabilities = 1	Σ likelihood > 1
	(indeed every likelihood is computed using a different probability distribution)

DEVIANCE